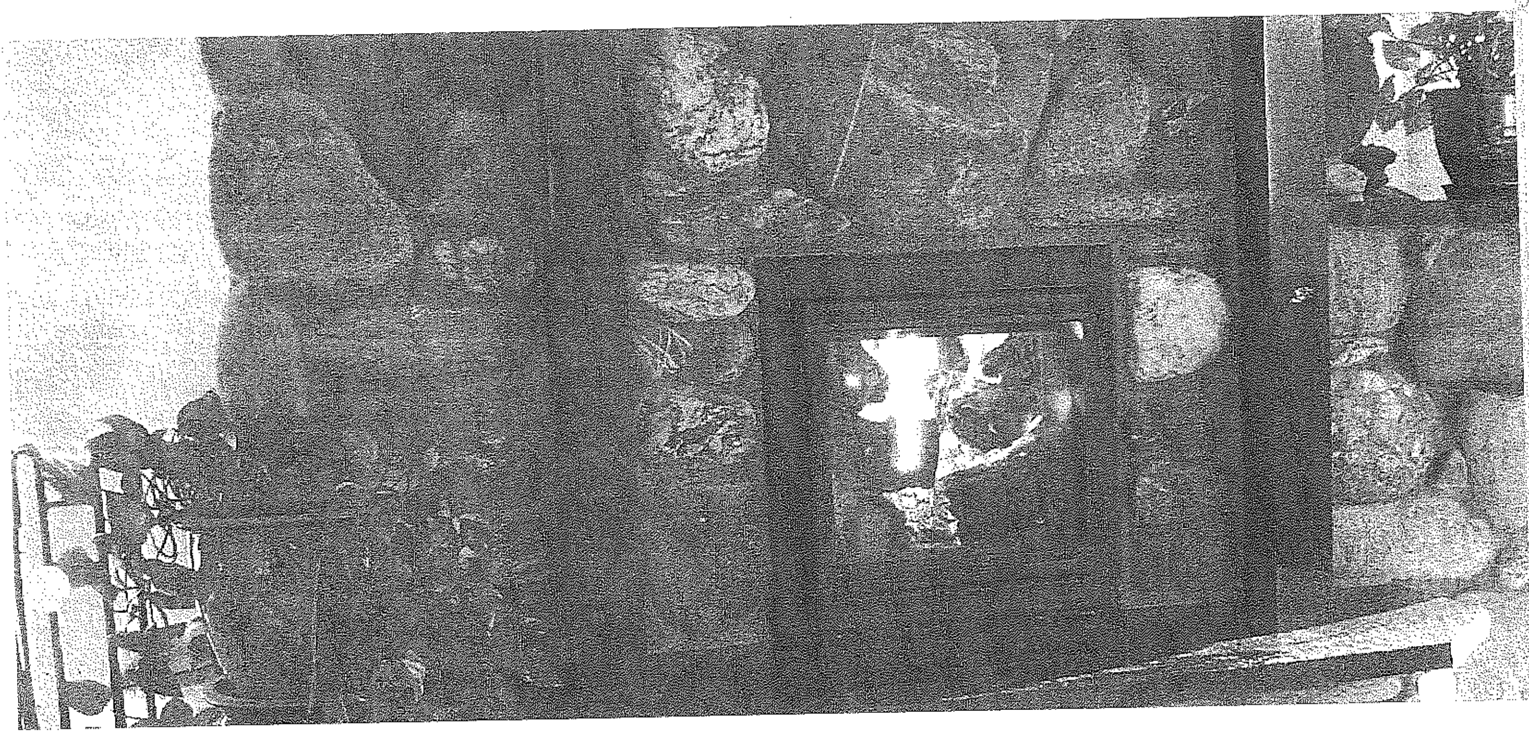
# Surface Area

# 3



A masonry stove stores heat in its stones or bricks. The surface area of the stove determines how much space it can heat.

**A.** Carys plans to finish her basement. She needs a stove with a surface area that is at least 80 sq ft to heat the basement. How can she determine whether a stove is large enough?

e.g., Carys can measure each dimension of the stove and calculate the surface area. If the surface area is equal to or greater than 80 sq ft, the stove is large enough.

**B.** Describe another job in finishing the basement for which Carys might need to determine a surface area.

e.g., Carys might need to determine an area to be drywalled or painted or how much wood is needed to make cabinets.

# 3 Getting Started

1. Match each area to an object it could describe.

| | |
|---|---|
| 4 cm$^2$ | a cellphone |
| 1 sq ft | a bed |
| 2 m$^2$ | a postage stamp |
| 8 sq in. | a window |
| 1 sq yd | a binder |

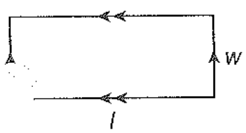
2. Evaluate.

a) $(23.5\text{ m})^2 =$ 552.25 m$^2$

b) $\sqrt{25\text{ cm}^2 + 144\text{ cm}^2} =$ 13 cm

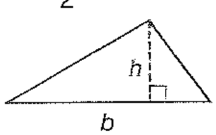
**areas of polygons**
**rectangle**

$A = \text{(length)(width)}$
$= lw$



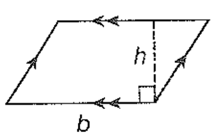

**triangle**

$A = \frac{1}{2}\text{(base)(height)}$
$= \frac{1}{2}bh$



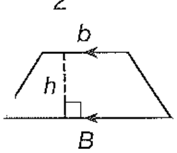

**parallelogram**

$A = bh$

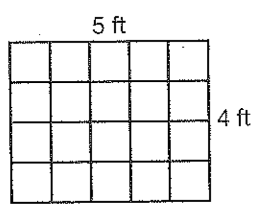



**trapezoid**

$A = \frac{1}{2}(b + B)h$

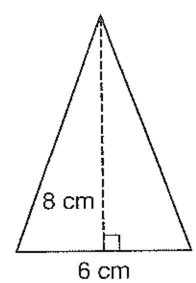



3. The area of a polygon is the number of square units of space that it covers. Name each polygon and determine its area.

a) 

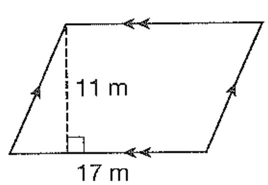

Name of polygon: rectangle

Area = 4 ft × 5 ft
= 20 sq ft

c) 

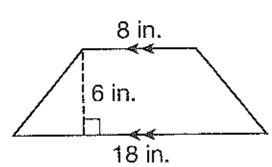

Name of polygon: triangle

Area = $\frac{1}{2}$(6 cm × 8 cm)
= 24 cm$^2$

b) 


Name of polygon: parallelogram

Area = 17 m × 11 m
= 187 m$^2$

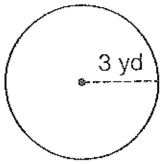
d) 


Name of polygon: trapezoid

Area = $\frac{1}{2}$(18 in. + 8 in.) × 6 in.
= 78 sq in.

4. Determine the circumference and area of each circle, to the nearest unit.

a)




$C = 2\pi(\underline{\ 3\ }\text{ yd})$

$= \underline{\ 18.849...\ }$ yd,

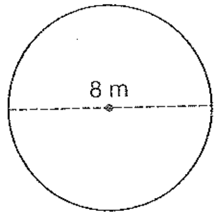
or about $\underline{\ 19\ }$ yd

$A = \pi(\underline{\ 3\ }\text{ yd})^2$

$= \underline{\ 28.274...\ }$ sq yd,

or about $\underline{\ 28\ }$ sq yd

b)




$C = \underline{\ \pi\ }(\underline{\ 8\ }\text{ m})$

$= \underline{\ 25.132...\ }$ m,

or about $\underline{\ 25\ }$ m

$r = \frac{1}{2}(\underline{\ 8\ }\text{ m})$

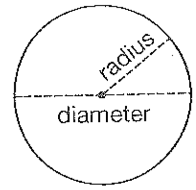
$= \underline{\ 4\ }$ m

$A = \underline{\ \pi\ }(\underline{\ 4\ }\text{ m})^2$

$= \underline{\ 50.265...\ }$ $\text{m}^2$,

or about $\underline{\ 50\ }$ $\text{m}^2$

**circle formulas**




**circumference**

$C = \pi \times (\text{diameter})$

or

$C = 2\pi \times (\text{radius})$

**area**

$A = \pi \times (\text{radius})^2$

**Tech Tip**

**Pi**

Some calculators have a special key for $\pi$. If your calculator does not, use 3.14 as an estimate for $\pi$.

