MathWorks 11
Teacher Resource

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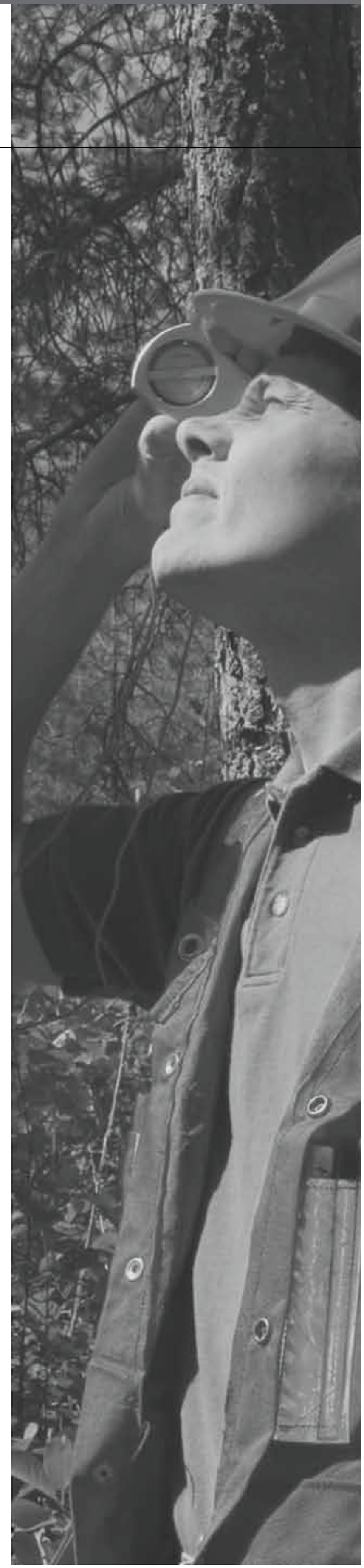

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INTRODUCTION

MathWorks 11 was developed to address the outcomes and philosophy of the Apprenticeship and Workplace Mathematics Grade 11 course.

The Workplace and Apprenticeship Mathematics pathway was designed for students who may want to pursue post-secondary studies in trades, certified occupations, or direct entry into the workforce. Consequently, *MathWorks 11* delivers the curriculum outcomes through projects, activities, and problems set in real-world contexts, enabling students to make connections between school mathematics and the workplace.

CONCEPTUAL FRAMEWORK

In keeping with the philosophy of the Common Curriculum Framework for Grades 10-12 Mathematics, the student textbook and teacher resource incorporate the following aspects of learning mathematics:

- communication
- connections
- mental mathematics and estimation
- problem solving
- reasoning
- technology
- visualization
- critical thinking
- cultural considerations
- adapting instruction for diverse student needs

Communication

Students are provided with opportunities to learn by reading, listening, doing, and speaking. Solving realistic workplace problems and engaging in a variety of hands-on activities will enable students to gather information and knowledge in various ways, express their learning, and communicate with others. The numerous opportunities for class or small group discussion of contextual problems encourage students to share their experiences and prior knowledge, and thereby develop mathematical understanding. Many features of the textbook are flexible, so teachers can decide which communication mode works best in their classroom.

Connections

The student textbook contains a wealth of real-world examples and problems, especially those related to apprenticeship programs and to employment that students can enter after completing secondary school. Connections between mathematical processes and real-world applications of those processes are made explicit. Concrete examples describe how math is used on the job, and word problems and activities are contextualized to ensure that students can make connections between the mathematical ideas and the workplace. In addition, connections are made across the chapters so that students will be able to apply mathematical ideas in different contexts when they encounter them.

Mental mathematics and estimation

Mental mathematics and estimation problems appear throughout the student textbook. Realistic problem scenarios show students that mental math and estimation are used in daily life as well as in the workplace.

Problem solving

Problem solving is fundamental in this textbook. Students are encouraged to critique given solutions, identify errors in given strategies, and develop their own strategies for approaching problems. They are given many opportunities to develop approaches to problems individually, in pairs, and in small groups. Examples with worked solutions range from simple to multi-step processes that build upon prior knowledge and skills. Students are challenged to see familiar mathematics in new scenarios and apply new mathematics to solve the multi-step questions.

Reasoning

Hands-on activities, puzzles, and projects in which there is no one set method and no one set solution challenge students to use analytical skills to find a solution. Group discussion of mathematics problems develops students' ability to make predictions and conjectures and encourages participation by students who have difficulty with rote algebraic mathematics. It also helps students to connect the abstract math to a familiar, concrete workplace situation.

Technology

A variety of technologies can be used to complete the projects and solve many of the problems in the textbook. However, technologies are not equally available to all students, so there is flexibility and choice. The use of communications technologies, such as the internet, presentation software, and spreadsheet programs will further expand students' abilities to collect data and to communicate mathematical ideas to others.

Visualization

The development of visualization skills, spatial sense, and measurement sense are fostered through the use of technology, graphic organizers, manipulatives, and diagrams. The culminating activities of many of the chapter projects are presented in a visual form, encouraging students to make the connection between abstract mathematical concepts and the physical world. In addition, the strong visual components of the textbook, including illustrations, photographs, graphs, and charts, enrich the presentation of the material.

Critical thinking

Critical thinking is key to problem solving. The textbook includes many opportunities for students to develop analytical and critical thinking skills by strategizing solutions to problems and evaluating the options presented.

Cultural considerations

To reflect the educational interests of western Canadian students, the images, problems, activities, and projects incorporate realistic western and northern contexts. This text is mindful of the multi-ethnic composition of Canadian schools. In particular, First Nations, Métis, Inuit, and francophone perspectives are represented.

Adapting instruction for diverse student needs

Many students learn best through experiential learning. With a range of hands-on activities and opportunities to adapt teaching strategies, the textbook accommodates these learners. The resources are flexible and adaptable to a variety of learning styles. Hands-on activities, discussion topics, and projects that can be completed by pairs or small groups as well as individually maximize opportunities to customize the course for particular classrooms. Alternative instructional strategies described in the teacher resource support this as well. In some cases, students may not have mastered mathematics from earlier grades. The teacher resource lists essential mathematics students may know from earlier grades and includes review materials. Teachers can decide whether or not students would benefit from a review.

ASSESSMENT

Teachers use assessment as an investigative tool to find out as much as they can about what their students know and can do and what confusions, preconceptions, or gaps in learning they might have. *MathWorks 11* supports Workplace and Apprenticeship Mathematics Grade 11 by incorporating assessment for learning, assessment as learning, and assessment of learning.

Assessment for learning

Teachers use assessment for learning to uncover what students believe to be true and to learn more about the connections students are making and their prior knowledge, preconceptions, knowledge gaps, and learning styles. In this textbook, assessment for learning is addressed through:

- ongoing dialogue that allows the student to reflect on his or her work and the teacher to uncover the student's mathematics misconceptions;
- group discussions of math from prior grades as well as the new concepts, which enable the teacher to gauge a student's prior knowledge of the topic and decide how much review is necessary;
- group discussions of applications of mathematics to real-world examples, which enable students to compare the processes they would use to answer the question and see that there are multiple ways to solve a problem. This sharing allows students to clarify confusions they may have about the mathematics.

Assessment as learning

Assessment as learning is an active process of cognitive restructuring that occurs when individuals interact with new ideas. For students to be actively engaged in creating their own understanding, they must become adept at personally monitoring what they are learning, and they must use what they discover from the monitoring to make adjustments, adaptations, and even major changes in their thinking. In this textbook, assessment as learning is addressed through:

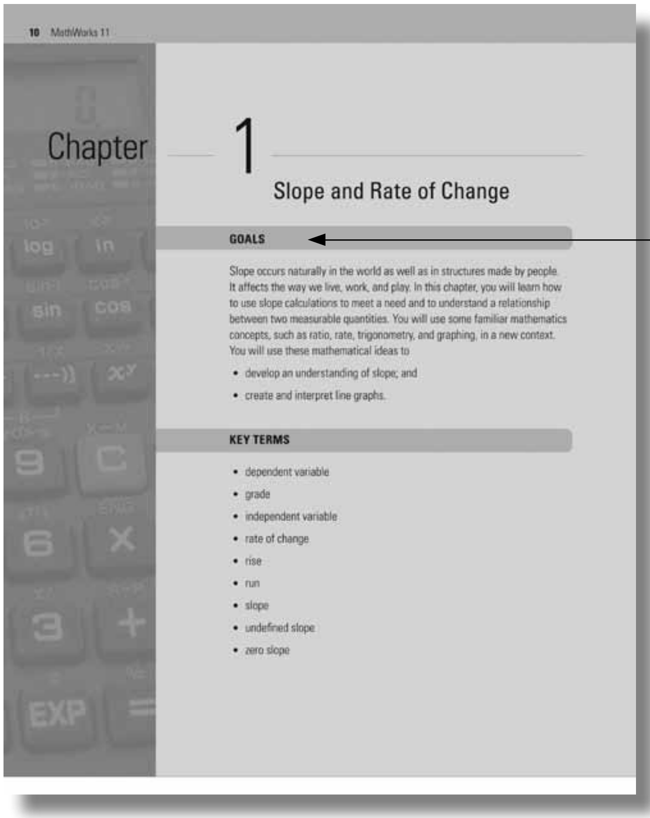
- student reflection on the information they gather and the decisions they make to complete activities and projects;
- hands-on activities and projects that allow students to learn through discovery, see patterns, make connections, draw conclusions, and make predictions;
- hands-on activities and projects that require students to work with mathematics in a non-algebraic format that challenges their preconceived notions of mathematics, helping them to discover a new way of conceptualizing math;
- puzzles with multiple possible solutions that encourage students to try to find a solution in any manner that suits their needs;
- detailed worked examples that allow students to see a step-by-step algebraic process to solve a problem;
- review and practice questions with an answer key, so students can gauge their progress.

Assessment of learning

Assessment of learning includes strategies designed to confirm what students know, demonstrate whether or not they have met curriculum outcomes or the goals of their individualized programs, or certify proficiency and make decisions about students' future programs or placements. It is designed to provide evidence of achievement to parents, other educators, the students themselves, and sometimes to outside groups (such as other educational institutions). In this textbook, assessment of learning is addressed through:

- project presentations that give students the opportunity to demonstrate their understanding of the math concepts using visuals, technology, and written or oral reports;
- chapter tests that give students the opportunity to demonstrate their mathematical understanding in written form.

HOW TO USE THE STUDENT RESOURCE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Chapter 1

Slope and Rate of Change

GOALS

Slope occurs naturally in the world as well as in structures made by people. It affects the way we live, work, and play. In this chapter, you will learn how to use slope calculations to meet a need and to understand a relationship between two measurable quantities. You will use some familiar mathematics concepts, such as ratio, rate, trigonometry, and graphing, in a new context. You will use these mathematical ideas to

- develop an understanding of slope; and
- create and interpret line graphs.

KEY TERMS

- dependent variable
- grade
- independent variable
- rate of change
- rise
- run
- slope
- undefined slope
- zero slope

Introduction

Each chapter begins with an introduction to the mathematical concepts addressed in the chapter and their relevance to the workplace, the learning outcomes, and the key mathematical terms students will encounter.


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PROJECT—DESIGN A TERRAIN PARK

START TO PLAN

PROJECT OVERVIEW

Your team has decided to bid on a contract to design a terrain park for a new ski resort. The resort owners want to build a park that is challenging for all levels of skiers and snowboarders. As part of your bid for the contract, your team will present a design for the terrain park that will include three model jumps designed for beginners, intermediates, and experts.

GET STARTED

Your first step is to research terrain parks and plan the features of your park. You can find ideas on the internet, in magazines, or by asking classmates who have visited terrain parks. Make a list of all the things you will need to consider. You may ask yourself:

- How can the jumps be designed for different skill levels?
- How will you test your jumps and measure the results?
- In addition to the jumps, what other terrain park features might you include in your design (rails, quarter pipes, etc.)?

In a terrain park, snowboarders can practise moves like basic jumps, frontside turns, or ollies.

SKETCH YOUR DESIGNS

Once you have decided on the design of your terrain park, make some sketches of your jumps. Later, you will use your sketches to build models of your jumps that you will test to determine whether they meet your design criteria.

- Sketch your designs for a beginner, an intermediate, and an expert jump.
- How will you modify your jumps to create three different levels of difficulty?
- How will you construct your models? What materials could you use?

FINAL PRESENTATION CHECKLIST

When you have completed this project, you will make a presentation that includes the following:

- a sales pitch to convince the resort to award the contract to your team;
- models of a beginner, an intermediate, and an expert jump;
- scale drawings of your three model jumps labelled with the dimensions and showing the slope as a ratio, an angle, and a percent.

Chapter Project

Each chapter contains a project in which students apply the mathematical concepts in a real-world scenario. The project provides students with opportunities to reflect on their learning and draw connections between the mathematical ideas and tools they encounter and real-world applications.

Students will return intermittently to the project as they work through the chapter and will complete a culminating activity at the end that allows them to synthesize the various mathematical concepts they have learned to use.

Math on the Job

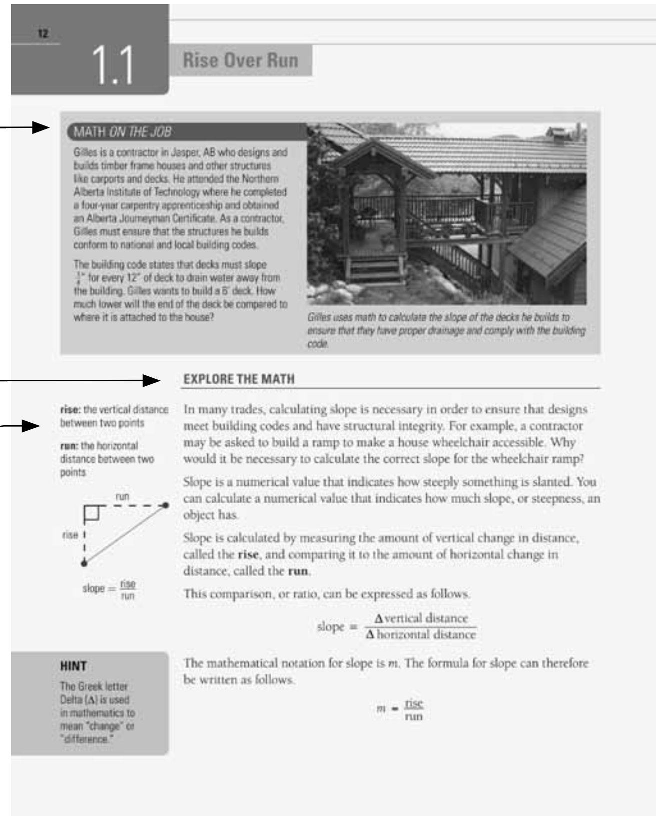
Each numbered section within the chapter begins with a Math on the Job scenario that briefly describes a job or workplace and specifically mentions the ways in which mathematics is used in that job. The scenario concludes with problems to be solved as a class, guided by the teacher.

Explore the Math

The lessons are called Explore the Math and contain a brief explanation of the mathematical ideas being considered and real-world contexts in which the math is applied.

Definitions

Definitions of mathematical terms relevant to the lesson are provided. Definitions are also included in the end-of-book glossary.



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1.1 Rise Over Run

MATH ON THE JOB

Gilles is a contractor in Jasper, AB who designs and builds timber frame houses and other structures like carports and decks. He attended the Northern Alberta Institute of Technology where he completed a four-year carpentry apprenticeship and obtained an Alberta Journeyman Certificate. As a contractor, Gilles must ensure that the structures he builds conform to national and local building codes.

The building code states that decks must slope $\frac{1}{4}$" for every 12" of deck to drain water away from the building. Gilles wants to build a 6' deck. How much lower will the end of the deck be compared to where it is attached to the house?

Gilles uses math to calculate the slope of the decks he builds to ensure that they have proper drainage and comply with the building code.

EXPLORE THE MATH

rise: the vertical distance between two points

run: the horizontal distance between two points

run

rise

slope = $\frac{\text{rise}}{\text{run}}$

In many trades, calculating slope is necessary in order to ensure that designs meet building codes and have structural integrity. For example, a contractor may be asked to build a ramp to make a house wheelchair accessible. Why would it be necessary to calculate the correct slope for the wheelchair ramp?

Slope is a numerical value that indicates how steeply something is slanted. You can calculate a numerical value that indicates how much slope, or steepness, an object has.

Slope is calculated by measuring the amount of vertical change in distance, called the **rise**, and comparing it to the amount of horizontal change in distance, called the **run**.

This comparison, or ratio, can be expressed as follows.

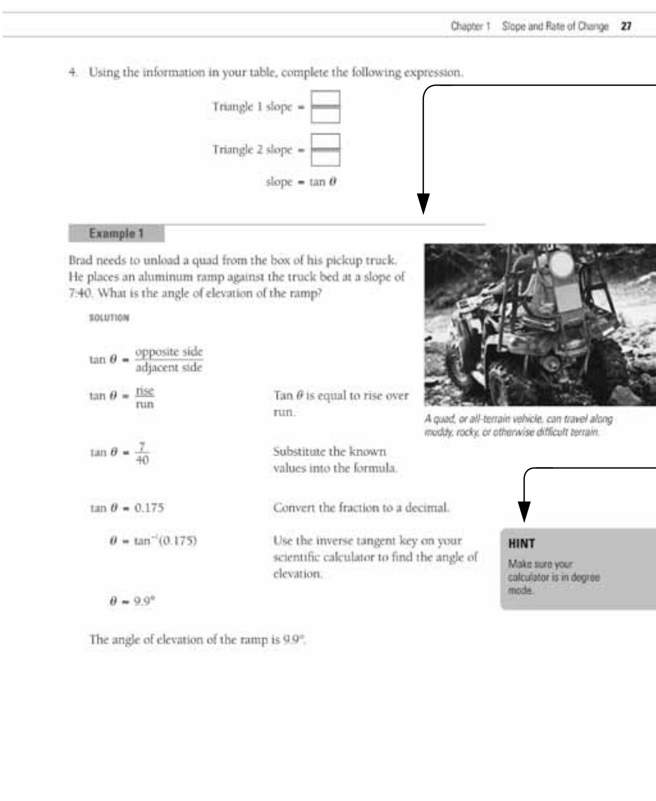
$$\text{slope} = \frac{\Delta\,\text{vertical distance}}{\Delta\,\text{horizontal distance}}$$

HINT

The Greek letter Delta (Δ) is used in mathematics to mean "change" or "difference."