5. Toula works at an aquarium store. She uses area formulas to estimate how many fish an aquarium can hold. For freshwater tropical fish, the formula is

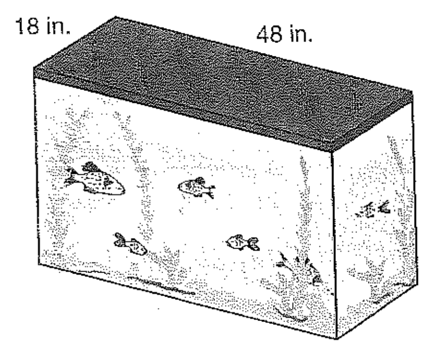
Area of top of tank ÷ 12 sq in. = number of fish

How many freshwater tropical fish can live in this aquarium?

48 in. × 18 in. = 864 sq in.

864 sq in. ÷ 12 sq in. = 72

About 72 fish can live in this aquarium.




6. Toula's aquarium, on the right, is 24 in. high. What is the total area of all of its outside surfaces in square inches? Include the top and bottom.

e.g., Top: 864 sq in. (from Question 5)

Top + bottom: 2 × 864 sq in. = 1728 sq in.

Front + back: 2 × (48 in. × 24 in.) = 2304 sq in.

Side + side: 2 × (18 in. × 24 in.) = 864 sq in.

Total surface area: 1728 sq in. + 2304 sq in. + 864 sq in. = 4896 sq in.

Total area is 4896 sq in.

**Hint**

Use the charts inside the back cover.

# 3.1 Relating Nets to Surface Area

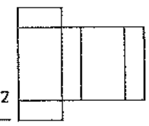
## Try These

**You will need**
- a small box
- a ruler
- a compass

Trace the faces of a small box, and draw its **net**.

**i)** How many faces does your box have? 6

**ii)** What is the total area of all the faces? e.g., raisin box: 127 $cm^2$

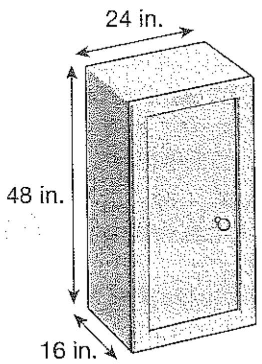


**net**

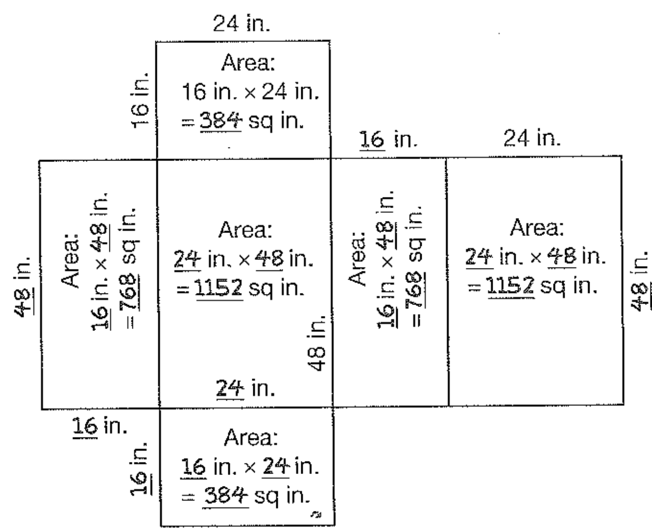
a composite 2-D shape that can be folded to create a 3-D object

Maia built this wooden cabinet. She plans to use 2 coats of maple stain on the outside surfaces. This includes the bottom and the door. A 1 L can of stain covers 80 sq ft.

Is one can enough to stain the cabinet?




**1** This is a net for the cabinet. Some measurements are done for you. Record the length and width of each rectangular face. Then record the area of each face in square inches.




**REFLECTING**

What are some other ways to draw the net for this cabinet?

**2** What is the total surface area of the cabinet in square inches?

2(768 sq in.) + 2(1152 sq in.) + 2(384 sq in.) = 4608 sq in.

**3** Will 1 can of stain be enough for 2 coats?

Stain needed: 2 × 4608 sq in. = 9216 sq in.

**Hint**

1 sq ft = 144 sq in.

One can covers: 80 sq ft × $\frac{\boxed{144} \text{ sq in.}}{1 \text{ sq ft}}$ = 11 520 sq in.

9216 sq in. < 11 520 sq in. So 1 can is enough.

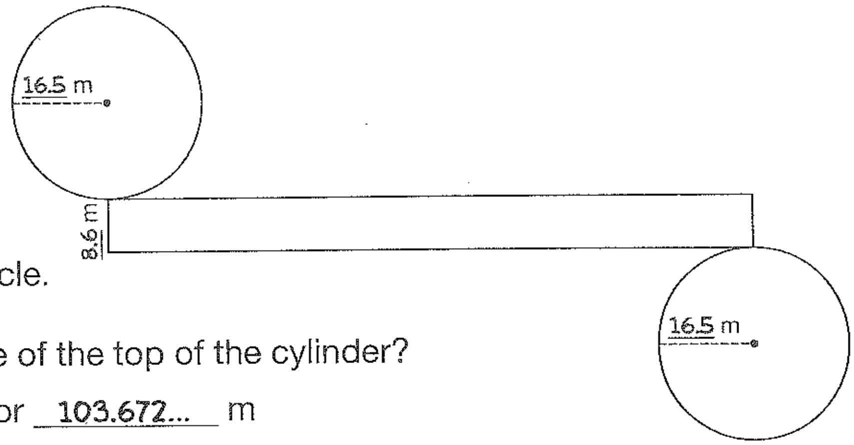
## Example

The North End Water Pollution Control Centre in Winnipeg has cylindrical tanks. Bacteria in the tanks clean wastewater. Each tank has a radius of 16.5 m and is 8.6 m high. Maintenance staff paint the tops and sides of the tanks.

What surface area needs to be painted for each tank, to the nearest square metre?

### Solution

**A.** This net represents a tank. Label the measurements of the width of the rectangle and the radius of each circle.




**B.** What is the circumference of the top of the cylinder?

$C = \pi \times 2 \times$ 16.5 m, or 103.672... m

**C.** What part of the rectangle has the same measurement as the circumference of the circle? the length

**D.** What are these areas?

$A_{\text{rectangle}} =$ 8.6 m $\times$ 103.672... m, or 891.583... m$^2$

$A_{\text{circle}} = \pi \times ($ 16.5 m$)^2$, or 855.298... m$^2$

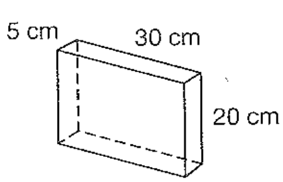
**E.** What is the surface area to be painted?

$SA_{\text{top + side}} =$ 855.298... m$^2$ + 891.583... m$^2$, or 1746.882... m$^2$

The surface area that needs to be painted is 1747 m$^2$, to the nearest square metre.

## Practice

**1.** Calculate the total surface area of this rectangular prism.




$SA = 2(5\text{ cm} \times 30\text{ cm}) + 2(5\text{ cm} \times 20\text{ cm}) + 2(20\text{ cm} \times 30\text{ cm})$

$= 2(150\text{ cm}^2) + 2(100\text{ cm}^2) + 2(600\text{ cm}^2)$

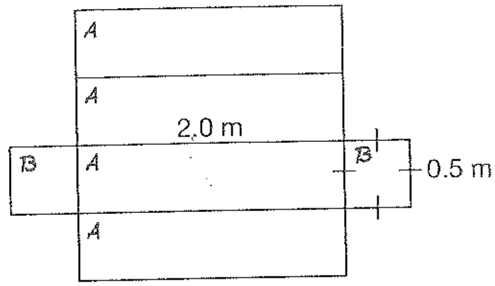
$= 300\text{ cm}^2 + 200\text{ cm}^2 + 1200\text{ cm}^2$

$= 1700\text{ cm}^2$

**REFLECTING**

Why do you need the areas of three faces to determine the surface area of a rectangular prism?

2. What is the surface area of the rectangular prism for this net?

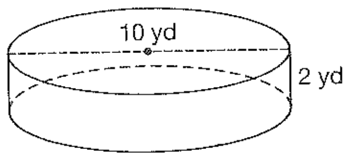


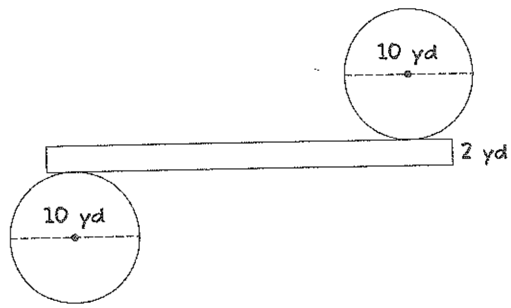

$$
\begin{aligned}
SA &= 4(A) + 2(B)\\
&= 4(2.0 \text{ m} \times 0.5 \text{ m}) + 2(0.5 \text{ m} \times 0.5 \text{ m})\\
&= 4(1.0 \text{ m}^2) + 2(0.25 \text{ m}^2)\\
&= 4.0 \text{ m}^2 + 0.5 \text{ m}^2\\
&= 4.5 \text{ m}^2
\end{aligned}
$$

**REFLECTING**

What does the surface area of a 3-D object have to do with the areas of 2-D shapes?

3. Draw a net for this cylinder. Record the dimensions on the net. Calculate the surface area, to the nearest square yard.




**Hint**

The radius is half the diameter.

e.g., $C = \pi \times 10$ yd, or 31.415... yd

$A_{rectangle} = 2$ yd $\times$ 31.415... yd, or 62.831... sq yd

$r = 10 \text{ yd} \div 2$

$= 5$ yd

$A_{circle} = \pi \times (5 \text{ yd})^2$, or 78.539... sq yd

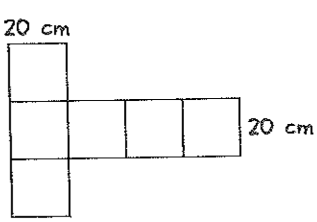
$SA = 2(78.539...$ sq yd$) + 62.831...$ sq yd, or 219.911... sq yd

The surface area is 220 sq yd, to the nearest square yard.

4. Jared is an engraver. A client has hired him to engrave slogans on novelty cubes. The side length of a cube is 20 cm. What is the total surface area for engraving?

a) Sketch a net for the cube. Record the dimensions.

4. a) e.g.,




b) What is the area of one square face?

$20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm}^2$

c) What is the total surface area of the cube?

$6 \times (400 \text{ cm}^2) = 2400 \text{ cm}^2$

The total surface area for engraving is $2400 \text{ cm}^2$.

d) How could you calculate the surface area of any cube?

$SA_{cube} =$ e.g., $6 \times (\text{side length})^2$

5. Jocelyn sells custom-made jewellery on her website. She is designing the box for shipping orders to her customers. She wants to make a box 9 in. long, 5 in. wide, and 3 in. tall. How much cardboard is needed, not including overlap?

e.g., $SA = 2(9 \text{ in.} \times 5 \text{ in.}) + 2(9 \text{ in.} \times 3 \text{ in.}) + 2(5 \text{ in.} \times 3 \text{ in.})$

$= 90 \text{ sq in.} + 54 \text{ sq in.} + 30 \text{ sq. in.}$

$= 174 \text{ sq in.}$

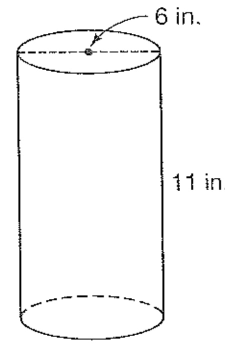
174 sq in. of cardboard is needed.

**REFLECTING**

How can sketching a net and recording the side lengths help you determine the surface area of an object?

6. Samantha looked up the formula for the surface area of a cylinder on two different websites. She found these formulas:

$SA = \pi d(r + h)$ $\qquad SA = 2\pi r^2 + 2\pi rh$




a) Show that both formulas give the same surface area for this cylinder.

$SA = \pi d(r + h)$
$= \pi(6 \text{ in.})(3 \text{ in.} + 11 \text{ in.})$
$= 263.893... \text{ sq in.}$

$SA = 2\pi r^2 + 2\pi rh$
$= 2\pi(3 \text{ in.})^2 + 2\pi(3 \text{ in.})(11 \text{ in.})$
$= 56.548... \text{ sq in.} + 207.345... \text{ sq in.}$
$= 263.893... \text{ sq in.}$

b) Explain why both formulas give the same answer for any cylinder.

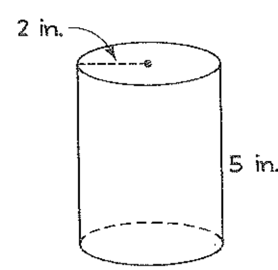
e.g., $\pi d$ is the circumference of a circle with radius $r$. The circumference can also be written as $2\pi r$. Substitute $2\pi r$ for $\pi d$ in $SA = \pi d(r + h)$ to get $2\pi r(r + h)$, or $2\pi r^2 + 2\pi rh$. The formulas give the same result.

**REFLECTING**

How is each part of the formula for the surface area of a cylinder related to the parts of the net?

7. Create and solve your own surface area problem.

e.g., Draw a container shaped like a rectangular prism or a cylinder. Record the dimensions on your drawing.




What is the surface area?

$C$ is $2 \times \pi \times 2 \text{ in.} = 12.566... \text{ in.}$

$A_{\text{rectangle}} = 5 \text{ in.} \times 12.566... \text{ in.}$, or $62.831... \text{ sq in.}$

$A_{\text{circle}} = \pi \times (2 \text{ in.})^2$, or $12.566... \text{ sq in.}$

$SA = 2(12.566... \text{ sq in.}) + 62.831... \text{ sq in.}$

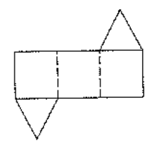
$= 87.964... \text{ sq in.}$

The surface area is about 88 sq in.

# 3.2 Surface Area: Prisms and Cylinders

## Try These

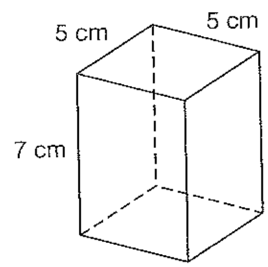
How can you make a net for an equilateral-triangle-based prism using 1 rectangle and 2 triangles?

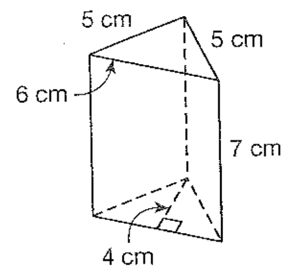


**Hint**
Prisms and cylinders have two congruent bases and a lateral area.

**lateral area**
combined area of the side faces of a 3-D object

Marcy works for a toy company that sells sets of painted blocks. The company wants to find out which type costs less to paint. Which block has less surface area?



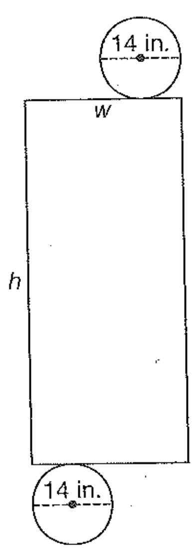

**1** Complete the chart.

| Type of block | Base area of block | Lateral area: base perimeter × height | Total surface area: 2 × (base area) + lateral area |
|---|---|---|---|
| square-based prism | 5 cm × 5 cm = 25 cm² | 4 (5 cm) × 7 cm = 140 cm² | 2 (25 cm²) + 140 cm² = 190 cm² |
| triangle-based prism | $\frac{1}{2}$(6 cm)(4 cm) = 12 cm² | 16 cm × 7 cm = 112 cm² | 2 ( 12 cm²) + 112 cm² = 136 cm² |

**2** Which block has less surface area?

136 cm² < 190 cm²

The triangle-based prism has less surface area.




### Example

Karen is designing a plastic wrapper for a cylindrical roll of R-12 insulation. The surface area of the wrapper is 2375 sq in. What are the width and height of the rectangular part of her design, to the nearest inch?

### Solution

**A.** What is the width of the rectangle?

Width = $\pi$ × 14 in., or 43.982... in.

The width of the rectangle is about 44 in.