The mathematical notation for slope is *m*. The formula for slope can therefore be written as follows.

$$m = \frac{\text{rise}}{\text{run}}$$



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4. Using the information in your table, complete the following expression.

$$\text{Triangle 1 slope} = \frac{\square}{\square}$$

$$\text{Triangle 2 slope} = \frac{\square}{\square}$$

$$\text{slope} = \tan\theta$$

Example 1

Brad needs to unload a quad from the box of his pickup truck. He places an aluminum ramp against the truck bed at a slope of 7:40. What is the angle of elevation of the ramp?

A quad, or all-terrain vehicle, can travel along muddy, rocky, or otherwise difficult terrain.

SOLUTION

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{\text{rise}}{\text{run}}$ — Tan θ is equal to rise over run.

$\tan\theta = \frac{7}{40}$ — Substitute the known values into the formula.

$\tan\theta = 0.175$ — Convert the fraction to a decimal.

$\theta = \tan^{-1}(0.175)$ — Use the inverse tangent key on your scientific calculator to find the angle of elevation.

$\theta = 9.9°$

The angle of elevation of the ramp is 9.9°.

HINT

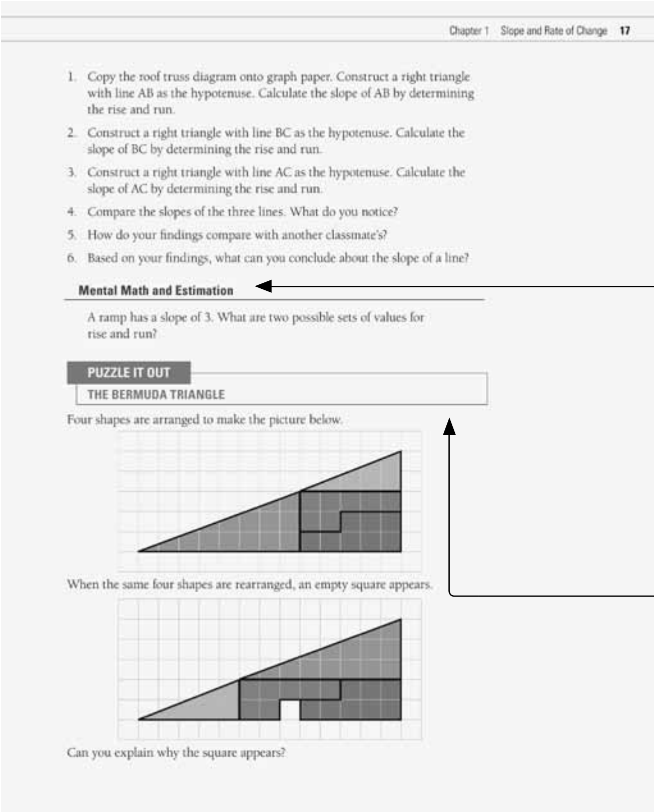
Make sure your calculator is in degree mode.

Examples

Each lesson includes one to four worked examples that model problem-solving strategies and techniques for students. Where appropriate, the worked examples include alternative solutions.

Hints

In some sections, hints are provided to help activate students' prior knowledge, remind them of concepts addressed in the chapter, or encourage reflection.



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1. Copy the roof truss diagram onto graph paper. Construct a right triangle with line AB as the hypotenuse. Calculate the slope of AB by determining the rise and run.
2. Construct a right triangle with line BC as the hypotenuse. Calculate the slope of BC by determining the rise and run.
3. Construct a right triangle with line AC as the hypotenuse. Calculate the slope of AC by determining the rise and run.
4. Compare the slopes of the three lines. What do you notice?
5. How do your findings compare with another classmate's?
6. Based on your findings, what can you conclude about the slope of a line?

Mental Math and Estimation

A ramp has a slope of 3. What are two possible sets of values for rise and run?

PUZZLE IT OUT

THE BERMUDA TRIANGLE

Four shapes are arranged to make the picture below.

When the same four shapes are rearranged, an empty square appears.

Can you explain why the square appears?

Mental Math and Estimation

Mental math problems are realistic situations in which estimation or mental math is required to arrive at a solution.

Puzzle It Out

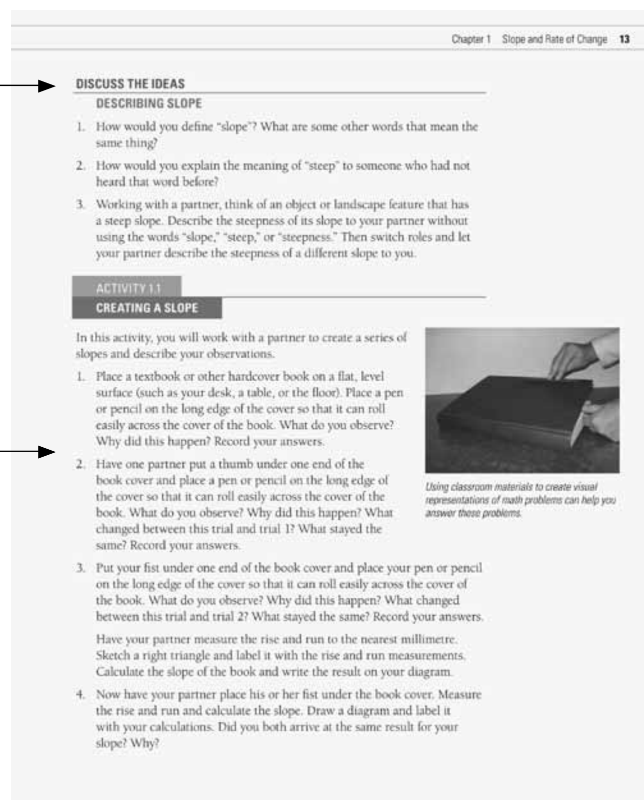
Each chapter contains a puzzle or game that incorporaates mathematical ideas and offers a light-hearted approach to mathematical strategy.

Discuss the Ideas

Once students have some familiarity with the material, they are presented with a contextual problem to consider and solve. Students can work on these in pairs or small groups or the teacher can lead a brief class discussion.

Activities

Each chapter contains several hands-on activities that provide opportunities for students to work collaboratively and apply their learning in a realistic context.



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DISCUSS THE IDEAS

DESCRIBING SLOPE

1. How would you define "slope"? What are some other words that mean the same thing?
2. How would you explain the meaning of "steep" to someone who had not heard that word before?
3. Working with a partner, think of an object or landscape feature that has a steep slope. Describe the steepness of its slope to your partner without using the words "slope," "steep," or "steepness." Then switch roles and let your partner describe the steepness of a different slope to you.

ACTIVITY 1.1

CREATING A SLOPE

In this activity, you will work with a partner to create a series of slopes and describe your observations.

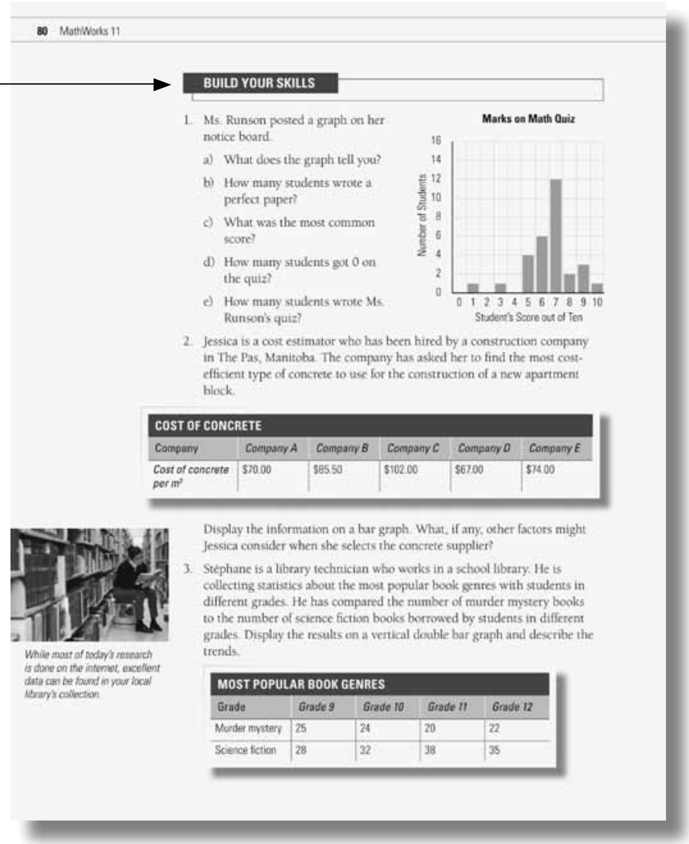
1. Place a textbook or other hardcover book on a flat, level surface (such as your desk, a table, or the floor). Place a pen or pencil on the long edge of the cover so that it can roll easily across the cover of the book. What do you observe? Why did this happen? Record your answers.
2. Have one partner put a thumb under one end of the book cover and place a pen or pencil on the long edge of the cover so that it can roll easily across the cover of the book. What do you observe? Why did this happen? What changed between this trial and trial 1? What stayed the same? Record your answers.
3. Put your fist under one end of the book cover and place your pen or pencil on the long edge of the cover so that it can roll easily across the cover of the book. What do you observe? Why did this happen? What changed between this trial and trial 2? What stayed the same? Record your answers.

   Have your partner measure the rise and run to the nearest millimetre. Sketch a right triangle and label it with the rise and run measurements. Calculate the slope of the book and write the result on your diagram.
4. Now have your partner place his or her fist under the book cover. Measure the rise and run and calculate the slope. Draw a diagram and label it with your calculations. Did you both arrive at the same result for your slope? Why?

Using classroom materials to create visual representations of math problems can help you answer these problems.

Build Your Skills

The practice problems in each chapter enable students to build their skills and gain confidence in their ability to strategize solutions. These problems can be used flexibly: they can be assigned as homework, completed in the classroom, or solved by pairs or small groups of students working collaboratively. Answers are included at the back of the student book, providing an opportunity for self-assessment.



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BUILD YOUR SKILLS

1. Ms. Runson posted a graph on her notice board.
   a) What does the graph tell you?
   b) How many students wrote a perfect paper?
   c) What was the most common score?
   d) How many students got 0 on the quiz?
   e) How many students wrote Ms. Runson's quiz?

2. Jessica is a cost estimator who has been hired by a construction company in The Pas, Manitoba. The company has asked her to find the most cost-efficient type of concrete to use for the construction of a new apartment block.

COST OF CONCRETE

Company	Company A	Company B	Company C	Company D	Company E
Cost of concrete per m³	$70.00	$85.50	$102.00	$67.00	$74.00

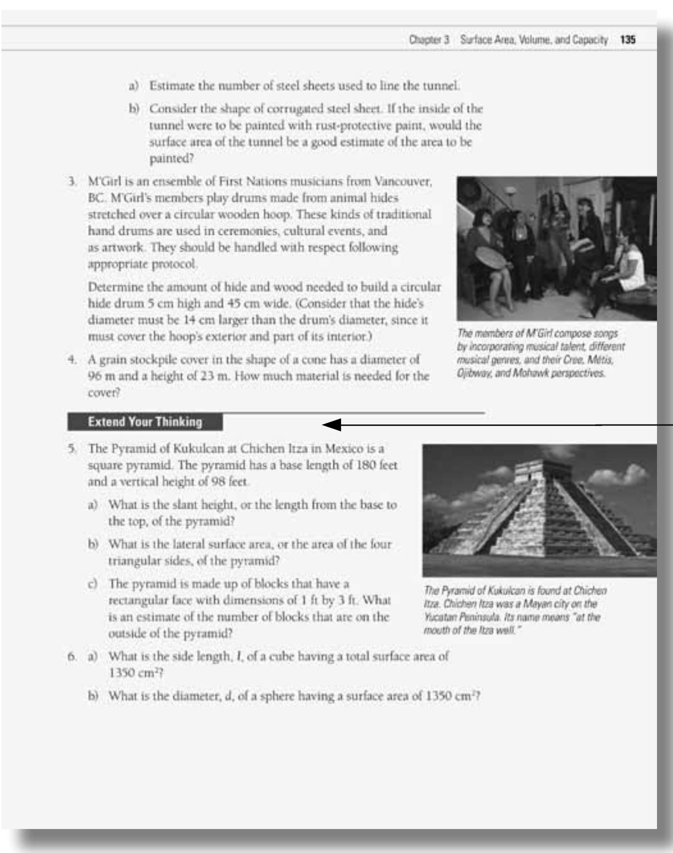
Display the information on a bar graph. What, if any, other factors might Jessica consider when she selects the concrete supplier?

While most of today's research is done on the internet, excellent data can be found in your local library's collection.

3. Stéphane is a library technician who works in a school library. He is collecting statistics about the most popular book genres with students in different grades. He has compared the number of murder mystery books to the number of science fiction books borrowed by students in different grades. Display the results on a vertical double bar graph and describe the trends.

MOST POPULAR BOOK GENRES

Grade	Grade 9	Grade 10	Grade 11	Grade 12
Murder mystery	25	24	20	22
Science fiction	28	32	38	35



Chapter 3 Surface Area, Volume, and Capacity 135

a) Estimate the number of steel sheets used to line the tunnel.
b) Consider the shape of corrugated steel sheet. If the inside of the tunnel were to be painted with rust-protective paint, would the surface area of the tunnel be a good estimate of the area to be painted?

3. M'Girl is an ensemble of First Nations musicians from Vancouver, BC. M'Girl's members play drums made from animal hides stretched over a circular wooden hoop. These kinds of traditional hand drums are used in ceremonies, cultural events, and as artwork. They should be handled with respect following appropriate protocol.

   Determine the amount of hide and wood needed to build a circular hide drum 5 cm high and 45 cm wide. (Consider that the hide's diameter must be 14 cm larger than the drum's diameter, since it must cover the hoop's exterior and part of its interior.)

The members of M'Girl compose songs by incorporating musical talent, different musical genres, and their Cree, Métis, Ojibway, and Mohawk perspectives.

4. A grain stockpile cover in the shape of a cone has a diameter of 96 m and a height of 23 m. How much material is needed for the cover?

Extend Your Thinking

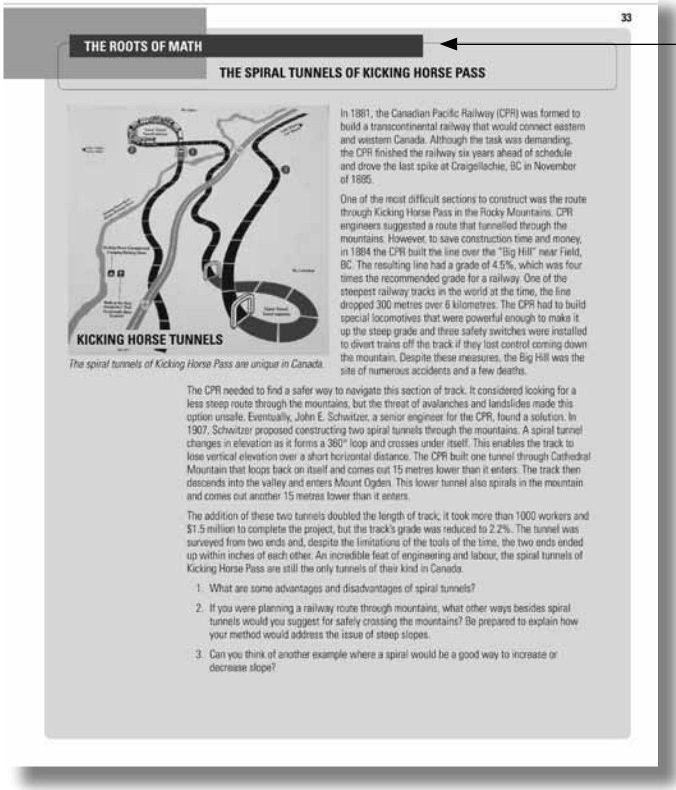
5. The Pyramid of Kukulcan at Chichen Itza in Mexico is a square pyramid. The pyramid has a base length of 180 feet and a vertical height of 98 feet.
   a) What is the slant height, or the length from the base to the top, of the pyramid?
   b) What is the lateral surface area, or the area of the four triangular sides, of the pyramid?
   c) The pyramid is made up of blocks that have a rectangular face with dimensions of 1 ft by 3 ft. What is an estimate of the number of blocks that are on the outside of the pyramid?

The Pyramid of Kukulcan is found at Chichen Itza. Chichen Itza was a Mayan city on the Yucatan Peninsula. Its name means "at the mouth of the Itza well."

6. a) What is the side length, l, of a cube having a total surface area of 1350 cm²?
   b) What is the diameter, d, of a sphere having a surface area of 1350 cm²?

Extend Your Thinking

Extension questions are in-depth problems students solve once they have completed the Build Your Skills questions.



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THE ROOTS OF MATH

THE SPIRAL TUNNELS OF KICKING HORSE PASS

KICKING HORSE TUNNELS

The spiral tunnels of Kicking Horse Pass are unique in Canada.

In 1881, the Canadian Pacific Railway (CPR) was formed to build a transcontinental railway that would connect eastern and western Canada. Although the task was demanding, the CPR finished the railway six years ahead of schedule and drove the last spike at Craigellachie, BC in November of 1885.