**B.** How can you use the formula for surface area to get the height?

$$SA_{\text{cylinder}} = 2 \times \pi \times (\underline{\ 7\ }\text{ in.})^2 + (\underline{43.982...}\text{ in.} \times h)$$

$$\underline{2375}\text{ sq in.} = \underline{307.876...}\text{ sq in.} + (\underline{43.982...}\text{ in.} \times h)$$

$$\underline{2067.123...}\text{ sq in.} = \underline{43.982...}\text{ in.} \times h$$

$$\frac{\boxed{2067.123...}\text{ sq in.}}{\boxed{43.982...}\text{ in.}} = h$$

$$\underline{46.998...}\text{ in.} = h$$

The height of the cylinder is ___47___ in., to the nearest inch.

**Hint**

Surface area formula for prisms and cylinders:

$SA = 2 \times$ base area + lateral area

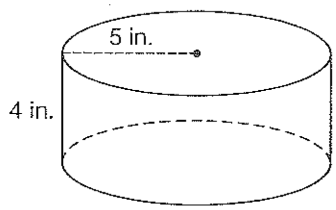
**REFLECTING**

How do you know the surface area formula above works for any prism or cylinder?

## Practice

**1.** What is the surface area of each object?

**a)**



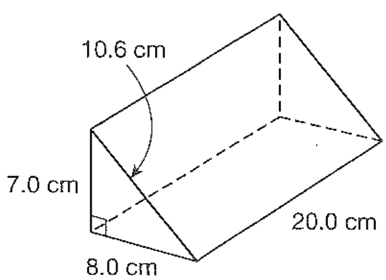

$SA = 2 \times (\text{base area}) + \text{lateral area}$

$= 2 \times \pi(5\text{ in.})^2 + \pi(10\text{ in.})(4\text{ in.})$

$= 157.079...\text{ sq in.} + 125.663...\text{ sq in.}$

$= 282.743...\text{ sq in., or about } 283\text{ sq in.}$

**b)**




$SA = 2 \times (\text{base area}) + \text{lateral area}$

$= 2 \times \left[\frac{1}{2}(8.0\text{ cm})(7.0\text{ cm})\right]$

$+ (7.0\text{ cm} + 8.0\text{ cm} + 10.6\text{ cm})(20.0\text{ cm})$

$= 56.0\text{ cm}^2 + 512.0\text{ cm}^2\text{, or } 568.0\text{ cm}^2$

**2.** A steel cylinder is 100 cm long and has a radius of 5 cm. Cal determined its surface area but made some mistakes. Describe the errors. Then correct the solution.

Base area $= \pi(5\text{ cm})^2$, or $15.707...\text{ cm}^2$

Base circumference $= 2\pi(5\text{ cm})$, or $31.415...$ cm

Lateral area $= 100\text{ cm} \times 31.415...\text{ cm}$, or $3141.592...\text{ cm}^2$

$SA = (2 \times \text{base area}) + \text{lateral area}$

$= 2(15.707...\text{ cm}^2) + (3141.592...\text{ cm}^2)$, or $3157.300...\text{ cm}^2$

e.g., Cal miscalculated the base area. He multiplied $\pi \times 5$ cm instead of $\pi \times 5\text{ cm} \times 5$ cm. He also forgot to multiply the base area by 2 in the last step.

Corrected solution:

Base area $= \pi \times 5\text{ cm} \times 5\text{ cm}$, or $78.539...\text{ cm}^2$

$SA = 2(78.539...\text{ cm}^2) + (3141.592...\text{ cm}^2)$, or $3298.672...\text{ cm}^2$

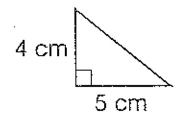


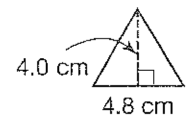
157.078

# 3.3 Surface Area: Pyramids and Cones

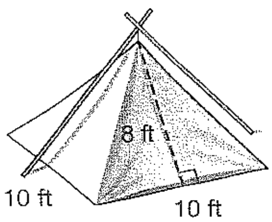
## Try These

Which triangle has a greater area? How do you know?

i) 


ii) 


Both have the same height. The triangle in Part i) has a longer base so it has a greater area.

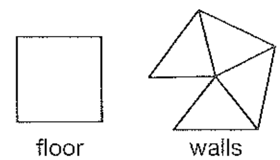



**slant height (pyramid)**
distance from the top of the pyramid to the centre of any side of the base

**REFLECTING**
The area of a triangle is $\frac{1}{2}$(base)(height). Why does it make sense that the formula for the lateral area of a pyramid is $\frac{1}{2}$(perimeter of base)(slant height)?

On canoe trips, Benoit packs two pieces of canvas to make this pyramid tent. One piece is for the walls. One is for the square floor.

What is the least amount of canvas for the tent?




1. Use this formula to determine the total area of the walls:

   Lateral area $= \frac{1}{2}$(perimeter of base)(**slant height**)

   Lateral area $= \frac{1}{2}$( 40 ft)( 8 ft), or 160 sq ft

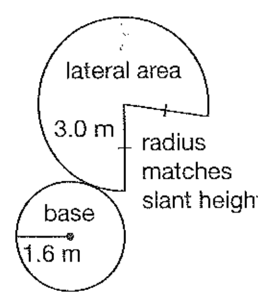
2. What is the total surface area of the tent, including the floor?

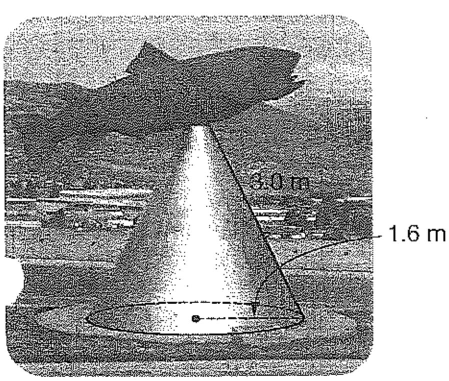
   100 sq ft + 160 sq ft = 260 sq ft

   The least amount of canvas Benoit needs is 260 sq ft.

### Example 1

A red cedar statue of a trout stands outside the Chamber of Commerce in Kamloops, British Columbia.




The stand is a metal cone. What area of metal is needed for the stand and its hidden base?

#### Solution

**A.** What is the area of the circular base of the cone?

Area of base $= \pi r^2$

$= \pi($ 1.6 m$)^2$, or 8.042... m$^2$

**B.** Use this formula to calculate the lateral area of the cone:

Lateral area = $\pi rs$, where $r$ is the radius of the circular base and $s$ is the **slant height of the cone**

Lateral area = $\pi rs$

= $\pi$( 1.6 m )( 3.0 m ), or 15.079... $m^2$

**slant height (cone)**
distance from the top of the cone to any point on the base circumference

**C.** What is the total surface area of the cone?

$SA_{cone} = \pi r^2 + \pi rs$

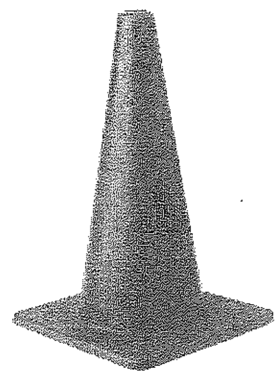
= 8.042... $m^2$ + 15.079... $m^2$, or 23.122... $m^2$

**D.** How much metal was used to make the cone?

About 23 $m^2$ of metal was used to make the cone.

## Example 2

Amrita's company rents traffic cones. One cone is 28 in. tall and has a base radius of 7 in. What is its lateral surface area?



### Solution

**A.** Use the Pythagorean theorem to determine the slant height, $s$.

$s^2$ = ( 28 in.$)^2$ + ( 7 in.$)^2$, or 833 sq in.

$s = \sqrt{833 \text{ sq in.}}$, or 28.861... in.

**B.** What is the lateral surface area of the traffic cone?

Lateral surface area = $\pi rs$

= $\pi$( 7 in.)( 28.861... in.)

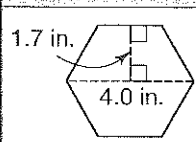
= 634.702... sq in., or about 635 sq in.

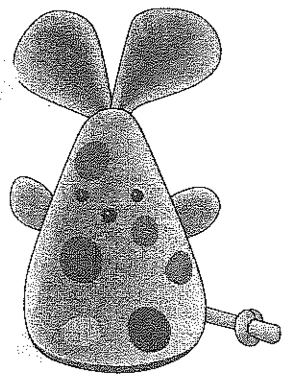
The lateral surface area is about 635 sq in.

**REFLECTING**
Create a surface area problem about a cone or a pyramid. How could you solve your problem?

# Practice

1. Complete the chart for this pyramid. The base has 2.0 in. sides. The slant height of each triangle is 10.0 in.

| Pyramid base (side = 2.0 in.) | Base area (sq in.) | Lateral area (sq in.) $\frac{1}{2}$(perimeter)(s) | Surface area (sq in.) (base area + lateral area) |
|---|---|---|---|
|   | $2\left(\frac{1}{2}\right)$(6.0 in.)(1.7 in.) = (6.0 in.)(1.7 in.) = 10.2 sq in. | $\frac{1}{2}$(12.0 in.)(10.0 in.) = 60.0 sq in. | 10.2 sq in. + 60.0 sq in. = 70.2 sq in. |



2. Marnie sells felt mice. Each mouse needs 9 sq in. of felt for the ears and arms, plus enough to make the cone. It takes 8 sq in. for the base. The slant height of the cone is 5 in.

   a) What is the radius of the base?

   $\pi r^2 = \underline{8}$ sq in.

   $\pi r^2 \div \underline{\pi} = \underline{8}$ sq in. $\div \underline{\pi}$

   $r^2 = \underline{2.546...}$ sq in.

   $r = \underline{1.595...}$ in.

   b) How much felt does Marnie need for each mouse, to the nearest square inch?

   $SA_{cone} = \pi rs + \pi r^2$

   $= \pi(\underline{1.595...}$ in.$)(\underline{5}$ in.$) + 8$ sq in.

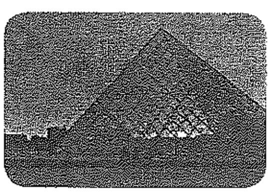
   $= \underline{33.066...}$ sq in.

   Cone: $\underline{33}$ sq in. Ears and arms: $\underline{9}$ sq in.

   Marnie needs $\underline{42}$ sq in. of felt for each mouse, to the nearest square inch.

3. Complete the chart for this cone. Round to whole numbers.

| Dimensions of cone | Base area ($\pi r^2$) | Lateral area ($\pi rs$) | Surface area ($\pi r^2 + \pi rs$) |
|---|---|---|---|
| Base radius: 6.5 cm<br>Slant height: 30.7 cm | $\pi(6.5 \text{ cm})^2$<br>$= 132.732... \text{ cm}^2$ | $\pi(6.5 \text{ cm})(30.7 \text{ cm})$<br>$= 626.904... \text{ cm}^2$ | $759.637... \text{ cm}^2$<br>$\doteq 760 \text{ cm}^2$ |



4. The entrance to the Louvre Museum in Paris, France is a giant glass pyramid.
   - The square base has sides 35 m long.
   - The lateral surface area is 1892 m².

   What is the slant height of the pyramid, to the nearest metre?

   e.g., Lateral area $= \frac{1}{2}$(base perimeter)(slant height)

   $1892 \text{ m}^2 = \frac{1}{2}(4 \times 35 \text{ m})$(slant height)

   $1892 \text{ m}^2 = (70 \text{ m})$(slant height)

   $1892 \text{ m}^2 \div 70 \text{ m} =$ slant height

   $27.028... \text{ m} =$ slant height

   The slant height is 27 m, to the nearest metre.

# Mid-Chapter Review

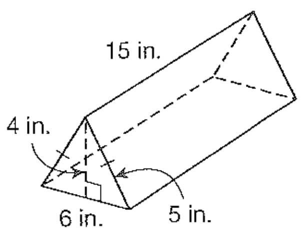
1. The world's largest raisin box was built by students in California. The box is a rectangular prism 12 ft high, 8 ft wide, and 4 ft deep. What is its surface area?

e.g., $SA = 2(12 \text{ ft} \times 4 \text{ ft}) + 2(8 \text{ ft} \times 4 \text{ ft}) + 2(12 \text{ ft} \times 8 \text{ ft})$

$= 2(48 \text{ sq ft}) + 2(32 \text{ sq ft}) + 2(96 \text{ sq ft})$, or 352 sq ft

The surface area of the giant raisin box is 352 sq ft.

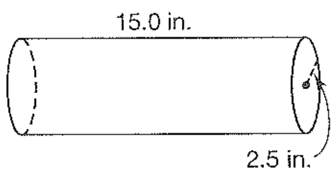
2. These two steel rods are to be case-hardened. The process will cost more for the rod with greater surface area. Which rod will cost more to case-harden?




e.g., $SA$ = 2(base area) + (lateral area)

$= 2 \times [\frac{1}{2}(6 \text{ in.})(4 \text{ in.})] + (6 \text{ in.} + 5 \text{ in.} + 5 \text{ in.})(15 \text{ in.})$

$= 264$ sq in.



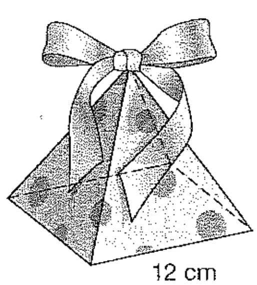

e.g., $SA$ = 2(base area) + (lateral area)

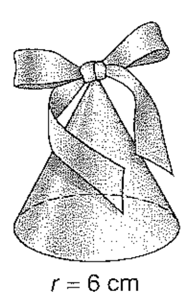
$= 2\pi(2.5 \text{ in.})^2 + 2\pi(2.5 \text{ in.})(15.0 \text{ in.})$

$= 274.889...$ sq in.

The cylinder has more surface area. So it will cost more.

3. Tim works at a chocolate store. The store sells candy in two types of gift boxes. One is a square-based pyramid. The other is a cone. The one that uses less cardboard costs less to make. Both boxes have a slant height of 13.5 cm. Which box costs less to make?




e.g., $SA = (12 \text{ cm})(12 \text{ cm}) + 4 \times [\frac{1}{2}(12 \text{ cm})(13.5 \text{ cm})]$

$= 468 \text{ cm}^2$

$SA = \pi(6 \text{ cm})^2 + \pi(6 \text{ cm})(13.5 \text{ cm})$

$= 367.566... \text{ cm}^2$

The cone box has less surface area. So it costs less to make.

# 3.4 Surface Area: Spheres

**You will need**
- a ball
- string and a ruler, or a measuring tape
- scissors
- paper for wrapping

## Try These

Use a small ball, such as a baseball or tennis ball.

i) Measure the circumference: e.g., 28 cm

Calculate the diameter to the nearest centimetre:
e.g., 28 cm ÷ $\pi$ ≐ 9 cm

ii) Cut a rectangle so length = circumference of ball and width ≐ diameter. Use the rectangle to wrap the ball. How do the two areas compare?

e.g., Without any overlap, the rectangle would cover the ball.

An official NBA basketball has a circumference of 29.5 in. How much leather, to the nearest square inch, is needed for the surface?

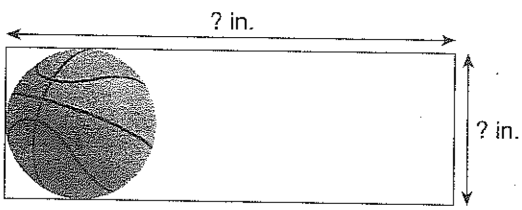
**REFLECTING**
If you know the surface area of a sphere, how can you determine the diameter? What does this have to do with the formula in Question 3?

1. The surface area of a sphere matches the area of a rectangle you could wrap around it. How long would a rectangle need to be to wrap around an NBA basketball? 29.5 in.

2. How wide would the rectangle need to be?

Diameter$_{ball}$ = (29.5 in.) ÷ $\pi$, or 9.390... in.

3. How much leather is needed for the surface?




$SA_{ball}$ = (length of rectangle)(diameter of ball)

$SA_{ball}$ = (9.390... in.)(29.5 in.), or 277.009... sq in.

To the nearest square inch, 277 sq in. of leather is needed.

### Example

The circumference of a spherical gas storage tank is 50 m. What is the surface area of the tank?

**Tech Tip**
When you divide by $2\pi$, use brackets:
50 ÷ ( 2 $\pi$ ) =

### Solution

**A.** What is the radius of the storage tank?

$2\pi r = 50$ m

$r$ = 50 m ÷ (2 $\pi$)

$r$ = 7.957... m

**B.** What is the surface area of the tank? Use this formula:

$SA_{sphere} = 4\pi r^2$

$= 4\pi(\underline{7.957...}\ m)^2$, or $\underline{795.774...}\ m^2$

The surface area of the tank is about $\underline{796}$ $m^2$.

**REFLECTING**

The surface area of the rectangle that wraps a sphere is $c \times d$, or $2\pi r \times 2r$.