One of the most difficult sections to construct was the route through Kicking Horse Pass in the Rocky Mountains. CPR engineers suggested a route that tunnelled through the mountains. However, to save construction time and money, in 1884 the CPR built the line over the "Big Hill" near Field, BC. The resulting line had a grade of 4.5%, which was four times the recommended grade for a railway. One of the steepest railway tracks in the world at the time, the line dropped 300 metres over 6 kilometres. The CPR had to build special locomotives that were powerful enough to make it up the steep grade and three safety switches were installed to divert trains off the track if they lost control coming down the mountain. Despite these measures, the Big Hill was the site of numerous accidents and a few deaths.

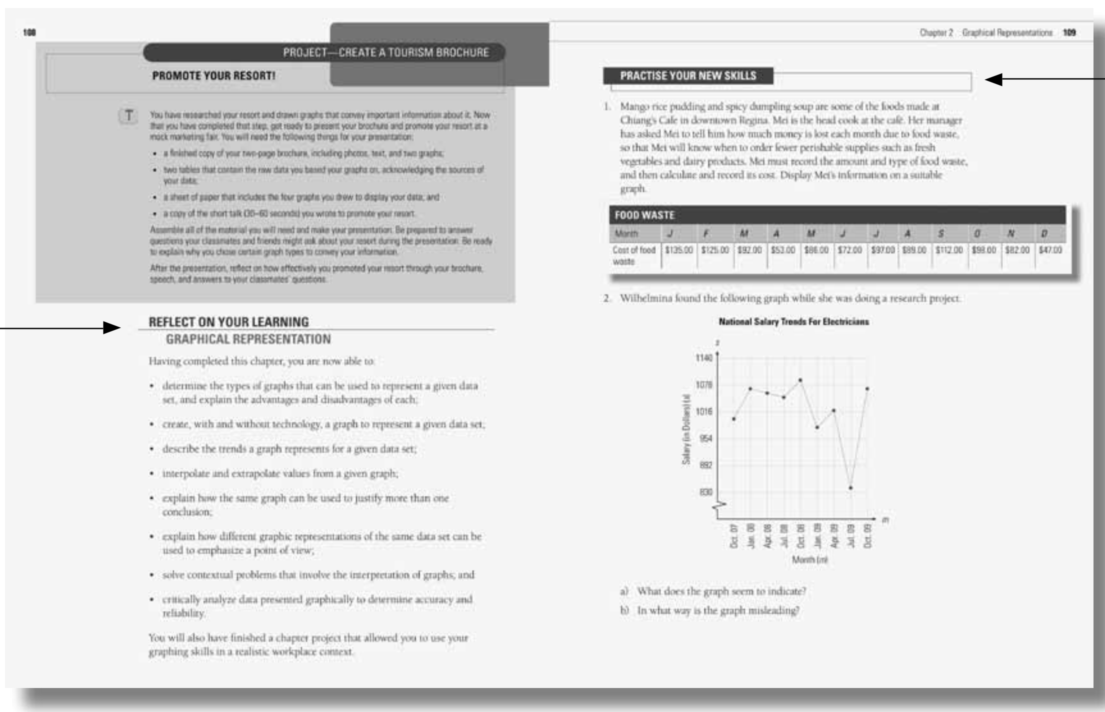
The CPR needed to find a safer way to navigate this section of track. It considered looking for a less steep route through the mountains, but the threat of avalanches and landslides made this option unsafe. Eventually, John E. Schwitzer, a senior engineer for the CPR, found a solution. In 1907, Schwitzer proposed constructing two spiral tunnels through the mountains. A spiral tunnel changes in elevation as it forms a 360° loop and crosses under itself. This enables the track to lose vertical elevation over a short horizontal distance. The CPR built one tunnel through Cathedral Mountain that loops back on itself and comes out 15 metres lower than it enters. The track then descends into the valley and enters Mount Ogden. This lower tunnel also spirals in the mountain and comes out another 15 metres lower than it enters.

The addition of these two tunnels doubled the length of track; it took more than 1000 workers and $1.5 million to complete the project, but the track's grade was reduced to 2.2%. The tunnel was surveyed from two ends and, despite the limitations of the tools of the time, the two ends ended up within inches of each other. An incredible feat of engineering and labour, the spiral tunnels of Kicking Horse Pass are still the only tunnels of their kind in Canada.

1. What are some advantages and disadvantages of spiral tunnels?
2. If you were planning a railway route through mountains, what other ways besides spiral tunnels would you suggest for safely crossing the mountains? Be prepared to explain how your method would address the issue of steep slopes.
3. Can you think of another example where a spiral would be a good way to increase or decrease slope?

The Roots of Math

Students are introduced to the history of mathematics through this short essay on a topic related to the chapter's focus. Where appropriate, Canadian history is emphasized.



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PROJECT—CREATE A TOURISM BROCHURE

PROMOTE YOUR RESORT!

T You have researched your resort and drawn graphs that convey important information about it. Now that you have completed that step, get ready to present your brochure and promote your resort at a mock marketing fair. You will need the following things for your presentation:

- a finished copy of your two-page brochure, including photos, text, and two graphs;
- two tables that contain the raw data you based your graphs on, acknowledging the sources of your data;
- a sheet of paper that includes the four graphs you drew to display your data; and
- a copy of the short talk (30–60 seconds) you wrote to promote your resort.

Assemble all of the material you will need and make your presentation. Be prepared to answer questions your classmates and friends might ask about your resort during the presentation. Be ready to explain why you chose certain graph types to convey your information.

After the presentation, reflect on how effectively you promoted your resort through your brochure, speech, and answers to your classmates' questions.

REFLECT ON YOUR LEARNING

GRAPHICAL REPRESENTATION

Having completed this chapter, you are now able to:

- determine the types of graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each;
- create, with and without technology, a graph to represent a given data set;
- describe the trends a graph represents for a given data set;
- interpolate and extrapolate values from a given graph;
- explain how the same graph can be used to justify more than one conclusion;
- explain how different graphic representations of the same data set can be used to emphasize a point of view;
- solve contextual problems that involve the interpretation of graphs; and
- critically analyze data presented graphically to determine accuracy and reliability.

You will also have finished a chapter project that allowed you to use your graphing skills in a realistic workplace context.

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PRACTISE YOUR NEW SKILLS

1. Mango rice pudding and spicy dumpling soup are some of the foods made at Chiang's Cafe in downtown Regina. Mei is the head cook at the café. Her manager has asked Mei to tell him how much money is lost each month due to food waste, so that Mei will know when to order fewer perishable supplies such as fresh vegetables and dairy products. Mei must record the amount and type of food waste, and then calculate and record its cost. Display Mei's information on a suitable graph.

FOOD WASTE

Month	J	F	M	A	M	J	J	A	S	O	N	D
Cost of food waste	$135.00	$125.00	$92.00	$53.00	$86.00	$72.00	$97.00	$89.00	$112.00	$98.00	$82.00	$47.00

2. Wilhelmina found the following graph while she was doing a research project.

National Salary Trends For Electricians

1140, 1078, 1016, 954, 892, 830

Salary (in Dollars) (s)

Oct. 07, Jan. 08, Apr. 08, Jul. 08, Oct. 08, Jan. 09, Apr. 09, Jul. 09, Oct. 09

Month (m)

a) What does the graph seem to indicate?
b) In what way is the graph misleading?

Reflect on Your Learning

Each chapter concludes with a summary of the concepts learned in the chapter.

Practise Your New Skills

Students complete the chapter by working through a series of problems to review and synthesize their learning.

TIME ALLOTMENT

MathWorks 11 is structured on the assumption that teachers have 90 instructional hours available. The following chart shows the estimated instructional time for each chapter, expressed as a percentage of total instructional time.

MATHWORKS 11 TIME ALLOTMENT

Chapter	*% Time*
1. Slope and Rate of Change	10%
2. Graphical Representations	20%
3. Surface Area, Volume, and Capacity	15%
4. Trigonometry of Right Triangles	10%
5. Scale Models and Representations	15%
6. Financial Services	15%
7. Personal Budgets	15%

HOW TO USE THE TEACHER RESOURCE

This teacher resource is a comprehensive resource for both new and experienced teachers. It outlines and discusses the repertoire of instructional and assessment strategies that may be used and identifies the features of the student book and their underlying rationale.

For each chapter in the student book, the teacher resource contains the following elements.

INTRODUCTION

The chapter introduction locates the chapter within the Workplace and Apprenticeship Mathematics 11 curriculum and maps it to the general and specific outcomes addressed in the chapter.

THE MATHEMATICAL IDEAS

In this section, the "big ideas" of the chapter are described, with examples. This provides some mathematical background for teachers, if needed, and explains the chapter's mathematical focus. The workplace relevance of the mathematical concepts is summarized under the heading Why Are These Concepts Important?

The section concludes with a list of the prior skills and knowledge that students are expected to bring to the chapter. You may choose to review these concepts with your students, depending on individual classroom needs.

PLANNING FOR INSTRUCTION AND ASSESSMENT

Rubrics have been provided to assist you in allocating class time, preparing materials, and designing your assessment strategy.

T The technology icon alerts you to activities in which the students may benefit from using technology such as the internet, scientific calculators, or presentation software.

CHAPTER PROJECT

A detailed description of the chapter project provides information on the project's goals, outcome, prerequisites, and activities. This overview will assist you to plan class time for project work during the course of the chapter. The teaching suggestions will assist you with integrating the mathematical concepts into the project. A project assessment rubric is provided, as well as a student self-assessment rubric.

Each chapter includes an alternative chapter project with Blackline Masters and a project assessment rubric, to accommodate different class interests and learning styles and to provide variety from year to year.

CHAPTER SUBSECTIONS

For each chapter subsection, the teacher resource follows the format of the student book.

Worked solutions have been provided for all questions, including alternative methods of arriving at solutions and, in some cases, extension activities for students ready for more in-depth work.

Teaching notes include alternative teaching strategies. For example, features such as Math on the Job and Discuss the Ideas can be used as discussion starters in the classroom, and the teacher resource contains numerous suggestions for connecting students' work and life experiences to the mathematical concepts.

The hands-on chapter activities allow for a range of teaching and learning strategies to be used to meet the needs of students with varying interests, backgrounds, and aptitudes.

PUZZLE IT OUT

Puzzles and games provide ample opportunities for students to demonstrate mathematical reasoning and to apply new skills in an engaging way. Many more spatial puzzles and games are available online, including on the website of the National Library of Virtual Manipulatives. Use the following key word searches: virtual math games, interactive math games, math puzzles, spatial puzzles, spatial games, spatial math games, and virtual math games.

ADDITIONAL MATERIALS

Each chapter concludes with a sample chapter test and worked solutions, graphic organizers, and other Blackline Masters for the chapter project and activities, and the alternative chapter project teacher and student materials with Blackline Masters and an assessment rubric.

Chapter 1

Slope and Rate of Change

INTRODUCTION

STUDENT BOOK, pp. 10–53

This first chapter in MathWorks 11 addresses the outcomes of developing statistical and algebraic reasoning for Apprenticeship and Workplace Mathematics 11. In this chapter, students will build on their knowledge of rate and ratio to explore different contexts where the mathematical concept of slope is expressed as rise over run, rate of change, percent grade, and angle of elevation. The chart below locates this chapter within the curriculum.

ALGEBRA, GRADES 10–12

This chart illustrates the development of the Algebra and Statistics strands in the Apprenticeship and Workplace Mathematics pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 1 addresses.

Grade 10	*Grade 11*	*Grade 12*
General Outcome Develop algebraic reasoning.	General Outcome Develop algebraic reasoning.	General Outcome Develop algebraic reasoning.
Specific Outcome It is expected that students will:	Specific Outcome It is expected that students will:	Specific Outcome It is expected that students will:
Solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.	Solve problems that require the manipulation and application of formulas related to slope and rate of change.	Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.
	Demonstrate an understanding of slope as rise over run and rate of change by solving problems.	
STATISTICS, GRADE 11		
	General Outcome Develop statistical reasoning.	
	Specific Outcome It is expected that students will:	
	Solve problems that involve creating and interpreting line graphs.	

CURRICULUM AND CHAPTER OVERVIEW

General outcome: Algebra
Develop algebraic reasoning.

General outcome: Statistics
Develop statistical reasoning.

Specific outcome:
Solve problems that require the manipulation and application of formulas related to slope and rate of change.

Specific outcome:
Demonstrate an understanding of slope as rise over run and rate of change by solving problems.

Specific outcome:
Solve problems that involve creating and interpreting line graphs.

Section 1.1:
Rise Over Run

- Math on the Job
- Explore the Math
- Discuss the Ideas: Describing Slope
- Activity 1.1: Creating a Slope
- Activity 1.2: A Property of Slope
- Mental Math and Estimation
- Puzzle It Out: The Bermuda Triangle
- Activity 1.3: Analyzing Slopes
- Discuss the Ideas: The Bugaboo Spire
- Build Your Skills

Section 1.2:
Grade, Angle of Elevation, and Distance

- Math on the Job
- Explore the Math
- Activity 1.4: Slope and Tangent
- Mental Math and Estimation
- Activity 1.5: Length of a Particular Slope
- Build Your Skills
- The Roots of Math: The Spiral Tunnels of Kicking Horse Pass

Section 1.3:
Rate of Change

- Math on the Job
- Explore the Math
- Activity 1.6: Climbing and Calculating
- Activity 1.7: Slope Direction
- Discuss the Ideas: Connections and Conversions
- Extend Your Learning
- Activity 1.8: Using Coordinates to Calculate Slope
- Discuss the Ideas: The Order of Coordinates
- Build Your Skills

Chapter Project:
Design a Terrain Park

- Reflect on Your Learning
- Practise Your New Skills

THE MATHEMATICAL IDEAS

SLOPE AS A RATIO

There are many instances in which two quantities are related and compared to each other. A relationship exists between them, and we can explore the changing relationship between them. Slope is a graphical way to represent and illustrate the relationship, or ratio, between the amount each quantity changes with respect to the other. It is used to measure change in physical objects as well as in measurable concepts.

This chapter includes a hands-on project in which students design and model a ski resort terrain park, which gives them the opportunity to apply the theoretical concepts as well as formulate and test hypotheses. The project is divided into activities that are carried out after the relevant theory has been taught. This ensures a contextual progression.

Each section starts with Math on the Job, which exposes students to workplace applications of the mathematical concepts addressed in the section. In addition, students work through a math problem typical of the job described. Students then work through activities that allow them to investigate the concepts and guide them to make connections themselves. Once students understand the theory, they can apply it by working through the examples. Mental Math and Estimation problems encourage students to make a quick calculation to reinforce the concepts. Discuss the Ideas gives students the opportunity to examine and discuss the concept in more depth as a class or in small groups.

The chapter begins with the concept of rise over run. Students discover that a slope value is the ratio of the change in the vertical over the change in the horizontal. With this basic understanding, students can calculate the slope of anything that is represented in the vertical and horizontal planes. As students progress through the sections, they should begin to make the connection that rise is the same as vertical change. Vertical change can also be expressed as the difference between the *y* coordinates, which is also the dependent variable in a graph. Furthermore, rise corresponds to the opposite side of a right triangle in the tangent ratio of the angle of elevation. The same holds true for the concept of run: the horizontal change is the difference in *x* coordinates, which is also the independent variable in a graph. Run also corresponds to the adjacent side of the right triangle in the tangent ratio of the angle of elevation.

A further connection that students will make as they progress through the sections is that the slope ratio of rise over run forms a right triangle. Students will activate their prior knowledge of trigonometry and realize that rise over run is equivalent to opposite side over adjacent side of the tangent ratio of the angle of elevation. They can also develop the formula for calculating the distance of a particular slope using their knowledge of the Pythagorean theorem.

The basic slope concepts are developed further in section 1.3, where the students apply what they have learned to solving rate of change problems.

WHY ARE THESE CONCEPTS IMPORTANT?

- It is important that students understand that slope is a way to measure the rate of change between two quantities. They can use this to calculate rates of change in their daily lives. For example, the amount of money they will earn depends on the number of hours they work.
- An understanding of slope and how to measure it is essential in many trades. For example, carpenters and plumbers must be able to create slopes that meet safety and functionality criteria.

PRIOR SKILLS AND KNOWLEDGE

Student work in this chapter will build on certain WNCP Common Curriculum Framework outcomes from earlier grades.

1. Concepts
   a) ratios, rates, proportional reasoning;
   b) interpolation, extrapolation;
   c) trigonometry;
   d) graphing;
   e) Pythagorean theorem;
   f) percent;
   g) calculating averages.

2. Mathematics Skills
   a) proportional reasoning;
   b) solving algebraic equations;
   c) identifying equivalent ratios;
   d) working with fractions, decimals, and percents.

T 3. Technology
   a) basic calculator functions;
   b) scientific calculator functions (tan and $\tan^{-1}$);
   c) internet research;
   d) presentation software.

REVIEWING PRIOR CONCEPTS

Some students may benefit from reviewing concepts that have been covered in prior years. You may want to give some students specific review exercises in the following concepts and processes.

Explore the Math 1.1 (Rise Over Run)

- expressing a numerical value as a fraction, ratio, or decimal, and converting between them;
- describing and drawing right triangles;
- using algebra to solve for one unknown variable; and
- proportional reasoning.

Explore the Math 1.2 (Grade, Angle of Elevation, and Distance)

- percent;
- the tangent ratio in trigonometry;
- working with squares and square roots;
- working with angles; and
- using a scientific calculator.

Explore the Math 1.3 (Rate of Change)

- creating and interpreting graphs;
- working with coordinates and the coordinate planes;
- working with formulas and equations; and
- creating data tables.

Blackline Master 1.9 contains review questions and solutions. It is found at the end of this chapter of the teacher resource (pp. 80–83).

PLANNING CHAPTER 1

This chapter will take three weeks of class time to complete. Class period estimates are based on a class length ranging from 60 to 75 minutes. These estimates may vary depending on individual classroom needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	11	Introduce the Chapter Project: "Design a Terrain Park"	20 minutes	internet, magazines Blackline Master 1.1 (p. 65), 1.4 (p. 68)
1.1	12 12 13	Math on the Job: Contractor Explore the Math Discuss the Ideas: Describing Slope	40 minutes	
1.1	13	Activity 1.1: Creating a Slope	15 minutes	For each pair of students: hardcover textbook, round pen or pencil that can roll easily
1.1	14	Examples 1, 2	10 minutes	
1.1	16 17	Activity 1.2: A Property of Slope Mental Math and Estimation	15 minutes	graph paper, ruler
1.1	17 18	Puzzle It Out: The Bermuda Triangle Activity 1.3: Analyzing Slopes	40 minutes	For each group of students: 1 cm graph paper (Blackline Master 1.3, p. 67), 30 cm ruler, 10 textbooks or a substitute, 3 different-coloured pens
1.1	19	Discuss the Ideas: The Bugaboo Spire	5 minutes	
1.1	19	Build Your Skills You may want to prepare a quiz for section 1.1.	1 class	
	24	Chapter Project: Test your Designs and Gather Data	1–2 classes	cardboard, scissors, glue, tape, rulers, marbles Blackline Master 1.2 (p. 66)
1.2	25 25	Math on the Job: Building Officer Explore the Math	30 minutes	
1.2	26	Activity 1.4: Slope and Tangent	10 minutes	Blackline Master 1.5 (p. 69)
1.2	27 28	Examples 1, 2 Mental Math and Estimation	10 minutes	
1.2	29	Activity 1.5: Length of a Particular Slope	10 minutes	Blackline Master 1.6 (p. 70)
1.2	29	Example 3	5 minutes	
1.2	30	Build Your Skills You may want to prepare a quiz for section 1.2.	1 class	
1.2	33	The Roots of Math: The Spiral Tunnels of Kicking Horse Pass	15 minutes	
	34	Chapter Project: Document your Design	30 minutes	

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PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
1.3	35 35	Math on the Job: Gardener Explore the Math	25 minutes	
1.3	37	Example 1	15 minutes	
1.3	39	Activity 1.6: Climbing and Calculating	20–30 minutes	graph paper, ruler, stopwatch, Blackline Master 1.7 (p. 71)
1.3	40	Activity 1.7: Slope Direction	30 minutes	graph paper, ruler
1.3	42	Discuss the Ideas: Connections and Conversions	10 minutes	
1.3	43	Extend Your Learning Example 2	15 minutes	
1.3	44	Activity 1.8: Using Coordinates to Calculate Slope	20 minutes	Blackline Master 1.8 (p. 72)
1.3	46	Discuss the Ideas: The Order of Coordinates	10 minutes	
1.3	46	Build Your Skills You may want to prepare a quiz for section 1.3	1 class	
	51	Chapter Project: Make a Presentation	1 class	
	51 52	Reflect on Your Learning Practise Your New Skills	1 class	
		Chapter Test (p. 58 of this resource)	1 class	

PLANNING FOR ASSESSMENT

Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Chapter launch • Project discussions (ongoing) • Math on the Job scenarios • Explore the Math • Activities • Discuss the ideas • Mental Math and Estimation • Puzzle It Out: The Bermuda Triangle	• Monitor how much work the students have done on their project activities. • Observe how students participate during discussions. • Observe how students interact during activities done as a group, in pairs, or individually.
Assessment as Learning	• Reflection and practice • Build Your Skills problems • Prompt students' self-assessment • Review student work, provide feedback	• Check homework and provide feedback on questions. • Challenge students to understand the relationship between different versions of the formulas and make connections. • Encourage reflection.
Assessment of Learning	• Chapter review • Chapter Project: Design a Terrain Park • Assignments/homework • Quizzes • Chapter Test	• Have students present their final project to the class and allow students to give feedback to presenters. • Give short quizzes as the chapter progresses to provide as much feedback as possible. • Review assessment records and add unit results to ongoing records.
Learning Skills/ Mathematical Disposition	• Observe and record throughout the unit how students are working with new language and concepts.	• Keep a log or journal of observations to aid in reporting.

PROJECT—DESIGN A TERRAIN PARK

GOALS: In this project, students will apply the concepts of slope and rate of change to build skills and synthesize learning in this chapter.

OUTCOME: In this project, students will integrate the concepts of slope and rate of change into a real-world scenario. Students will design a terrain park and model slopes with different levels of difficulty, build and test their models, and communicate their findings by creating scale drawings and presenting their terrain park design to the class.

PREREQUISITES: To complete this project, students will need to understand how to calculate slope. They must also be able to think creatively and critically to solve problems within given parameters.

ABOUT THIS PROJECT: This project is divided into four parts. Initially, students brainstorm ideas for how to construct their terrain park. They create sketches of the features and estimate the dimensions so that they will have a set of plans to build from. Next, they create three-dimensional models based on their sketches and test the effects of rolling a marble down their slopes to simulate a skier or snowboarder. They will construct slopes for three different skill levels and record the distance that the marble travels off the jumps. In the third activity, students will draw scale diagrams of each of the ramps they constructed and calculate the values for each of the slopes. As a final activity, students will present their terrain park design to the class. The presentations can be set up around the room or be presented individually in front of the class. Allow 3–5 minutes for each presentation.

You may wish to offer your students options to designing a terrain park. Other related settings that will work include snowmobile parks, dirt bike or mountain bike jumps, and skateboarder parks. The activities in the terrain park project will work equally well in these other settings.

Students should be given a few class periods to work on this project. This will allow students time to ask questions or discuss particular aspects of the project and will also give the teacher an opportunity to observe the quality of work being done and provide guidance, if needed. This project may be completed by small groups or pairs. A self-assessment rubric, Blackline Master 1.4 (p. 68), should be given to students early in the project. It outlines the criteria for evaluation of their project and suggests some ways for students to reflect on their learning.

An alternative project, "Wheelchair Accessibility," is included on p. 73.

1. Start to plan

STUDENT BOOK, p. 11

Introduce the project to the students as you begin the chapter. You can begin with a class discussion on terrain parks and skateboard parks. Some students may have visited and used them and can share their experiences with the class. Students can then brainstorm the "Get Started" questions in the student textbook in small groups. Students can use the checklist included as Blackline Master 1.1 (p. 65).

Part of this brainstorming should include making sketches of possible designs. The sketches should give enough detail and dimensions for students to use as plans for constructing their models. They do not have to be elaborate, but they should be done before the beginning of the class set aside for building. Therefore, if they do not complete the sketches during this class, have students finish them for homework. Students should have plans for three skill levels: beginner, intermediate, and expert. For the final presentation, teams should prepare a sales pitch for their design plan. This can be done during spare time in the upcoming classes or completed for homework.

2. Test your designs and gather data

STUDENT BOOK, p. 24

In this part of the project, students will build models from the sketches they have made. They can use materials such as cardboard, Bristol board, tape, or glue to construct them. Other materials could be used as well, such as empty cardboard rolls from paper towels or anything that students can think of that will allow a marble to roll down and up. Once they have constructed their models, students can test them by rolling a marble down one slope and up and off the other. They can measure the distance that the marble "flies" off the jumps. The distances should have a distinguishable difference between the three skill levels. Students may need to modify their models to adjust the distance the marble travels. Encourage students to think of ways to adapt their designs if they need to. Students can increase the steepness by putting books under the models they have made, by trimming their models, or by cutting out new models. You can also challenge students to conceptualize different ways to modify their designs. They should test their models until they get three trial results that are relatively close to each other for each skill level. Have students use Blackline Master 1.2 (p. 66) to organize their information.

3. Document your design

STUDENT BOOK, p. 34

Once students have completed testing their models, they will create scale drawings that illustrate the shape of each ramp as well as show the calculations of each of the slopes. Graph paper is provided as Blackline Master 1.3 (p. 67). Students should also be made aware that if they have a space between the decline and incline slopes, this space has a slope of zero and should also be included in the drawings. The slopes should be expressed as a fraction, a decimal, an angle, and a percentage, and recorded on the drawings.

4. Make a presentation

STUDENT BOOK, p. 51

In the final activity of the project, students will present their terrain park proposal to the class as if they were presenting a bid to win a contract. They can also share their findings and experience of the project to the class, describe any problems they encountered, and explain the solutions they came up with. They can demonstrate one of their models to the class or they can have their models stationed around the room and have the class go around to view them. Students may use presentation software or any other ideas they have for their presentation.

ASSESSING THE PROJECT

1. Start to plan

- Record your observations. Provide students with numeric information on how they will be assessed, using a scheme that meets your reporting needs. Look for completeness and clarity in the sketches.

2. Test your designs and gather data

- Provide students with numeric information on how they will be assessed. Look for quality of construction and reliability of the models.
- Ensure that they have enough models for the three different skill levels.

3. Document your design

- Check that students have completed scale drawings for all the models and included all slope calculations as fractions, decimals, angles, and percentages.

4. Make a presentation

- Use the rubric on p. 28 as a gauge to accompany a numerical grading rubric you have created that meets your needs.
- Ask students to self-assess their projects using Blackline Master 1.4 (p. 68).
- Projects can be arranged around the room so that students can provide constructive feedback to their peers. If some students have created electronic presentations, arrange to have a projector available.

CHAPTER PROJECT EXTENSION

Students who wish to have more of a challenge can be asked to construct their models so that the distance travelled from the intermediate jump is exactly twice the distance travelled from the beginner jump, and the distance travelled from the expert jump is exactly three times the distance from the beginner jump. To accomplish this, students would have to establish a baseline distance on one of the models, then calculate twice the distance and three times the distance. Then they would have to create models that meet those parameters.

PROJECT ASSESSMENT RUBRIC

	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Explanations show understanding of creating and calculating slope	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are often incomplete or somewhat confusing	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and thorough
Procedural Knowledge				
• Accurately: • sketches designs and plans that can be constructed • constructs models with three distinct skill levels • tests and records data • calculates the slope values • draws scale diagrams of slopes • compiles information for a presentation	limited accuracy; major errors or omissions For example: • sketches have no dimensions or are incomplete • models do not work or are incomplete • data are missing • many calculation errors • diagrams not to scale and missing key information • project is incomplete	partially accurate; some errors or omissions For example: • sketches missing some information • models work but are poorly constructed • some data are missing • a few calculation errors • diagrams not to scale and missing some information • project could use more work to ensure information is complete and accurate	generally accurate; few errors or omissions For example: • sketches missing little information • models work and are constructed well • very few data are missing • very few calculation errors • diagrams to scale, but missing some information • project is complete and meets minimum requirements	accurate and precise; very few or no errors For example: • sketches complete, with dimensions • models work and are constructed well • data complete • no calculation errors • diagrams to scale and information is complete • extra creativity added to project
Problem-Solving Skills				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
Communication				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

1.1 Rise Over Run

TIME REQUIRED FOR THIS SECTION: 3 CLASSES

STUDENT BOOK, pp. 12–23

MATH *ON THE JOB*

STUDENT BOOK, p. 12

Have students read the Math on the Job as a group or individually. Then discuss the importance of proper water drainage. What will happen if the deck slopes towards the house? Water will collect and pool against the house. If water is allowed to pool, it can rot the structure, making the house unsafe. This can lead to accidents and costly repairs. Ask students for other examples of slope in house construction. They may suggest roofs, stairs, decks, drain pipes, and driveways. Ask students what would happen if the deck had a really steep slope. It would be uncomfortable and unsafe to walk on.

SOLUTION

The deck slopes $\frac{1}{4}''$ for every foot of deck. To find out how much it slopes over 6 feet, create equivalent fractions.

$$\frac{0.25}{1} = \frac{x}{6}$$

$$6 \times \frac{0.25}{1} = \frac{x}{6} \times 6$$

$$1.5 = x$$

The free end of the deck is $1\frac{1}{2}''$ lower than the end attached to the house.

EXPLORE THE MATH

STUDENT BOOK, p. 12

Use this section as an opportunity to activate students' prior knowledge about slope. Start by asking students for examples of slopes that they have encountered. You may wish to divide the class into small groups and ask them to list as many examples as they can of things that slope. Have students compare their lists. They may suggest mountains, hills, slides, drainage pipes, stairs, roads, roofs, ramps, and aqueducts.

Most students will relate the concept of slope to hills and ramps. Ask them in what ways hills can differ from each other. Students should suggest the idea of varying steepness and height. On the board or an overhead projector, draw two ramps with different slopes. Ask students how they could determine the steepness of each ramp. They may say they could tell by looking at it. Explain that it's possible to calculate a numerical value, and ask them what information they would need to calculate the slope. This should lead them to think about drawing vertical and horizontal lines to create a right triangle. To calculate the amount of slope, you need to compare the vertical component (rise) to the horizontal component (run). The relationship is the ratio of rise over run.

The example of the loading ramp (student book, p. 15) can be used to start a discussion about the purpose of calculating slope value and the different trades that use slope calculations on the job. If the loading ramp were too steep, it would be very difficult to move goods up the ramp. If the ramp were not steep enough, a longer ramp would be required to reach the loading dock.

DISCUSS THE IDEAS

DESCRIBING SLOPE

STUDENT BOOK, p. 13

This Discuss the Ideas provides an opportunity for students to explore their prior knowledge and understanding of the terms "slope" and "steep." It encourages students to think of other ways to express the concept of slope and to think about a variety of real-world examples of slope.

T

To give students another opportunity to reflect on their learning, ask them to look online for definitions of the concept of slope in mathematics and then to write their own understanding of the term.

SOLUTIONS

1. Answers will vary, but students may suggest that a slope is a slanted surface, an incline, a tilt, or an angled line.
2. Answers will vary; for example, students may suggest that "steep" means the degree or amount of slant.
3. Students may use descriptors such as "the incline has a large amount of slant," or "the ramp's angle is very shallow."

ACTIVITY 1.1
CREATING A SLOPE

STUDENT BOOK, p. 13

In this activity, students work in pairs to create a simple slope and examine the effects of different magnitudes of slope. Each pair of students will need a flat, level surface such as a table, desk, or the floor; a hardcover textbook or alternative, such as a short piece of board, to act as a ramp; and a pencil or pen. Students can take turns rolling the pen or pencil and observing.

The students learn that there are three components to slope:
- rise (vertical component)
- run (horizontal component)
- slope (angled component)

This activity is designed to show how rise, run, and slope are interrelated. You cannot change only one component. When one component changes, at least one other component changes as well. To illustrate this, lay a metre stick flat on a table. Ask what the value of the rise is. Since it is flat on the table, there is no rise; therefore, the value is zero. Ask what the value of the run is. Since the metre stick is flat on the table, the run will be the full length of the metre stick, or 1 metre. Now, to create a slope with the metre stick, what must we do? We must raise one end. Now we have created a value for the rise. Has the length of the metre stick changed? The answer is no. Therefore, if the length of the slope is the same, and we have created a rise, what must have happened to the run? The length of the run must have become smaller or decreased. To get a steeper slope with the metre stick, the rise must increase and the run must decrease.

This concept can be extended by asking students how they would make a slope steeper or shallower if the rise did not change. To make the slope steeper, they would need a shorter slope length, which would make the run shorter as well. To make a shallower slope, a longer slope length would be needed, which would also increase the length of the run.

By understanding how a change in the value of one component affects the other two components, students will be able to make the necessary adjustments to create a desired slope.

SOLUTIONS

1. Students may say that nothing is happening; however, something *is* happening. The pen sits on the textbook and does not move. Reasons given for this will vary. Students may say the pen does not roll because the textbook is level or horizontal or because the textbook doesn't slant. Note that students may say that the pen does not roll because there is no slope; however, a horizontal line does have slope, which they will discover in Activity 1.3 (student book, p. 18).
2. By placing a finger under the textbook, students create an angled slope, a slant, a ramp, or an

incline. They should observe that the pen rolls down the textbook. They may also observe that it rolls slowly all the way down and off the textbook. The changes between this trial and trial 1 are that the vertical height of one end of the textbook increases, and the horizontal component of the ramp decreases very slightly. The component that does not change is the length of the slope or ramp because the textbook length remains constant.

3. When a student places a fist under the textbook, the pen rolls down quickly. The pen also spends less time on the ramp. This happens because the ramp is steeper. The height of one end of the textbook is higher than it was in trial 2, so the vertical component of the slope increases and the horizontal component decreases, creating a steeper slope. The component that does not change is the length of the slope or ramp because the textbook length remains constant.

   Answers will vary. Ensure that students have correctly labelled the rise and run and have used the rise over run formula to calculate the slope of their textbook.

4. Answers will vary. Students should realize that other students will get different results for the slope values if their fists (the "rise") are different sizes.

ACTIVITY 1.2
A PROPERTY OF SLOPE

STUDENT BOOK, p. 16

This activity enables students to discover that the slope of a straight line is constant. In other words, the slope value is the same anywhere along the length of a line.

Prior to having students work through the steps in this activity, engage them in a discussion about why triangular trusses give a roof strength. Ask if any students have ever worked on building a roof with trusses. What are the advantages of having a steep roof versus a flat or shallow-sloped roof? Each student will need a pencil, ruler, and graph paper.

SOLUTIONS

1. Ensure that students have copied the roof truss diagram accurately.

   $$\text{slope AB} = \frac{3}{3}$$
   $$\text{slope AB} = 1$$

2. $$\text{slope BC} = \frac{5}{5}$$
   $$\text{slope BC} = 1$$

3. $$\text{slope AC} = \frac{8}{8}$$
   $$\text{slope AC} = 1$$

4. The students should notice that all three slopes have the same value.
5. The students should all get the same result.
6. The students should see that the slope of a straight line is the same all along its length.

When students have worked through the activity, they should realize that the slope of a straight line is the same between any two points on that line. Ask them to suggest a term that describes a value that remains the same and does not change. You may wish to write the following sentence on the board: The slope of a straight line is ______ , which means it is the same for any two points on the line. Have them fill in the blank with the word "constant."