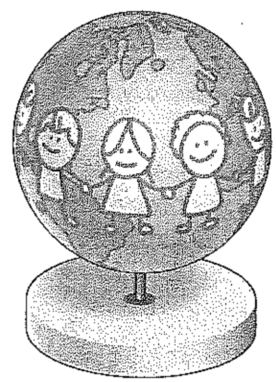
What does this have to do with the formula in Question B?

## Practice

**1.** Complete the chart.

| Radius ($r$) | Diameter ($2r$) | Circumference ($\pi d$, or $2\pi r$) | Surface area ($4\pi r^2$) |
|---|---|---|---|
| 50 cm | $2 \times 50$ cm $= 100$ cm | $\pi(100\ cm)$ $= 314.159...$ cm | $4\pi(50\ cm)^2$ $= 31\ 415.926...\ cm^2$ |
| 26 ft $\div$ 2 $=$ 13 ft | 26 ft | $\pi(26\ ft)$ $= 81.681...$ ft | $4\pi(13\ ft)^2$ $= 2123.716...$ sq ft |

**2.** Cool Globes is a project designed to raise awareness of global warming. Artists decorate white globes to show ways to protect the environment. Each globe has a diameter of 5 ft. What area needs to be decorated for each sphere?



e.g., Diameter: 5 ft, so radius = 2.5 ft

$SA_{sphere} = 4\pi(2.5\ ft)^2$, or 78.539... sq ft. About 79 sq ft needs to be decorated.

**3.** Jacques claims that another formula for surface area of a sphere is $SA_{sphere} = \pi d^2$. Is he correct? Justify your decision.

Yes. He is correct.

e.g., $\pi d^2 = \pi(2r)^2$, or $4\pi r^2$

**4.** Sarah is installing security cameras. Each security camera is a sphere. The camera's surface area is 12.5 sq in. What is its diameter?

$SA_{sphere} = \pi d^2$

$12.5\ sq\ in. = \pi d^2$

$3.978...\ sq\ in. = d^2$

$\sqrt{3.978...\ sq\ in.} = d$

$1.994...\ in. = d$ The diameter of the camera is about 2.0 in.

# 3.5 Estimating Surface Area

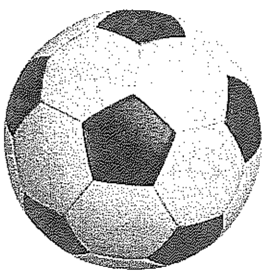
## Try These

Estimate the surface area of this workbook.

e.g., 2(20 cm)(30 cm) + 2(1 cm)(30 cm) + 2(1 cm)(20 cm) is about 1300 $cm^2$

**You will need**
- a ruler
- 3-D objects (prisms, cylinders, pyramids, cones, spheres)

Jeff is a cake decorator. A client ordered a cake shaped like a soccer ball for her son's 8th birthday. About how much surface area needs to be iced for the cake?



**referent**
a known measure used for comparing and estimating

**REFLECTING**
Why is it helpful to use a referent to estimate surface area?

1. What square unit will you use to estimate the surface area of the cake? Give reasons for your choice.

   e.g., Square centimetres; I can estimate the diameter of the cake in centimetres. Then I can use the formula to estimate the surface area in square centimetres.

2. What is the surface area of the cake?

   e.g., $Diameter_{cake}$: about 20 cm

   $SA_{cake} = \pi d^2$, or $\pi(20)^2$ About 1260 $cm^2$

3. What **referent** did you use to estimate? How did you use it?

   e.g., My pen; It is about 10 cm long. The diameter of the cake is about 2 pen lengths. So it is about 20 cm.

### Example

Choose an object that is like a prism or a cylinder. How can you use a referent to estimate its surface area?

### Solution

**A.** Name the object. Describe the referent you plan to use.

Object: e.g., classroom door (rectangular prism)

Referent: e.g, The part of the door from the floor to the doorknob is about 1 $m^2$.

**B.** Estimate the surface area of the object. Explain your thinking.

e.g., The front is about 2 $m^2$. The side is about $\frac{1}{10}$ of 1 $m^2$, or about 0.1 $m^2$. The bottom is about $\frac{1}{2}$ of the side, or about 0.05 $m^2$.

$SA_{door}$ is about $2(2\ m^2) + 2(0.1\ m^2) + 2(0.05\ m^2) = 4.3\ m^2$.

## Practice

**1.** The front of a binder is about 1 sq ft. You can use this to help you estimate areas in square feet. Explain how to estimate the surface area of an object you can see in square feet.

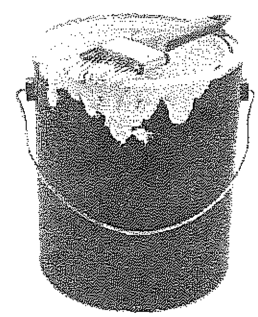
e.g., The top of the filing cabinet could be covered by 2 binders. So it is about 2 sq ft. The side could be covered by 4 binders. So it is about 4 sq ft. The front could be covered by 3 binders. So it is about 3 sq ft. The surface area, including the bottom, is about 18 sq ft.

**2.** Name a referent you could use to estimate areas in each unit. Describe the area of each referent.

**a)** square metres: e.g., The bottom half of the interactive whiteboard is about 2 $m^2$.

**b)** square inches: e.g., My hand covers about 30 sq in.

**3.** One gallon of paint covers about 200 sq ft. About how many gallons of paint would it take to repaint the walls and ceiling of your classroom?



e.g., The front and back are about 25 ft by 10 ft, or 250 sq ft. The left side is about 30 ft by 10 ft, or 300 sq ft. The right side is mostly windows and will not be painted.

The ceiling is about 25 ft by 30 ft, or 750 sq ft. The surface area to be painted is about 2(250 sq ft) + 300 sq ft + 750 sq ft = 1550 sq ft.

Amount of paint: 1550 sq ft ÷ 200 sq ft = 7.75, so it would take about 8 gal of paint for one coat.

**4.** Describe how a drywaller might use a formula to estimate the number of sheets of drywall needed for a room.

Estimate the dimensions of the room. Use the formula area = length × width to estimate of the area of each wall and the ceiling. Add the areas to calculate the total surface area. Divide the total by the area of one sheet of drywall.

# 3.6 Surface Area: Dimension Changes

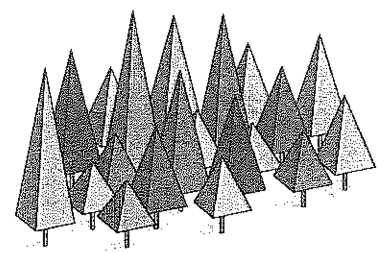
### Try These

**You will need**
- a ruler

How does the area of a 2-D shape change when you double the side lengths? Check some examples. Then explain a rule.

The area is multiplied by 4. e.g., This happens because doubling the width doubles the area, and doubling the length doubles it again.

Liana is a carpenter. She is making model pyramid-shaped trees out of plywood for a pavilion at a fair.



- All the pyramids have a square base with sides 4.9 m long.
- The slant heights of the pyramids vary from 5.5 m to 18.2 m.

How much more plywood is needed for the largest tree than for the smallest?

1. Which face is the same on all the pyramid trees? the square base

2. Which faces increase in size? the triangles

**REFLECTING**
Why can you ignore the base areas when you are comparing the surface areas of the trees? Can you always ignore the base areas? Explain.

3. What is the surface area of the smallest pyramid tree?

$SA_{\text{smallest tree}} = (\text{base area}) + \frac{1}{2}(\text{perimeter})(s)$
$= (\underline{4.9}\text{ m})^2 + \frac{1}{2}(4 \times \underline{4.9}\text{ m})(\underline{5.5}\text{ m})$
$= \underline{77.91}\text{ m}^2$

4. What is the surface area of the largest pyramid tree?

$SA_{\text{largest tree}} = (\text{base area}) + \frac{1}{2}(\text{perimeter})(s)$
$= (\underline{4.9}\text{ m})^2 + \frac{1}{2}(4 \times \underline{4.9}\text{ m})(\underline{18.2}\text{ m})$
$= \underline{202.37}\text{ m}^2$

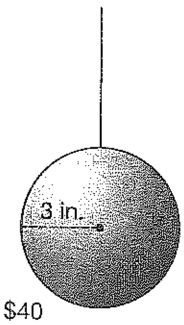
5. How much more plywood is needed for the largest tree, to the nearest square metre?

$\underline{202.37}\text{ m}^2 - \underline{77.91}\text{ m}^2 = \underline{124.46}\text{ m}^2$

About $\underline{124}$ $\text{m}^2$ more is needed for the largest tree.

### Example

Maurice is an interior designer. He sees this spherical light fixture that costs $40. Maurice wants to use a spherical light fixture with triple the radius of this one. What would be a fair price for it?




$40

### Solution

**A.** What area of glass is needed for the smaller light?

$SA_{\text{sphere}} = 4\pi r^2$

$= \underline{\ 4\ }\pi(\underline{\ 3\ }$ in.$)^2$, or $\underline{113.097...}$ sq in.

**B.** What area of glass is needed for the larger light?

Larger radius = $\underline{\ 3\ }(\underline{\ 3\ }$ in.), or $\underline{\ 9\ }$ in.

$SA_{\text{sphere}} = \underline{\ 4\pi\ }(\underline{\ 9\ }$ in.$)^2$, or $\underline{1017.876...}$ sq in.

**C.** What would be a fair price for the larger light? Explain.

The larger light has <u>9</u> times as much glass as the smaller one, so the price should be <u>9 times as much.</u>

<u>9 × $40 = $360, so $360 would be a fair price.</u>

**REFLECTING**

Yvonne said the amount of glass tripled when the radius tripled. Why is this not correct?

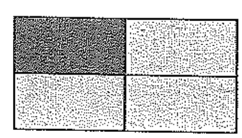
(Hint: Think of the rectangle that wraps the sphere.)

## Practice

**1. a)** A manufacturer sells marbles in rectangular boxes. He wants to increase the size of the box. Complete the chart.

| Box size | Base area | Lateral area (base perimeter × height) | Surface area 2(base area) + lateral area |
|---|---|---|---|
| original size: base: 4 in. by 5 in. height: 3 in. | 4 in. × 5 in. = 20 sq in. | 18 in. × 3 in. = 54 sq in. | 2(20 sq in.) + 54 sq in. = 94 sq in. |
| all sides doubled: base: 8 in. by 10 in. height: 6 in. | 8 in. × 10 in. = 80 sq in. | 36 in. × 6 in. = 216 sq in. | 2(80 sq in.) + 216 sq in. = 376 sq in. |
| all sides tripled: base: 12 in. by 15 in. height: 9 in. | 12 in. × 15 in. = 180 sq in. | 54 in. × 9 in. = 486 sq in. | 2(180 sq in.) + 486 sq in. = 846 sq in. |

**b)** Suppose you double the length and width of each face of the box. By what factor does the original surface area of each face change by? <u>4</u>



**c)** Suppose you triple the length and width of each face. By what factor does the original surface area of each face change by? <u>9</u>

d) Suppose the manufacturer multiplied each side of the original box by 4. What would the new surface area be?

e.g., The surface area would be multiplied by $4 \times 4 = 16$.

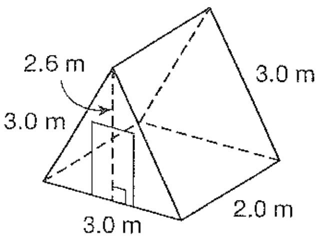
So it would increase to $16 \times 94$ sq in., or 1504 sq in.

**Hint**

Use the data from Question 1.

2. What happens to the surface area of a rectangular prism if you divide all of the side lengths by 2?

The surface area is divided by 4.




3. Kiran is a carpenter. He designed this prism-shaped tree house for a client. The cost to build it will be \$40 per square metre of surface area. The client wonders how much it would cost to increase the depth from 2.0 m to 3.0 m.

a) What is the surface area of the smaller tree house?

$SA = 2(\text{base area}) + (\text{perimeter})(\text{depth})$

$= 2(\frac{1}{2})(\underline{3.0}\text{ m})(\underline{2.6}\text{ m}) + (\underline{9.0}\text{ m})(\underline{2.0}\text{ m})$

$= \underline{25.8\text{ m}^2}$

b) What is the surface area of the larger tree house?

$SA = 2(\text{base area}) + (\text{perimeter})(\text{depth})$

$= 2(\frac{1}{2})(\underline{3.0}\text{ m})(\underline{2.6}\text{ m}) + (\underline{9.0}\text{ m})(\underline{3.0}\text{ m})$

$= \underline{34.8\text{ m}^2}$

c) What is the difference in the surface areas?

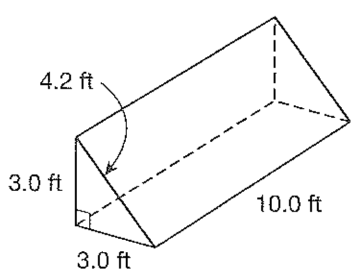
$34.8\text{ m}^2 - 25.8\text{ m}^2 = 9.0\text{ m}^2$

d) How much would it cost to increase the depth?

$(\underline{9.0}\text{ m}^2)(\$\underline{40}/\text{m}^2) = \$\underline{360}$

It would cost \$360 to increase the depth.

4. The 10.0 ft dimension of this prism is doubled. The side lengths of the base stay the same. How much will the surface area increase?




e.g., The lateral area will double. The base area will stay the same. The increase will equal the original lateral area.

$(3.0\text{ ft} + 3.0\text{ ft} + 4.2\text{ ft}) \times 10.0\text{ ft}$

$= 102.00$ sq ft

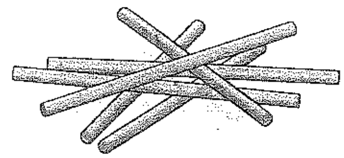
The surface area will increase by 102 sq ft.

# Solving a Surface Area Puzzle

**3.7**

Sal has 6 sticks. Each stick is 6 cm long.

- Sal is going to make the frame of a gift box without bending or breaking any of the sticks.
- Then he will cover the frame with paper.



**You will need**
- 6 sticks of equal length
- a ruler

**A.** What 3-D object can Sal make?

a triangle-based pyramid

**B.** What is the least amount of paper Sal needs to cover the frame?

e.g., The net is a parallelogram:

$(3.0 \text{ cm})^2 + h^2 = (6.0 \text{ cm})^2$

$h^2 = 36.0 \text{ cm} - 9.0 \text{ cm}$

$h = \sqrt{27.0 \text{ cm}}$

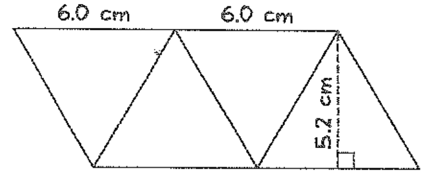
$h = 5.2 \text{ cm}$

Area of parallelogram is:

$A = bh$

$= 12.0 \text{ cm} \times 5.2 \text{ cm}$

$= 62.4 \text{ cm}^2$




The least amount of paper Sal needs to cover the frame is $62.4 \text{ cm}^2$.

**C.** Suppose Sal used sticks that were twice as long. What would be the least amount of paper he could use?

e.g., When you double the lengths, the surface area is multiplied by 4. The new paper area would be $4(62.4 \text{ cm}^2) = 249.6 \text{ cm}^2$, or about $250 \text{ cm}^2$.

**D.** Discuss your solutions with a partner.

- What strategies did you use to solve the puzzle?
- Did you both get the same solutions?

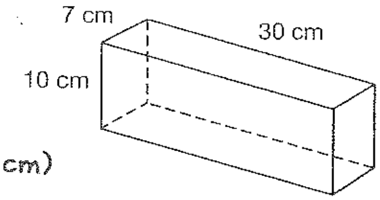
e.g., Strategies might include guess and check, draw or model, simplify the problem, or work backwards.

**E.** Marla said that if each stick used had a length of 3.0 cm, the least amount of paper would be $31.2 \text{ cm}^2$. Do you agree or disagree? Explain.

Disagree. e.g., Marla divided the area by 2. She should have divided the area by $2^2$ or 4. The area would be $62.4 \text{ cm}^2 \div 4 = 15.6 \text{ cm}^2$, or about $16 \text{ cm}^2$.