Mental Math and Estimation

STUDENT BOOK, p. 17

In the previous examples, the students have seen slope as a ratio with a numerator and denominator that correspond to rise and run. Now, they are given the integer 3. Give students a few minutes to try the problem on their own, and if they need assistance, ask them to identify the difference between this slope value and the slope values they saw before. Then ask how 3 can be expressed as a fraction. Ask whether they can now determine the rise and run. Then ask how they would find another similar rise and run. They should realize that they can make equivalent ratios, such as the following.

SOLUTION

$$\frac{3}{1} = \frac{6}{2}$$

or

$$\frac{3}{1} = \frac{9}{3}$$

PUZZLE IT OUT

THE BERMUDA TRIANGLE

STUDENT BOOK, p. 17

If some students require help with this puzzle, first ask them how they could use their knowledge of slope to analyze the diagrams. They should reason that they can calculate the slope values for the three triangles. When they see that they all have different values, ask them what this could mean or how this could affect what is happening in the puzzle. Then, ask them to look at the slopes and see if they can find any subtle differences between the top diagram and the bottom. If they cannot see a difference, ask them to compare the two hypotenuses. Ask them how many sides there are on these polygons. When they realize that there are actually four sides, they should realize that one quadrilateral has a greater area than the other, and the difference between the areas equals the area of the missing square.

SOLUTION

Although the two pictures appear to be right triangles with a rise of 5 and a run of 13, in fact they are quadrilaterals, not triangles. This can be shown by calculating the slopes of the yellow and green triangles.

Slope of yellow triangle $= \frac{3}{8}$

slope $= 0.375$

Slope of green triangle $= \frac{2}{5}$

slope $= 0.4$

The green triangle has a steeper slope. In other words, the "hypotenuse" of each picture is not a straight line and the two pictures are not right triangles. Because the green triangle has a steeper slope than the yellow triangle, in the top picture, the hypotenuse is slightly concave. When the green and yellow triangles are switched in the bottom picture, the hypotenuse is slightly convex. The difference between the two areas along the hypotenuse equals the area of the missing square.

Extension

For highly motivated and gifted math students, suggest that they create their own math puzzle relating to slope and rate of change for other students to solve.

ACTIVITY 1.3

ANALYZING SLOPES

STUDENT BOOK, p. 18

Prior to having groups of students work through the steps in this activity, you might ask them an open-ended question, such as: How could you determine how rise and run affect the steepness of a slope? Give them 10 minutes to come up with some possible approaches (using concrete models, pictorial models, or symbolic models) and discuss these with the whole class.

In this activity, students create, diagram, and examine different slopes to discover how different rise and run lengths create the amount of steepness in a slope. Each group of students will need a 30-cm ruler, 10 textbooks or other hardcover books of approximately the same size (or an alternative such as short pieces of board), coloured pens or pencils, and 1 cm graph paper (Blackline Master 1.3, p. 67). If the quantity of items needed proves unwieldy, this activity can be done by organizing stations and dividing students into teams to complete the steps.

It is important that students examine the relationship between the length of the rise and the length of the run of each slope. Encourage them to be aware of how each change in the length of the rise and run affects the amount of steepness in the resulting slope. They should discover that a steep slope will have a large rise and a small run, and a slope that is not very steep will have a small rise and a large run.

SOLUTIONS

Part A

1. One student should hold the ruler so that the end just touches the textbook, while another student measures the rise and run. Answers will vary depending on the thickness of the textbook used and the accuracy of students' measurements.
2. It will be easier for students to examine their slopes if they draw the line segments representing slope in a different colour than the rise and run. Ensure that students have drawn their triangles to scale, so that they will be able to see that each slope is different.
3. Encourage students to draw their slope diagrams in alphabetical order, one beneath the other. This will make it easier for them to answer the remaining questions.
4. Check that students are placing the end of the ruler against the top textbook in the pile, thus creating a steep slope.
5. Because the rise and run are equal, the slope value for Slope D is 1. Students will not need to use the full length of the ruler.
6. Slope A has a small rise and a large run. Slope C has a large rise and a small run. A large rise and small run makes the slope steeper. A small rise and a large run will make a slope less steep.
7. The rise of A is much smaller than that of B. The run of A is much greater than that of B. A small rise value and large run value produce a small, or gentle, slope. A large rise value and a small run value produce a large, or steep, slope.
8. A line whose rise and run are equal lengths has a slope value of 1. The position of Slope D is at 45°, which is equidistant from the *x* and *y* axes (it bisects the 90° angle of the axes). Therefore, a 45° line has a slope of 1.
9. As the slopes get closer to the horizontal axis, the values get smaller (closer to zero).
10. As the slopes get closer to the vertical axis, the values get larger.

Part B

1. The run is 14 units. There is no change in the vertical component of the horizontal line; therefore, the value of the rise is zero.

   $$\text{slope} = \frac{0}{14}$$

   $$\text{slope} = 0$$

2. The run is 4.5 units. There is no change in the vertical component. The value of the rise is zero.

   $$\text{slope} = \frac{0}{4.5}$$

   $$\text{slope} = 0$$

   Students should conclude that the slope of all horizontal lines is zero.
3. Predictions may vary. Students may say that the slope will be a very large number. They may predict it will be zero. They may predict that it is impossible to calculate.
4. There is no change in the horizontal component of a vertical line, so the run is zero. The rise is 14. The slope cannot be calculated. A calculator will give an error message. This happens because it is not possible to divide by zero.
5. Students should conclude that slope cannot be calculated for vertical lines. Explain that the term for the slope of a vertical line is "undefined." We say that it is undefined because the slope value cannot be calculated.

DISCUSS THE IDEAS

THE BUGABOO SPIRE

STUDENT BOOK, p. 19

Encourage students to examine the photograph and discuss the questions with a partner.

To give students another opportunity to reflect on their learning, ask them to research steep mountains, identify five of the steepest, and support their findings.

SOLUTIONS

1. The right side is steeper. Students may say that by looking at the slopes, they can tell that the slope on the right is closer to being vertical than the slope on the left.
2. The difference in steepness depends on either the amount of change in the rise for each unit of run or the amount of change in the run for each unit of rise.

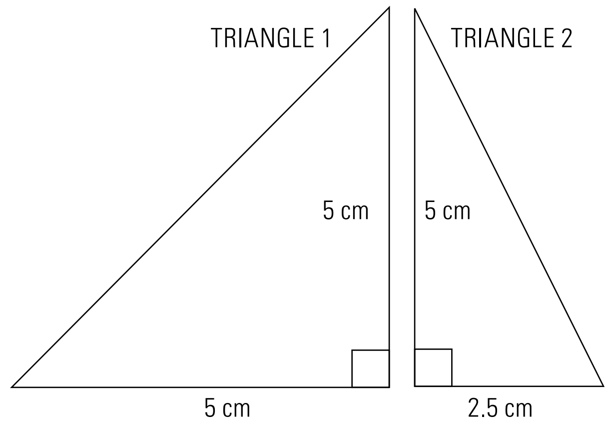
Extension

Work in pairs to solve the following problems. Referring to the photograph of the Bugaboo Spire, draw two right triangles, one representing the slope on the left (Triangle 1) and the other representing the slope on the right (Triangle 2). The vertex of both triangles is the highest point of the spire. Assume that the rise for both triangles is 5 cm, the run for Triangle 1 is 5 cm, and the run for Triangle 2 is 2.5 cm.

1. One partner explains to the other partner how to prove which triangle has a steeper slope. Draw triangles and provide the necessary calculations. Explain your reasoning in terms of a climber ascending the Bugaboo Spire.
2. The second partner draws a right triangle similar to Triangle 2 but increases both the run and the rise by 50%. Label this Triangle 3. Explain to partner one how the slope of Triangle 3 compares with the slope of Triangle 2. Explain why the slopes are different or the same.

SOLUTIONS

1. Triangles 1 and 2 should be sketched as follows.



To prove which triangle has a steeper slope, calculate the rise over run.

Triangle 1

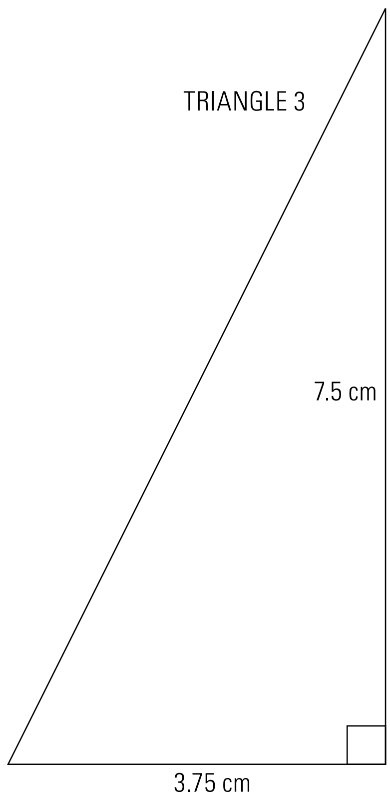
$$\text{rise} = 5 \text{ cm}$$
$$\text{run} = 5 \text{ cm}$$
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$\text{slope} = \frac{5}{5}$$
$$\text{slope} = 1$$

Triangle 2

$$\text{rise} = 5 \text{ cm}$$
$$\text{run} = 2.5 \text{ cm}$$
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$\text{slope} = \frac{5}{2.5}$$
$$\text{slope} = 2$$

Triangle 2 has a steeper slope because the vertical distance (rise) compared to the horizontal distance (run) is higher than in Triangle 1, in which the two distances are the same. If you travel the same distance vertically in a shorter distance horizontally, you are climbing a steeper slope, so climbing the right side of the spire would be steeper.

2. Triangle 3
   Sketch a right triangle that looks like the following.



The slope of Triangle 3 would be the same as the slope of Triangle 2 because the rise and the run were increased proportionally. Therefore, their relationship remains constant.

BUILD YOUR SKILLS SOLUTIONS

STUDENT BOOK, p. 19

1. Use the rise over run formula to find the slope of each section of highway.

   Kootenay Pass:

   $$\text{slope} = \frac{610}{12\,100}$$

   $$\frac{610}{12\,100} \approx 0.05$$

   The slope of the Kootenay Pass is 0.05.

   Coquihalla Pass:

   $$\text{slope} = \frac{845}{19\,880}$$

   $$\frac{845}{19\,880} \approx 0.04$$

   The slope of the Coquihalla Pass is 0.04.

   The section of the Kootenay Pass has a greater slope than the Coquihalla Pass, so the Kootenay Pass is steeper.

2. Count the number of squares on the grid to find the rise and run.

   a) rise = 1
      run = 4

      $$\frac{1}{4} = 0.25$$

      Roof A has a slope of 0.25.

      $$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$ Use proportional reasoning to convert the slope to roof pitch.

      Roof A is a 3:12 roof.

   b) rise = 5
      run = 6

      $$\frac{5}{6} \approx 0.83$$

      Roof B has a slope of 0.83.

      $$\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

      Roof B is a 10:12 roof.

   c) rise = 4
      run = 3

      $$\frac{4}{3} \approx 1.3$$

      Roof C has a slope of 1.3.

      $$\frac{4 \times 4}{3 \times 4} = \frac{16}{12}$$

      Roof C is a 16:12 roof.

3. To find the run, divide the overall span of the truss in half.

   $$\text{run} = \frac{23}{2}$$

   Calculate the slope.

   $$\frac{\text{rise}}{\text{run}} = \frac{5.75}{11.5}$$

   $$\text{slope} = 0.5$$

   The truss has a slope of 0.5. This truss is for the house.

4. Count the grid squares to determine the rise and run. Measure the run at the points where the rail intersects the grid.

   Slope of rail A: $\frac{1}{7} \approx 0.14$

   Slope of rail B: $\frac{1}{6} \approx 0.17$

   Slope of rail C: $\frac{1}{5} = 0.20$

   Only rail C is within the intermediate range.

5. A 1:4 slope can also be written as $\frac{1}{4}$ or as 0.25. A 4:1 slope can also be written as $\frac{4}{1}$ or as 4. Because 4 is greater than 0.25, the 4:1 slope would be steeper. Amy is right.

6. a) Convert the rise and run measurements to a decimal and use the slope formula.

   $\frac{21.75}{69.375} \approx 0.31$

   The slope of the staircase is 0.31.

   b) Divide the total rise and run by the number of steps. There are 3 steps, so there are 3 runs and 3 rises.

   rise for each step:

   $\frac{21.75}{3} = 7.25$

   run for each step:

   $\frac{69.375}{3} = 23.125$

   The rise of each step is 7.25 inches; the run of each step is 23.125 inches.

   c) $\frac{7.25}{23.125} \approx 0.31$

   The slope of each step is 0.31.

   d) The slope of the staircase is equal to the slope of each and every step. The slope is constant.

   e) Answers will vary.

7. The run of $3\frac{1}{2}''$ is the same for all the ramps on each gumball machine. The total rise on each machine is 6″.

   a) Machine with 6 ramps:

   $\frac{6}{6} = 1$ Divide the total rise by the number of ramps to find the rise of one ramp.

   $\frac{1}{3.5} \approx 0.29$ Calculate the slope.

   The average slope of the ramps is 0.29.

   b) Machine with 5 ramps:

   $\frac{6}{5} = 1.2$

   $\frac{1.2}{3.5} \approx 0.34$

   The average slope of the ramps is 0.34.

8. a) The run of each staircase is 1.3 m. The total rise of the staircase is 2.5 m. The landing is halfway between the two staircases.

   $\frac{2.5}{2} = 1.25$ Divide the total rise by 2 to find the rise of each staircase.

   $\frac{1.25}{1.3} \approx 0.96$ Calculate the slope.

   The slope of each staircase is 0.96.

   b) Substitute the building code requirements into the slope formula to find the steepest slope permitted.

   $\frac{200}{210} \approx 0.95$

   Karen's staircases are steeper than the building code allows, so they do not meet the building code.

   c) Answers will vary.

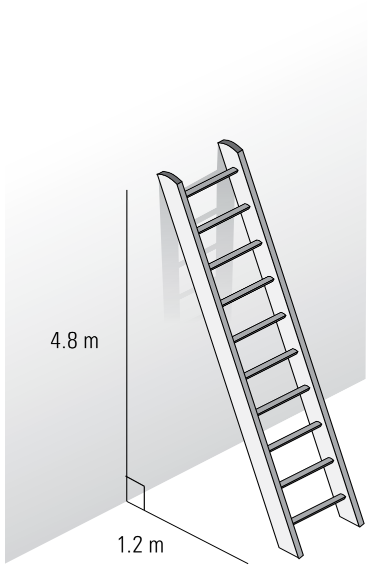
9. a) According to the ladder safety rule, the run should be $\frac{1}{4}$ of the rise.

   $\text{run} = \frac{1}{4} \times \text{rise}$

   $\frac{4}{1} = \frac{\text{rise}}{\text{run}}$ Divide both sides by run.

   The slope of a safely leaning ladder is 4.

b)



$$m = \frac{\text{rise}}{\text{run}}$$

$\frac{4.8}{1.2} = 4$ Substitute the rise and run into the slope formula.

The slope of Daisuko's ladder is 4, so his ladder is placed safely.

c) Find the run.

$\frac{10.68}{x} = 4$ Substitute the known value into the slope formula.

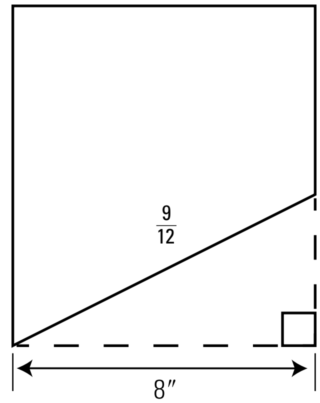
$x\left(\frac{10.68}{x}\right) = 4x$ Multiply both sides by x.

$\frac{10.68}{4} = \frac{4x}{4}$ Divide both sides by 4.

$2.67 = x$

The ladder must be placed 2.67 m from the house to be safe.

10. The slope of the angled cut at the bottom of the tube must equal the slope of the roof. A side view of the tube illustrates that the cut will form a right triangle. The diameter of the tube equals the run. The distance from the bottom of the tube to the point where Sharon will begin the cut equals the rise.



$\frac{9}{12} = \frac{\text{rise}}{8}$ Substitute the known values into the slope formula.

$(8)\frac{9}{12} = \frac{\text{rise}}{8}(8)$ Multiply by 8 to isolate rise.

$\frac{72}{12} = \text{rise}$ Simplify the fraction.

$6 = \text{rise}$

Sharon must measure 6″ from the bottom of the pipe to cut the pipe at the correct angle.

Extend Your Thinking

11. The contour intervals on the topographic map represent vertical distance, or rise. Each interval equals 1 metre. The grid squares represent horizontal distance, or run. Each grid square equals 2 metres. Count the intervals and grid squares and use rise over run to calculate the slopes.

a) Line AB crosses 3 contour intervals.

$3 \times 1 = 3$

The rise is 3 m.

Line AB crosses 7 grid squares.

$7 \times 2 = 14$

The run is 14 m.

$$\text{slope} = \frac{3}{14}$$

$\text{slope} \approx 0.21$

The slope of line AB is 0.21.

b) Line BC crosses 4 contour intervals and 11 grid squares.

$$4 \times 1 = 4$$

$$11 \times 2 = 22$$

$$\text{slope} = \frac{4}{22}$$

$$\text{slope} \approx 0.18$$

The slope of line BC is 0.18.

c) Line CD does not cross a contour interval; points C and D are within the same interval. Because the line does not cross an interval, the vertical change is 0.

$$0 \times 1 = 0$$

Line CD crosses 7 grid squares.

$$7 \times 2 = 14$$

$$\text{slope} = \frac{0}{14}$$

$$\text{slope} = 0$$

The slope of line CD is 0.

d) Line DA crosses 1 contour interval and 11 grid squares.

$$1 \times 1 = 1$$

$$11 \times 2 = 22$$

$$\text{slope} = \frac{1}{22}$$

$$\text{slope} \approx 0.05$$

The slope of line DA is 0.05.

1.2 Grade, Angle of Elevation, and Distance

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 25–33

MATH *ON THE JOB*

STUDENT BOOK, p. 25

Begin by asking a student to read aloud the story of Richard Hall, the CMHC building officer. You may wish to have a class discussion about the responsibilities of a building inspector. Why do we need building inspectors? Students may suggest that building inspectors ensure that structures are built correctly, that they are safe, or that they are constructed to a certain standard of quality. Students who have worked in the construction industry can be encouraged to share what they know about industry standards and building code compliance.

To solve this problem, first ask whether it is possible to make a comparison with the values as given. Students should note that there is a difference in the way the information is presented. The slope value is given as a percentage, but the ramp is measured in centimetres. Ask what needs to be done to make a comparison. Students should answer that they need to make a conversion; they would either convert the percentage to a decimal or convert the slope of the ramp to a percentage.

SOLUTION

Convert the slope of the ramp to a percentage.

$$\text{slope} = \frac{32}{370}$$

$$\text{slope} \approx 0.086$$

$$0.086 \times 100 \approx 8.6\%$$

A legal wheelchair ramp has a maximum slope of 8.3%. This wheelchair ramp has a slope of 8.6%, so it is not legal.

EXPLORE THE MATH

STUDENT BOOK, p. 25

Before reading the Explore the Math section, ask students to name the different ways to write a slope value that they have learned so far. Students have learned to express slope as a fraction, integer or decimal, and ratio. Ask them whether they can think of other ways to express slope value. Guide them towards percent and angles by asking questions like, “Have you seen road signs on steep hills?” and “What do we use in trigonometry that could indicate the amount or value of a slope?” Once they have come up with “percent” and “angle,” read the Explore the Math section.

Most students will have seen a percent grade road sign on hills. The road sign depicted warns drivers that the road has an 8% grade. The steeper the road, the higher the percent grade. Truck drivers, in particular, need this information because a steep road will be more difficult to climb or make it more difficult to slow down when driving downhill.

To help students make the connection between percent grade and slope, ask students what the slope formula is. They should say “rise over run.” In what form is this formula written? The answer is as a fraction or ratio. Then ask, “What needs to be done to the 8% to express it in the rise over run form?”

Convert percentage to a fraction.

$$8\% = \frac{8}{100}$$

$$\frac{8}{100} = \frac{2}{25}$$

Now ask, “What does this fraction represent?” It represents a slope with a rise of 2 and a run of 25.

To introduce slope as an angle of elevation, activate students’ prior knowledge of the tangent ratio by drawing a right triangle on the board and labelling the acute angle with θ. Ask students to identify its opposite and adjacent sides and the formula they would use to find θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

To introduce the concept that the line representing a particular slope has distance or length, activate students' prior knowledge of the Pythagorean theorem. Draw a right triangle and label one side 6 and another side 8. Ask the students how they would find the hypotenuse.

$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = 10$$

ACTIVITY 1.4

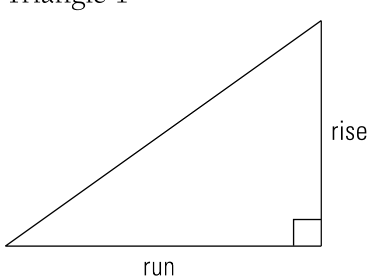
SLOPE AND TANGENT

STUDENT BOOK, p. 26

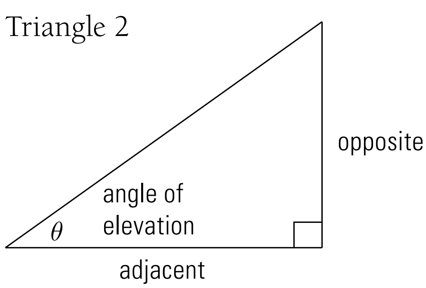
This activity will help students to visualize the relationship between slope and the tangent ratio. Ensure that students draw their two congruent triangles next to each other. By comparing the two triangles, students should see that the slope ratio is the same as the tangent ratio. Students can use Blackline Master 1.5 (p. 69) to record their calculations.

SOLUTIONS

1. Triangle 1



2. Triangle 2



3.

SLOPE AND TANGENT RATIO		
Triangle 1		*Triangle 2*
rise	is the same as	opposite side
run	is the same as	adjacent side

4. To complete the expression, have students write the formula for slope and the formula for tan θ. Then tell them to combine the two formulas into one expression so that all the terms are equivalent.

Triangle 1 slope $= \frac{\text{rise}}{\text{run}}$

Triangle 2 slope $= \frac{\text{opposite}}{\text{adjacent}}$

$$\frac{\text{rise}}{\text{run}} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\frac{\text{opposite}}{\text{adjacent}} = \tan\theta$$

$$\text{slope} = \tan\theta$$

Extension

For highly motivated and gifted math students, suggest that they design a hands-on activity to teach the relationship between slope and tangent ratio to fellow students.

Mental Math and Estimation

STUDENT BOOK, p. 28

SOLUTION

To find the percent grade, convert $\frac{5}{8}$ to a decimal and multiply by 100.

$$\frac{5}{8} = 0.625$$

$$0.625 \times 100 = 62.5\%$$

A slope of $\frac{5}{8}$ is a 62.5% slope.

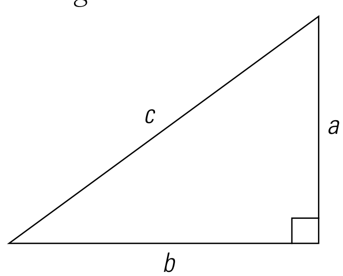
ACTIVITY 1.5
LENGTH OF A PARTICULAR SLOPE

STUDENT BOOK, p. 29

In this activity, students draw congruent triangles to visualize the relationship between the Pythagorean theorem and the formula for calculating the length of a particular slope. By examining their triangles, students see that this formula is a manipulation of the Pythagorean theorem, written in a slightly different form. Students can use Blackline Master 1.6 (p. 70) to record their calculations.

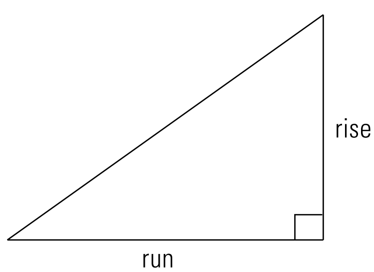
SOLUTIONS

1. Triangle 1



$a^2 + b^2 = c^2$

2. Triangle 2



3.

LENGTH OF A PARTICULAR SLOPE		
Triangle 1		*Triangle 2*
a	is the same as	rise
b	is the same as	run
c	is the same as	slope

Note that *a* could be rise or run, as could *b*, depending on how students label their triangles.

4. Students are used to seeing the Pythagorean theorem in the $a^2 + b^2 = c^2$ form. Ask them how they would rewrite the formula with c^2 as the only term on the left side.

$c^2 = a^2 + b^2$

Now, ask them, "What represents the length of the hypotenuse?" It's important that they realize that it's c and not c^2. What needs to be done to the formula to express it in terms of c and not c^2? You must take the square root of both sides.

$$c = \sqrt{a^2 + b^2}$$

5. Now they can repeat the steps in question 4, substituting rise, run, and slope for ***a, b***, and ***c***.

$$\text{slope}^2 = \text{run}^2 + \text{rise}^2$$

$$\text{slope} = \sqrt{\text{run}^2 + \text{rise}^2}$$

BUILD YOUR SKILLS SOLUTIONS

STUDENT BOOK, p. 30

1. The diagram shows five sections of pipe: three vertical sections and two diagonal sections. Count the grid squares to find the length of the vertical sections.

3 units + 1 unit + 2 units = 6 units

Use the formula for calculating the length of a slope to find the length of the diagonal sections.

$$\text{length} = \sqrt{\text{run}^2 + \text{rise}^2}$$

The top diagonal section has a rise of 2 units and a run of 6 units.

$$\text{length} = \sqrt{6^2 + 2^2}$$

$$\text{length} = \sqrt{40}$$

$$\text{length} = 6.324\,555\,32$$

The top diagonal section is 6.324 555 32 units long.

The bottom diagonal section has a rise of 1 unit and a run of 5 units.

$$\text{distance} = \sqrt{5^2 + 1^2}$$
$$\text{distance} = \sqrt{26}$$
$$\text{distance} = 5.099\,019\,514$$

The bottom diagonal section is 5.099 019 514 units long.

Now add the vertical and diagonal units together.

$$6 + 6.324\,555\,32 + 5.099\,019\,514 = 17.423\,574\,83$$
$$17.423\,574\,83 \times 15 = 261.353\,622\,5$$

Because she is measuring materials, Shelley will need to round up. She will need 262 cm of pipe.

2. $3\frac{3}{4} = 3.75$ Convert $3\frac{3}{4}$ to a decimal.

$9 \times 12 = 108$ Convert 9 feet to inches.

$\text{percent grade} = \frac{3.75}{108} \times 100$ Substitute the known values into the percent grade formula.

$\text{percent grade} \approx 3.47$

The centre line crown has a 3.47% grade.

3. a) Use the tangent formula to find the angle of the bend.

$$\tan\theta = \frac{5.3}{2.8}$$
$$\theta = \tan^{-1}\left(\frac{5.3}{2.8}\right)$$
$$\theta \approx 62.2$$

He needs to bend the copper at an angle of 62°.

b) First, use the formula for the length of a slope to find the length of the front of the frame.

$$\text{length} = \sqrt{5.3^2 + 2.8^2}$$
$$\text{length} = \sqrt{28.09 + 7.84}$$
$$\text{length} = \sqrt{35.93}$$
$$\text{length} = 5.99$$

Add the length of the front of the frame to the depth of the base.

$5.99 + 1.9 = 7.89$

He will need 7.89 cm of copper sheet metal, or 8 cm, rounded up.

4. a) Use the rise over run formula to find the drop (rise).

$\frac{1}{4} = 0.25$ Convert $\frac{1}{4}$ to a decimal.

$11 \times 12 = 132$ Convert feet to inches.

$\frac{0.25}{12} = \frac{\text{drop}}{132}$ Set up a proportion.

$(132)\frac{0.25}{12} = \frac{\text{drop}}{132}(132)$ Multiply by 132 to isolate the variable.

$2.75 = \text{drop}$

The drainage pipe drops 2.75″ over 11 feet.

ALTERNATIVE SOLUTION

$\frac{0.25}{1} = \frac{\text{drop}}{11}$ Set up a proportion.

$(11)\frac{0.25}{1} = \frac{\text{drop}}{11}(11)$ Multiply by 11 to isolate the variable.

The drainage pipe drops 2.75″ over 11′.

b) Substitute the known values into the percent grade formula.

$\text{percent grade} = \frac{0.25}{12} \times 100$

$\text{percent grade} \approx 2.08$

The drainage pipe has a 2% grade.

ALTERNATIVE SOLUTION

Substitute the known values into the percent grade formula.

$\text{percent grade} = \frac{2.75}{132} \times 100$

$\text{percent grade} \approx 2.08$

The drainage pipe has a 2% grade.

5. a) $\text{slope} = \frac{3}{18}$

$\text{slope} = \tan^{-1}\left(\frac{3}{18}\right)$ Use the inverse tangent function to convert the slope to degrees.

$\text{slope} \approx 9.5°$

The angle of the Cardinals' roof is 9.5°. They should use tar and gravel.

b) $\text{slope} = \frac{6}{16}$

$\text{slope} = \tan^{-1}\left(\frac{6}{16}\right)$

$\text{slope} \approx 20.6°$

The angle of the Sebastians' roof is 20.6°. They should use asphalt shingles.

6. Calculate the rise (drop) of the pipe length and add the height of the intake point.

The slope of the pipe is 0.8°. Convert the slope to a fraction.

$\tan(0.8°) \approx 0.014$

$0.014 = \frac{14}{1000}$

$\frac{14}{1000} = \frac{\text{drop}}{750}$ Create equivalent fractions.

$\frac{(750)14}{1000} = \text{drop}$

$10.5 = \text{drop}$

$10.5 + 1.5 = 12$

The pipe enters the reservoir 12 m above the ground.

7. $31.35 - 30.60 = 0.75$ Find the rise by finding the difference in elevation between the two pipe openings.

$0.75 \times 12 = 9$ Convert the rise to inches.

$120 \times 12 = 1440$ Convert the run to inches.

$1440 + 8 = 1448$

$\text{percent grade} = \frac{9}{1448} \times 100$

Substitute the known values into the percent grade formula.

$\text{percent grade} \approx 0.62$

The laser level should be set at 0.62%.

8. First, find the rise and run of a 2.1% grade.

$0.021 = \frac{21}{1000}$ Convert the percent grade to a fraction.

The rise and run of a 2.1% slope is $\frac{21}{1000}$.

Use the length formula to find the length of the line segment representing a generic 2.1% slope.

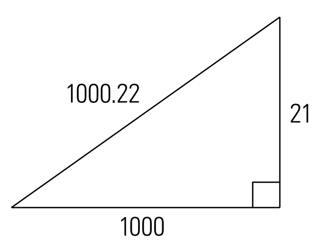
$\text{length} = \sqrt{\text{run}^2 + \text{rise}^2}$

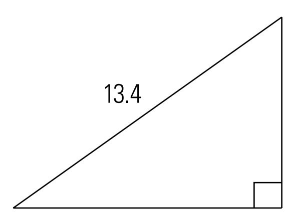
$\text{length} = \sqrt{1000^2 + 21^2}$

$\text{length} = \sqrt{1\,000\,441}$

$\text{length} \approx 1000.22$

Once the values for a 2.1% slope are known, set up a proportion to find the rise of the railway track.





$$\frac{13.4}{1000.22} = \frac{\text{rise}}{21}$$

$$(21)\frac{13.4}{1000.22} = \frac{\text{rise}}{21}(21)$$

$$0.281 \approx \text{rise}$$

$0.281 \times 1000 = 281$ Convert kilometres to metres.

The rise of the railway track is 281 metres.

Extend Your Thinking

9. First calculate the elevation gain (rise) in a 10° slope.

$\tan(10°) \approx 0.176$ Convert 10° to a decimal.

$0.176 = \frac{176}{1000}$ Convert the slope to a fraction.

$\frac{176}{1000} = \frac{22}{125}$ Simplify the fraction.

The rise and run of a generic 10° slope is $\frac{22}{125}$.

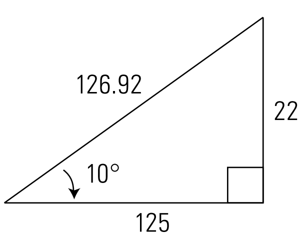
Use the length formula to find the length of a generic 10° slope.

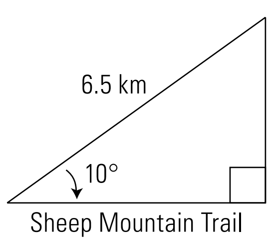
$$\text{length} = \sqrt{125^2 + 22^2}$$

$$\text{length} = \sqrt{16\,109}$$

$$\text{length} \approx 126.92$$

Once the values for the generic 10° slope are known, set up a proportion to find the rise of the Sheep Mountain Trail.





$$\frac{6.5}{126.92} = \frac{\text{rise}}{22}$$

$$(22)\frac{6.5}{126.92} = \frac{\text{rise}}{22}(22)$$

$$1.127 \approx \text{rise}$$

The rise, or elevation gain, of the Sheep Mountain Trail is 1.127 km.

Calculate the elevation gain (rise) in an 8° slope.

$\tan(8°) \approx 0.141$ Use a calculator to find the slope.

$0.141 = \frac{141}{1000}$ Convert the slope to a fraction.

The rise and run of a generic 8° slope is $\frac{141}{1000}$.

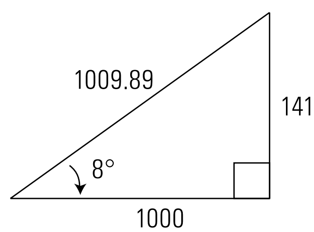
Use the length formula to find the length of a generic 8° slope.

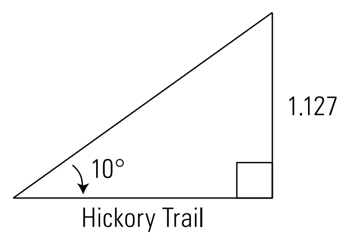
$$\text{length} = \sqrt{141^2 + 1000^2}$$

$$\text{length} = \sqrt{1\,019\,881}$$

$$\text{length} \approx 1009.89$$

Once the values for the generic 8° slope are known, set up a proportion to find the length of the Hickory Trail if its rise is the same as Sheep Mountain Trail.





$$\frac{\text{length}}{1009.89} = \frac{1.127}{141}$$

$$(1009.89)\frac{\text{length}}{1009.89} = \frac{1.127}{141}(1009.89)$$

$$\text{length} \approx 8.07$$

Hickory Trail is 8.07 km long.

$$8.07 - 6.5 = 1.57$$

Hickory Trail is 1.57 km longer than Sheep Mountain Trail.

THE ROOTS OF MATH

THE SPIRAL TUNNELS OF KICKING HORSE PASS

STUDENT BOOK, p. 33

This is a teacher-led activity with the entire class. Have students take turns.

Have students take turns reading the passage aloud. Some students may have visited the area of Field, BC, have been on the Rocky Mountaineer train that travels through the spiral tunnels, or have ridden on a train. Ask them to share their experiences with the class.