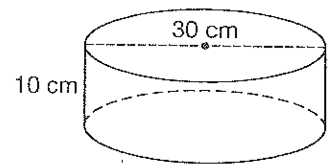
# Chapter Review

**1.** Which block of cheese has more surface area to wrap: the rectangular prism or the cylinder?




$SA_{\text{rectangular prism}} = 2(10 \text{ cm})(30 \text{ cm}) + 2(10 \text{ cm})(7 \text{ cm}) + 2(7 \text{ cm})(30 \text{ cm})$

$= 600 \text{ cm}^2 + 140 \text{ cm}^2 + 420 \text{ cm}^2$, or $1160 \text{ cm}^2$




Area of 2 bases $= 2\pi(15 \text{ cm})^2$, or $1413.716... \text{ cm}^2$

Circumference $= \pi(30 \text{ cm})$, or $94.247... \text{ cm}$

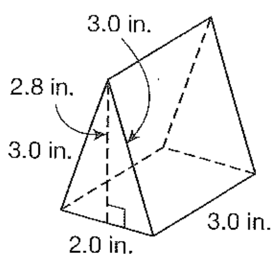
Lateral area $= (94.247... \text{ cm})(10 \text{ cm})$, or $942.477... \text{ cm}^2$

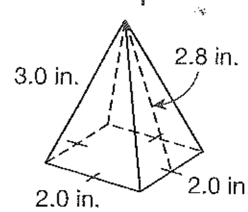
$SA_{\text{cylinder}} = 1413.716... + 942.477... \text{ cm}^2$

$= 2356.194... \text{ cm}^2$

The cylinder has more surface area to wrap.

**2.** Which wooden block has more surface area to paint?




$SA = 2(\text{base area}) + \text{lateral area}$

$= 2\left(\frac{1}{2}\right)(2.0 \text{ in.})(2.8 \text{ in.}) + (8.0 \text{ in.})(3.0 \text{ in.})$

$= 29.6 \text{ sq in.}$

$SA = \text{base area} + \text{lateral area}$

$= 4.0 \text{ sq in.} + \left(\frac{1}{2}\right)(8.0 \text{ in.})(2.8 \text{ in.})$

$= 15.2 \text{ sq in.}$

The triangular prism has more surface area to paint.

**3.** A box-shaped chest freezer has a surface area of $5.1 \text{ m}^2$. The freezer is 1.2 m long and 0.7 m wide. How tall is it?

e.g., $SA = 2(\text{base area}) + \text{lateral area}$

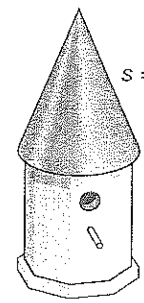
$5.1 \text{ m}^2 = 2(1.2 \text{ m})(0.7 \text{ m}) + (\text{base perimeter})(\text{height})$

$5.1 \text{ m}^2 = 1.68 \text{ m}^2 + (3.8 \text{ m})(\text{height})$

$3.42 \text{ m}^2 = (3.8 \text{ m})(\text{height})$

$0.9 \text{ m} = \text{height}$ The freezer is 0.9 m tall.

4. Melissa is making this bird feeder with a metal roof for her craft store. She is leaving the cone open at the bottom. She needs to calculate the area of metal needed for the roof.




Melissa made an error when she calculated the area of metal. Describe the error. Correct the solution.

*SA*(roof) is the lateral area of cone

$SA = \pi rs$

$= \pi(20\text{ cm})(30\text{ cm})$

$= 1884.955...\text{ cm}^2$

I will need about 1880 $\text{cm}^2$ of metal for the roof.

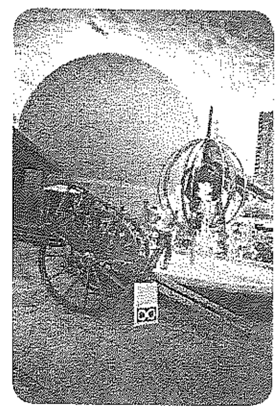
e.g., Melissa used the diameter instead of the radius in her calculations.

Correct solution:

$\pi(10\text{ cm})(30\text{ cm}) = 942.477...\text{ cm}^2$

She will need about 942 $\text{cm}^2$ of metal.

5. The Aboriginal Pavilion at the 2010 Winter Olympics in Vancouver included a giant sphere with a radius of 32.5 ft. About how many square feet of material were used to cover the surface of the sphere?



e.g., $SA_{\text{sphere}} = 4\pi r^2$

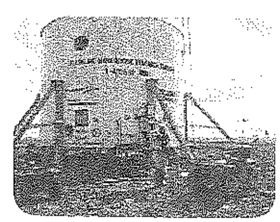
$= 4\pi(32.5\text{ ft})^2$, or 13 273.228... sq ft

About 13 273 sq ft, or about 13 300 sq ft of material were used.

6. Choose an object you can see from your desk that looks like a prism, cylinder, sphere, pyramid, or cone. Estimate the surface area of the object. Show your steps.

e.g., Softball: I know my hand covers about 100 $\text{cm}^2$. It takes three of my hand areas to cover the softball. So the surface area is about 300 $\text{cm}^2$.

7. The Flashline Mars Arctic Research Station (FMARS) on Devon Island in Nunavut is a research station where scientists explore what it might be like to live and work on Mars.



height = 9 m
diameter = 8 m

Suppose the station had a flat roof and no openings. What would be the surface area of the station, to the nearest square metre? Include the floor.

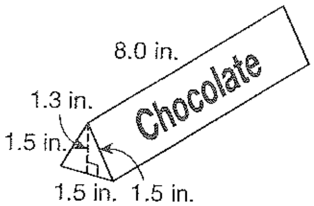
e.g., $SA = 2\pi r^2 + (\pi d)(\text{height})$

$= 2\pi(4\text{ m})^2 + \pi(8\text{ m})(9\text{ m})$

$= 326.725...\text{ m}^2$

The surface area would be 327 $\text{m}^2$, to the nearest square metre.

# Chapter Test

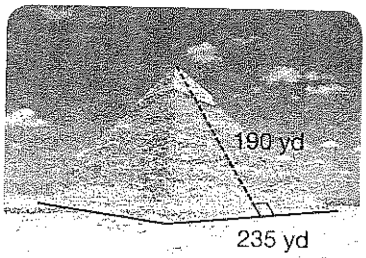



1. Determine each surface area.

   a) this giant chocolate bar

   $SA = 2\left(\frac{1}{2}\right)(1.5 \text{ in.})(1.3 \text{ in.}) + (3 \times 1.5 \text{ in.})(8.0 \text{ in.})$

   $= 37.95$ sq in., or about 38.0 sq in.




   b) the Pyramid of Khafre, built on a square base

   $SA = (235 \text{ yd})^2 + \frac{1}{2}(4)(235 \text{ yd})(190 \text{ yd})$

   $= 144\,525$ sq yd, or about $145\,000$ sq yd

   c) a cone with a slant height of 8 cm and a base radius of 3 cm

   $SA = \pi(3 \text{ cm})^2 + \pi(3 \text{ cm})(8 \text{ cm})$

   $= 103.672... \text{ cm}^2$, or about $104 \text{ cm}^2$

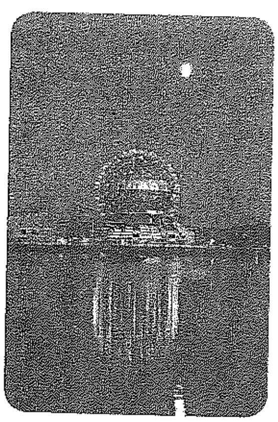
2. A cylindrical grain storage bin has a base radius of 5 m. The surface area is $1099 \text{ m}^2$. What is the height of the bin, to the nearest metre?

e.g., $1099 \text{ m}^2 = 2\pi(5 \text{ m})^2 + 2\pi(5 \text{ m})(h)$

$1099 \text{ m}^2 = 157.079... \text{ m}^2 + 31.415... \text{ m}(h)$

$941.920... \text{ m}^2 = (31.415... \text{ m})(h)$

$29.982... \text{ m} = h$ The height of the bin is about 30 m, to the nearest metre.



3. The diameter of the Omnimax Theatre at Science World in Vancouver is 47 m. If the theatre were a perfect sphere, what would its surface area be?

e.g., Radius: $\left(\frac{47 \text{ m}}{2}\right) = 23.5 \text{ m}$ $\quad SA = 4\pi r^2$

$= 4\pi(23.5 \text{ m})^2$,

or $6939.778... \text{ m}^2$

The surface area of the theatre would be about $6940 \text{ m}^2$.

4. Suppose the length of the chocolate bar in Question 1 increased from 8.0 in. to 10.0 in. How would the surface area increase?

..g., The ends would not change, but a 2.0 in. by 4.5 in. rectangular surface would be added to the lateral surface area. The surface area would increase by 9.0 sq in.