SOLUTIONS

1. Answers will vary. From reading the text, students will understand that the main advantage of a spiral tunnel is that it allows the track to lose vertical elevation over a small horizontal distance. In the case of Kicking Horse Pass, the spiral tunnels also reduce the trains' exposure to avalanches and landslides. As well, using spiral tunnels allows the line to maintain the most direct route, instead of looking for a route that is less steep but may be more circuitous. Spiral tunnels reduce the need for specially equipped trains. For example, to climb a steeper track, trains often need highly powerful engines or a second engine that pushes them from behind. The main disadvantage of spiral tunnels is cost. The construction costs, the time required, and the amount of labour required to build such tunnels are all significant. If an accident occurs inside the tunnel, getting to the cars may be difficult. For example, in 1997, a freight train carrying grain derailed near Field, and 18 train cars were wedged in the upper tunnel. It took more than 100 workers to remove the cars from the tunnel, and each of the cars had to be cut up and removed in pieces. Students may also suggest that tunnelling through mountains can be cause for environmental concern.
2. This question provides students with the opportunity to connect what they've learned in this chapter with what they know from experience. Their answers will vary, and students should be encouraged to explore a range of possibilities—with all answers being acceptable as long as the student can defend them.
   - Students may suggest tunnelling through a different mountain or finding a different, less steep route through the region. This answer can connect to a geography discussion about the region's terrain or a social studies discussion about the politics of mapping a route that must consider local investors, access to large communities, and proximity to an international border.
   - Students may suggest using bridges. This answer may lead to a discussion about trestle bridges such as the Kinsol Trestle on Vancouver Island, Lethbridge, Alberta's High Level Bridge—the highest and longest trestle in the world—and the trestle bridges of the Kettle Valley in BC.
   - Another option is switchbacks, also called zigzag tracks, that allow the train to zigzag back and forth up a steep slope. Students may not be familiar with the idea of switchbacks. If that is the case, relate the idea of a switchback to skiing. To control their speed, skiers travel down the mountain in a zigzag rather than going straight down. By moving back and forth, skiers increase the total distance they travel, which is the run. As students know from slope calculations, an increased run results in a decreased slope, which means a slower speed.
3. Answers will vary. Possible examples are:
   - a pinball machine
   - a roller coaster
   - an on-ramp to a highway
   - a car exit ramp from a parkade

1.3 Rate of Change

TIME REQUIRED FOR THIS SECTION: 3 CLASSES

STUDENT BOOK, pp. 35–50

MATH *ON THE JOB*

STUDENT BOOK, p. 35

Before the students read this Math on the Job, ask them to suggest different ways of presenting data or information. They may suggest that information can be presented verbally, in a picture, or by using a list, a table, or a chart. Ask what method of presentation would convey the information quickly and clearly. They should reason that a graph is one way to achieve this effectively.

Have students read the Math on the Job individually or as a class. Ask them to look at the graph and identify the difference between this graph and the slope diagrams they worked with in sections 1.1 and 1.2. They should recognize that this graph has axes and scales and realize that the graph is showing the relationship between profits earned and the number of flats of herbs sold.

Work through the Explore the Math section so that students become familiar with the concept of rate of change and understand that a linear relationship has slope. Then revisit the problem posed in this Math on the Job. Have students work with a partner to discuss how to calculate the slope of the line segment that graphs Arlette's profits compared to the number of flats sold. They will be able to use the formula for slope. They will be able to see that the rise is 300 and that the run is 10 and use the slope formula to find the solution. They should also be able to reason that the more flats sold, the greater the profit.

SOLUTION

$$\text{slope} = \frac{300 - 150}{10 - 5}$$

$$\text{slope} = 30$$

The slope of the line is 30.

The slope represents Arlette's profit for one flat of herbs ($30.00).

EXPLORE THE MATH

STUDENT BOOK, p. 35

Read the Explore the Math section aloud as a class and discuss the question in the first paragraph. Students should recognize that if you work at a steady pace, then your output will be related to the number of hours you work. If you work more hours, you will make more skateboard decks.

An example that will likely be familiar to students will be an amount paid compared to hours worked. Activate prior learning by asking students to suggest an hourly rate of pay, for example, $10.00. Draw a graph that shows 4 hours' pay at this rate. Discuss the concept of dependent and independent variables. Some students may have some difficulty identifying which variable is which. Ask them to try out a few combinations to ensure that they can identify the dependent and independent variables (for example, speed compared to distance travelled or the cost of meat purchased compared to the amount of meat purchased). In the case you have graphed, hours are the independent variable and pay is the dependent variable. The dependent variable will be on the y-axis. The independent variable will be on the x-axis. Example 1 (student book, p. 37) works through a similar calculation.

If students have difficulty recalling how to graph a linear relationship, remind them that points on a graph have a set of coordinates, one for each axis. Ask them which coordinates represent the starting point before any work is done or pay earned. They will likely be able to suggest that (0, 0) is the starting point. After one hour, how much has been earned? After two hours? Mark the points on the graph. Continue graphing the values up to four hours. Example 1 on p. 37 of the student book leads students through the process of making a data table, expressing the data as an equation, and then graphing the equation. This is followed by an analysis of the information the graph shows. Students may note that the values in the data table represent the coordinates of points on the graph.

Now, relate rate of change to slope. Ask students to consider the sample graph showing skateboard deck output on p. 36 of the student book. Ask them how they could find the slope of the line segment shown on the graph. They should recognize that they can use the same method they used earlier in the chapter, using the formula $m = \frac{\text{rise}}{\text{run}}$. Ensure that they observe that rise (the number of units you move up or down) is a change in the y-values on a graph, and that run (the number of units you move left or right) is a change in the x-values on a graph. Ask them to calculate the slope of line segment AB.

$$\text{slope} = \frac{160}{8}$$

$$\text{slope} = 20$$

Then ask them to calculate the slope of line segment CD.

$$\text{slope} = \frac{80}{8}$$

$$\text{slope} = 10$$

Point out that students could use the coordinates for a different point and find the same value for slope.

Discuss what students notice about the relationship between the rate at which the variables change compared to the slope value. They will notice that one line segment rises more steeply than the other and conclude that the steepness of the slope and the resulting higher slope value indicates a greater rate of change.

Introduce students to the idea that slope can be positive or negative. Have them examine the positive and negative graphs on p. 36 of the student book and consider the relationship of the slope to the rate of change. They will recognize that a line segment rising up and to the right is positive and that a line segment falling to the right is negative. Some examples to discuss could include an airplane landing (amount of height lost compared to distance travelled); you could also show students a positive slope on take-off for comparison. There are many examples in the workplace where a dropping temperature can produce a negative slope if it is graphed, such as the amount of time it takes paint or glues to dry, or concrete and asphalt to set, and so on. Ask students to explain in their own words what negative and positive slope means. They should be able to express the idea that a positive slope means that the dependent variable is rising when the slope is positive and falling when it is negative.

ACTIVITY 1.6
CLIMBING AND CALCULATING

STUDENT BOOK, p. 39

Activity 1.6 is an experiential team activity allowing students to investigate real-life examples of slope in their school environment. Students may use Blackline Master 1.7 (p. 71) to record their data.

PART B SOLUTIONS

1. Answers will vary, depending on the possibilities available in your school, and the choices made by the students.
2. Answers for slope and rate of change will vary, depending on the objects students choose to measure. It is a good idea to remind students of right triangles, since staircases and individual stairs form right triangles, with the rise and run of a staircase or individual stair making up the legs of a right triangle.
3. Possible answers regarding the real-life application of this scenario in the construction industry could include the following. The ability to calculate slope is necessary for builders and carpenters to build staircases and complete other tasks in their fields. The slope of staircases is governed by building codes, so people working in the construction industry must ensure that the slopes of the staircases and ramps they are constructing adhere to these codes. If a staircase or ramp is built with a slope that is too steep, it will be dangerous. If a staircase or ramp is built with a slope that is too gentle, it could be impractical.

4. Students here have an opportunity to reflect on their learning and share it with their fellow students.

Extension

As an extension, suggest that students evaluate how accessible the stairs and other inclines throughout their school are. Is their school easily accessible for people with limited mobility or those in wheelchairs, or does accessibility need to be improved? Did they find it easy to climb the stairs and ramps they used in their calculations? If accessibility needs to be improved, how would students recommend doing this?

SOLUTIONS

Answers will vary. Students will probably suggest some creative ideas regarding issues of accessibility around their school.

ACTIVITY 1.7
SLOPE DIRECTION

STUDENT BOOK, p. 40

In this activity, students will use coordinates to sketch slopes to visualize the possible directions of slope and how positive and negative slopes can be created using combinations of positive and negative runs and rises.

To start this activity, some students may need to review some material. Ask students how many types of slope there are. They should be able to identify four types of slopes: positive, negative, zero, and undefined. Ask which directions slopes can be oriented. Positive slopes slant up and to the right, negative slopes slant down and to the right, zero slopes are horizontal lines, and undefined slopes are vertical lines. Then ask them to think about how a negative run or rise may affect the slope direction.

SOLUTIONS

1. Check that students have drawn a coordinate plane with four quadrants and have numbered at least 8 increments.
2. a) The arrow would be at the end of the line at point B.
   b) Line AB is positive.
3. a) The arrow would be at the end of the line at point D.
   b) Line CD is negative.
4. a) Line EF slopes the same way as line CD.
   b) The arrow would be at the end of the line at point E.
   c) Line EF is negative.
5. a) Line GH slopes the same way as line AB.
   b) The arrow would be at the end of the line at point G.
   c) Line GH is positive.
6. a) The arrow could be at point I or J.
   b) Line IJ has a slope of zero.
7. a) The arrow could be at point L or K.
   b) Line KL has an undefined slope.
8. a) The rise is 4 and the run is 11.
   b) This slope points up to the right.
   c) Another rise is –4 and the corresponding run is –11.
9. a) The rise is –7 and the run is 3.
   b) This slope points down to the right.
   c) Another rise is 7 and the corresponding run is –3.
10. a) A positive slope increases.
    b) A negative slope decreases.

DISCUSS THE IDEAS

CONNECTIONS AND CONVERSIONS

STUDENT BOOK, p. 42

This Discuss the Ideas allows students an opportunity to make connections among the different ways that slope has been considered so far in this chapter and to visualize the progression from the general formula for slope (rise over run) to the variety of ways that slope can be represented. To start the discussion, ask students to list the different ways they have seen the slope formula expressed. Then ask them whether these different expressions mean that the slope is different in each case. They should realize that they are equivalent to each other.

Slope can be represented as a decimal, fraction, ratio, percent, and angle. Because slope can be expressed using different forms, it is important for students to be able to solve problems and express the slope in any of these forms. Therefore, they should be able to convert between any two.

Start by asking students whether they can easily compare the steepness of a slope expressed as an angle and a slope expressed as a percentage. Then ask them what the problem is in doing this type of comparison. Students should realize that it is difficult to compare quantities with different units, so they must convert to the same units. By working in pairs, they can discuss and work out the conversions.

SOLUTIONS

1. Slope can be represented as a decimal, fraction, ratio, percent, and angle.
2. a) $\dfrac{\text{percent grade}}{100} = \text{slope}$

   b) $\tan^{-1}(\text{slope}) = \text{angle}$

   c) $\tan(\text{angle}) = \text{slope}$

   d) $\tan(\text{angle}) \times 100 = \text{percent grade}$

   e) $\tan^{-1} \dfrac{\text{percent grade}}{100} = \text{angle}$

Extend Your Learning

STUDENT BOOK, p. 43

Using the coordinates of pairs of points to calculate slope and then using the Pythagorean theorem to find the length of a particular slope is an extension to the outcomes identified in the Apprenticeship and Workplace Mathematics 11 curriculum, but many students will benefit from exploring this method of solving for slope.

Ask students to look at the graph on p. 43 of the student book and suggest how they might use the coordinates of the two points to find the rise and the run of the line segment illustrated. They will observe that they can find the values for rise and run by finding the coordinates of two points and calculating the amount of vertical change and the amount of horizontal change by subtracting the value of y_1 from y_2 and the value of x_1 from x_2.

If some students have difficulty understanding how to find values for rise and run, work through the process with them, using the graph "Using Coordinates to Calculate Slope" on p. 43 of the student book. Point out that when using a graph, it is easiest to work with points that lie on the intersection of the grid. Ask students to find the values of two points that lie on an intersection. The points marked on the sample graph in the student book are (4, 20) and (12, 60). Ask students to identify which values would represent the rise (the y values) and the run (the x values). Have them identify which number in each coordinate pair they will use. Then ask them to consider how they would find the amount of change between the points. The amount of change is found by subtraction. Work through the calculation with your students.

$$m = \frac{60 - 20}{12 - 4}$$

$$m = \frac{40}{8}$$

$$m = 5$$

The slope of the line segment is 5.

Ask students to rewrite the formula for slope using this calculation in place of "rise/run." They will likely be able to suggest that the formula can also be represented as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Suggest that in this case they can check their work by looking at the value of one point on the *y*-axis and divide it by the corresponding *y*-value.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{60}{12}$$

$$m = 5$$

The slope of the line segment is 5.

Now discuss with students how they can use the coordinates of this slope to find the length of the line segment it represents. Remind them that the slope forms the hypotenuse of a right triangle and thus the Pythagorean theorem can be used to solve for length. Example 2 works through one good example where this method of calculating a distance can be used.

ACTIVITY 1.8

USING COORDINATES TO CALCULATE SLOPE

STUDENT BOOK, p. 44

In this activity, students apply their knowledge of rise and run to the coordinate system.

Prior to having pairs of students work through the steps in this activity, you might ask them an open-ended question: How could you use the coordinates of line segments in a triangle to determine slope? Suggest they work on graph paper to determine possible solutions.

Introduce the activity by reviewing the rise over run formulas for slope and length. Students will use this information to develop slope and length formulas that use *x* and *y* coordinates. Students can use Blackline Master 1.8 (p. 72) to record their calculations.

From this activity, students will realize that the run is the difference of the *x* coordinates, $x_2 - x_1$ and that rise is the difference in the *y* coordinates, $y_2 - y_1$. They will then be able to use these forms to derive the slope and length formulas.

SOLUTIONS

1. Check that students have correctly recorded the *x* and *y* values in their table.
2. The *x* coordinate of C will be same as the *x* coordinate of B, which is 10. The *y* coordinate of C will be same as the *y* coordinate of A, which is 2.
3. To calculate rise, subtract the *y* coordinates of points B and C.

   8 – 2 = 6
4. To calculate run, subtract the *x* coordinates of points A and C.

   10 – 3 = 7
5. slope $= \frac{6}{7}$
6. The pattern is that the coordinates of point C are the *x* coordinate of point B and the *y* coordinate of point A. Students should see that they don't need to find point C and make a right triangle to calculate rise and run. The rise and run can be calculated directly using the coordinates of A and B. See solution for step 6 below.
7. The rise is the difference between the *y* coordinates $(y_2 - y_1)$. The run is the difference between the *x* coordinates $(x_2 - x_1)$.
8. $m = \frac{\text{difference between } y \text{ coordinates}}{\text{difference between } x \text{ coordinates}}$

   $m = \frac{y_2 - y_1}{x_2 - x_1}$

x coordinate of point A	*y coordinate of point A*	*x coordinate of point B*	*y coordinate of point B*	*Coordinates of point C*	*Rise*	*Run*	*Slope*
3	2	10	8	(10, 2)	6	7	0.857
–3	12	5	1	(–3, 1)	–11	8	–1.375

9. length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

DISCUSS THE IDEAS

THE ORDER OF COORDINATES

STUDENT BOOK, p. 46

In this Discuss the Ideas, students explore the importance of maintaining a consistent order when they substitute the coordinates into the slope formula. Start the discussion by asking students how they decide which coordinate is x_1 or x_2 and whether it matters how the values are substituted into the formulas. Then have students collaborate on the questions with a partner.

SOLUTIONS

1. When students calculate the slope, they will find that they obtain the same result as in Example 2. Therefore, it is acceptable to reverse the coordinates so long as both the x and y coordinates are reversed.
2. The y coordinates were reversed but the x coordinates were not, so the slope calculation gives a different result. Therefore, it is not acceptable to do this because you will get the opposite value for the slope.

$$\frac{12 - 28}{14 - 4} = \frac{-16}{10}$$

$$\frac{-16}{10} = \frac{-8}{5}$$

3. The order of the coordinates doesn't matter, so long as the x and y coordinates are substituted in the same order. That is, it does not matter which coordinate is x_1 or x_2 so long as students use the corresponding y values.
4. The order in which the coordinates are substituted into the formula for calculating the length of a particular slope doesn't matter because the difference is squared; therefore, the result will always be a positive value.

BUILD YOUR SKILLS SOLUTIONS

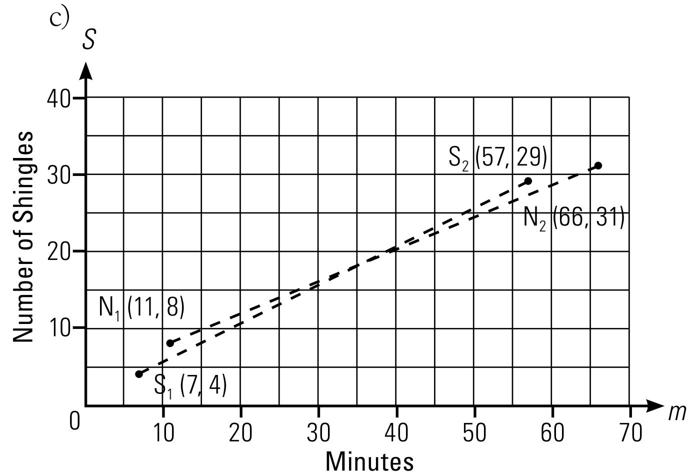
STUDENT BOOK, p. 46

1. a) This problem can be illustrated using a rate of change graph that compares the number of shingles laid to the time in minutes. Therefore, the independent variable would be time in minutes on the x-axis, and the dependent variable would be the number of shingles on the y-axis.

b) Nick's work as coordinates would be $N_1(11, 8)$ and $N_2(66, 31)$.

Sergio's work as coordinates would be $S_1(7, 4)$ and $S_2(57, 29)$.

c)



d) The rate of work is the number of shingles laid per minute. Use the formula for slope to find the rate of work.

Nick: $m = \frac{31 - 8}{66 - 11}$

$m = \frac{23}{55}$

$m \approx 0.42$

Sergio: $m = \frac{29 - 4}{57 - 7}$

$m = \frac{25}{50}$

$m = 0.5$

Nick lays 0.42 shingles/minute and Sergio lays 0.5 shingles/minute.

e) To find out how long it takes each person to lay one shingle, divide one shingle by the person's rate of work.

Nick: $\frac{1}{0.42} \approx 2.38$

Sergio: $\frac{1}{0.5} = 2$

It takes Nick 2.38 minutes to lay one shingle, and it takes Sergio 2 minutes to lay one shingle.

f) $5 \times 60 = 300$ Convert 5 hours to minutes.

Multiply the rate of work by 300 minutes.

Nick:

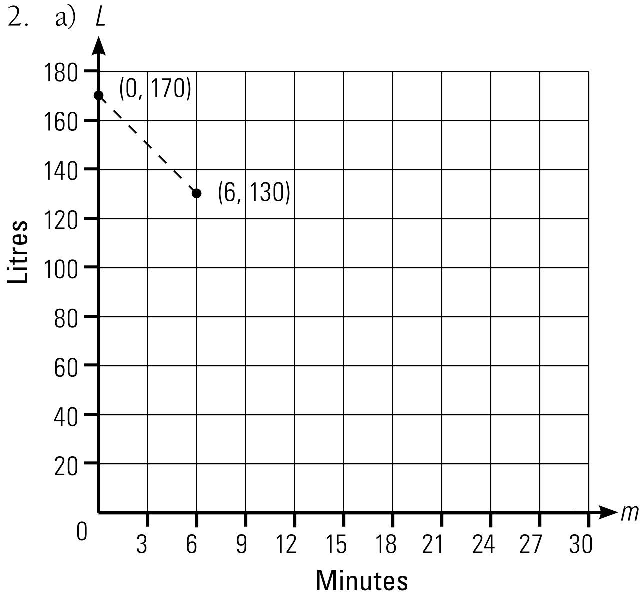
$0.42 \times 300 = 126$

Sergio:

$0.5 \times 300 = 150$

In 5 hours, Nick lays 126 shingles and Sergio lays 150 shingles.

2. a)



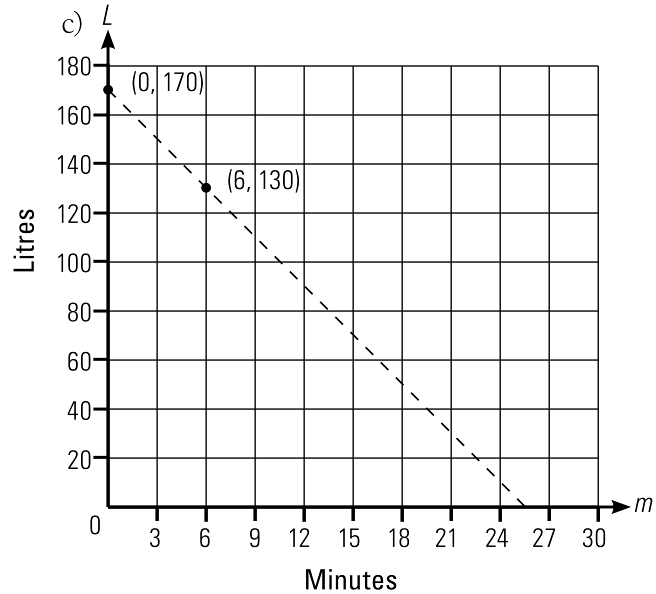
b) $m = \frac{170 - 130}{0 - 6}$

$m = \frac{40}{-6}$

$m \approx -6.7$

This slope means that the level of the water is decreasing at a rate of 6.7 litres every minute.

c)



Point plotted will vary, but the value of the slope will be the same. Therefore, the slope will be constant.

d) Students should be able to read the result from their graph. After 12 minutes, there will be 90 L left in the tank.

e) The graph shows that after 18 minutes, there will be 50 L left in the tank. Subtract the amount left from the total amount to find the amount that will have drained.

$$170 - 50 = 120$$

After 18 minutes, 120 L will have drained.

f) Students can read the result from their graph, which indicates that it will take about 25 minutes to drain the tank. Alternatively, they can calculate the amount of time as follows.

$\frac{170}{6.7} \approx 25.4$

g) The equation for this graph could be written as follows.
$w = -6.7(t) + 170$

3. a) Use the coordinates to find the slope.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{(-3) - (-13)}{(-4) - (-12)}$ Substitute the known values into the slope formula.

$m = \frac{10}{8}$

$m = \frac{5}{4}$ Simplify the fraction.

$m = 1.25$ Convert the fraction to a decimal.

The slope of the angled section is 1.25.

b) Use the coordinates to find the length of the angled section.

$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\text{length} = \sqrt{\left[(-4) - (-12)\right]^2 + \left[(-3) - (-13)\right]^2}$

$\text{length} = \sqrt{(8)^2 + (10)^2}$

$\text{length} = \sqrt{64 + 100}$

$\text{length} = \sqrt{164}$

$\text{length} \approx 12.8$

The length of the angled section is 12.8 mm.

4. a) Use the formula for the length of a line representing a specific slope to find the length of AC. Substitute the coordinates of A and C into the formula.

$\text{length} = \sqrt{(24 - 4)^2 + \left[(-1) - 1\right]^2}$

$\text{length} = \sqrt{(20)^2 + (-2)^2}$

$\text{length} = \sqrt{400 + 4}$

$\text{length} = \sqrt{404}$

$\text{length} \approx 20.1$

AC is 20.1 m.

Find the length of BD.

$\text{length} = \sqrt{(15 - 13)^2 + \left[(-10) - 10\right]^2}$

$\text{length} = \sqrt{(2)^2 + (-20)^2}$

$\text{length} = \sqrt{4 + 400}$

$\text{length} = \sqrt{404}$

$\text{length} \approx 20.1$

BD is 20.1 m.

The diagonal distances are equal, so the corners are 90°.

b) Use the formula for finding the length of a specific slope and the coordinates of line AB to find the width of the garage.

$\text{width} = \sqrt{(13 - 4)^2 + (10 - 1)^2}$

$\text{width} = \sqrt{9^2 + 9^2}$

$\text{width} = \sqrt{81 + 81}$

$\text{width} = \sqrt{162}$

$\text{width} \approx 12.7$

Alternatively, the width can be found using the coordinates of line CD, as follows.

$\text{width} = \sqrt{(24 - 15)^2 + \left[(-1) - (-10)\right]^2}$

$\text{width} = \sqrt{(9)^2 + (9)^2}$

$\text{width} = \sqrt{81 + 81}$

$\text{width} = \sqrt{162}$

$\text{width} \approx 12.7$

Find the length of the garage using the coordinates of line BC.

$\text{length} = \sqrt{(24 - 13)^2 + \left[(-1) - 10\right]^2}$

$\sqrt{(11)^2 + (-11)^2}$

$\text{length} = \sqrt{(11)^2 + (-11)^2}$

$\text{length} = \sqrt{121 + 121}$

$\text{length} = \sqrt{242}$

$\text{length} \approx 15.6$

Alternatively, find the length using the coordinates of line AD.

$$\text{length} = \sqrt{(15-4)^2 + (-10-1)^2}$$

$$\text{length} = \sqrt{(11)^2 + (-11)^2}$$

$$\text{length} = \sqrt{121 + 121}$$

$$\text{length} = \sqrt{242}$$

$$\text{length} \approx 15.6$$

The garage is 12.7 m wide and 15.6 m long.

c) The diagonal measurement is the hypotenuse of the right triangle formed.

5. a) Substitute the coordinates of lines A, B, C, D, and E into the slope formula.

$$m_A = \frac{6-10}{1-0}$$

$$m_A = \frac{-4}{1}$$

$$m_A = -4$$

$$m_B = \frac{2-6}{4-1}$$

$$m_B = \frac{-4}{3}$$

$$m_B \approx -1.3$$

$$m_C = \frac{2-2}{4-6}$$

$$m_C = \frac{0}{-2}$$

$$m_C = 0$$

$$m_D = \frac{4-2}{7-6}$$

$$m_D = \frac{2}{1}$$

$$m_D = 2$$

$$m_E = \frac{6-4}{7.5-7}$$

$$m_E = \frac{2}{0.5}$$

$$m_E = 4$$

b) A: The marble is rolling down a steep slope.

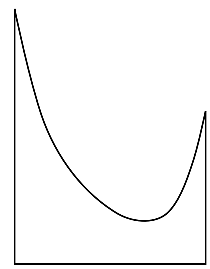
B: The marble is rolling down a less steep slope.

C: The marble is rolling horizontally.

D: The marble is rolling up a slope.

E: The marble is rolling up a steeper slope.

c)



6. At first glance, Graph B's slope appears steeper. However, Suzanne's simple application of a rule is incorrect. She did not consider the different scales on the axes, which will affect how the slope will appear on the graph. The calculations for the slopes are as follows.

$$m_A = \frac{200-100}{8-2}$$

$$m_A = \frac{100}{6}$$

$$m_A \approx 16.7$$

$$m_B = \frac{4-1}{4-1}$$

$$m_B = \frac{3}{3}$$

$$m_B = 1$$

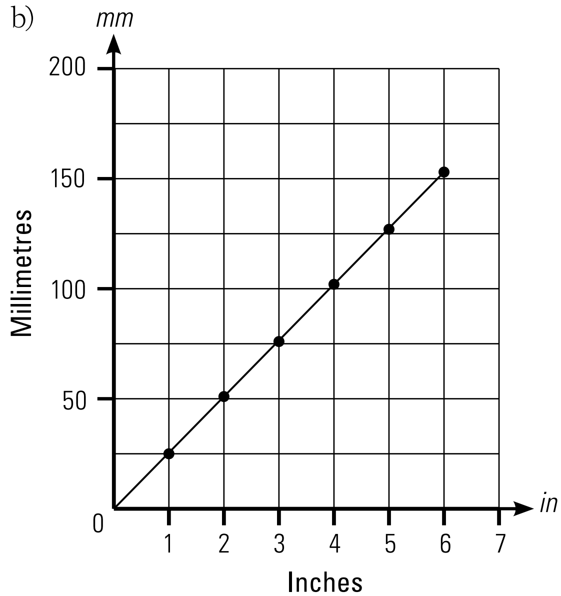
Graph A has a greater slope. You need to be careful when comparing graphs. Make sure that the scales are the same.

7. They are both correct. Line segment AB is negative for Frank because, from his perspective, the roof slopes down from left to right. Line segment DE is positive for Maxine because, from her perspective, the roof slopes up from left to right. Positive and negative refer to the direction of the line segment representing a slope, which is relative to your point of reference.

8. a) Answers may vary by 1 mm depending on how accurately students read the ruler.

Inches	*Millimetres*
1	25
2	51
3	76
4	102
5	127
6	153

b)



c) Any two points on the graph can be used to calculate the slope. Answers may vary depending on the accuracy of reading the ruler and also how students rounded the measurements. For example,

$$m = \frac{153 - 51}{6 - 2}$$

$$m = \frac{102}{4}$$

$$m = 25.5$$

The slope is the conversion factor between inches and millimetres.

d) The slope value is quite close to the actual conversion factor of 25.4 but is not exact. With the scale of the graph and the ruler, it is difficult to read millimetres to the precision of 1 decimal place.

e) The equation for this graph would be stated as: $m = 25.5i$.

9. a) Graph A and Graph B appear to have the same steepness.

b) Graph A:

$$m = \frac{4 - 1}{6 - 1}$$

$$m = \frac{3}{5}$$

$$m = 0.6$$

Graph B:

$$m = \frac{4 - 1}{1 - 6}$$

$$m = \frac{3}{-5}$$

$$m = -0.6$$

Graph A and Graph B have the same amount of slope. The negative sign here does not refer to quantity or magnitude; it is used to indicate the direction of the line representing the slope.

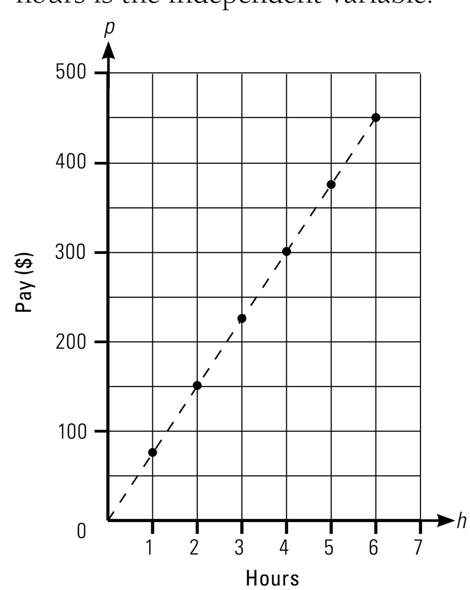
Extend Your Thinking

10. a)

Hours	*Pay*
1	\$75.00
2	\$150.00
3	\$225.00
4	\$300.00
5	\$375.00
6	\$450.00

b) Pay is the dependent variable; number of hours is the independent variable.

c)



Any two points on the graph can be used to calculate the slope. For example,

$$m = \frac{375 - 150}{5 - 2}$$

$$m = \frac{225}{3}$$

$$m = 75$$

The slope is 75.

d) The slope represents the rate of pay an hour, which is $75.00/hour.

REFLECT ON YOUR LEARNING

SLOPE AND RATE OF CHANGE

Students have completed activities and questions that allowed them to construct and measure rise, run, and slope. They have also learned how tradespeople use slope and rate of change to help them do their jobs.

Ask students to read through the list of items they have learned about in this chapter. Ask them to give examples of where they encounter these items in their day-to-day lives. You can also ask them to define some of the key terms included in this list, such as rise, run, or rate of change. Asking students to explain how they used these items when they are completing the chapter project will also help them to see how this chapter's learning can be applied in a practical way.

PRACTISE YOUR NEW SKILLS

STUDENT BOOK, p. 52

SOLUTIONS

1. a) negative
   b) zero
   c) positive
   d) undefined
2. a) Since the run is the same for AF, BF, and CF, it is the difference in rise that influences the steepness of the lines.
   b) Since the rise is the same for CF, DF, and EF, it is the difference in run that influences the steepness of the lines.
   c) There is not one particular factor. Steepness requires both a rise and run; its amount will depend on how each changes with respect to the other.
3. Count the squares to find the slope. The rise is 10 units and the run is 6 units.

   slope: $\frac{10}{6} = \frac{5}{3}$

   fraction: $\frac{10}{6} = \frac{5}{3}$

   decimal: $\frac{5}{3} = 1.67$

   percent: $1.67 \times 100 = 167\%$

   angle: $\tan^{-1}\left(\frac{5}{3}\right) \approx 59°$
4. Use the coordinates to find how far apart the two flag poles are:

   $$\text{length} = \sqrt{(127 - 371)^2 + (444 - 85)^2}$$

   $$\text{length} = \sqrt{59\,536 + 128\,881}$$

   $$\text{length} = \sqrt{188\,417}$$

   $$\text{length} \approx 434 \text{ m}$$

   They are 434 m apart.
5. Convert both slopes to a decimal in order to compare them.

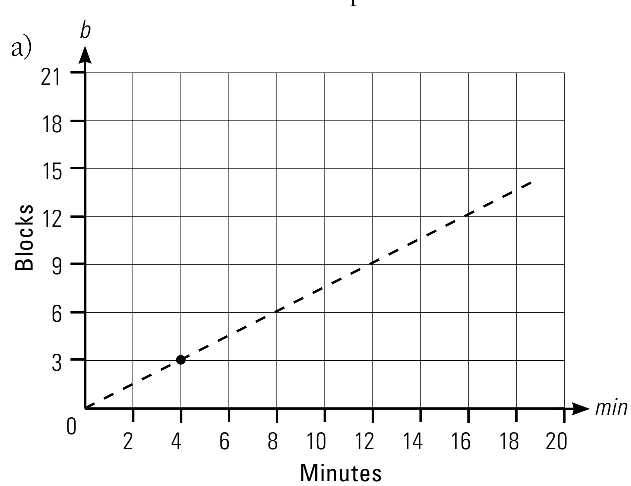
   Fir Street:

   $\tan(18.1°) = 0.3269$

   Balsam Street:

   $\frac{32.6}{100} \approx 0.326$

   The hill on Fir Street is steeper.
6. a)



b) $m = \dfrac{3-0}{4-0}$

$m = \dfrac{3}{4}$

$m = 0.75$

c) The slope represents Shizuko's distance from the house, which is increasing at a rate of 0.75 blocks per minute.

d) Answers may vary depending on how carefully students have drawn their graph, but they should state that she is around 10 blocks from the house.

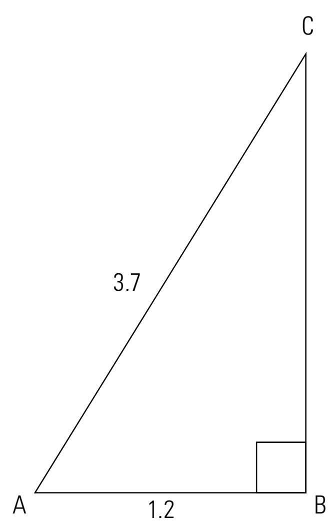
e) $d = 0.75t$

f) $d = 0.75(13)$

$d = 9.75$ blocks

If students have not obtained the same answer as 6d), they could improve the accuracy of their graph by marking the axes in smaller increments.

7. To find the slope, we need to find the rise, BC. Use the Pythagorean theorem.



$$AC^2 = AB^2 + BC^2$$
$$3.7^2 = 1.2^2 + BC^2$$
$$3.7^2 - 1.2^2 = BC^2$$
$$\sqrt{3.7^2 - 1.2^2} = BC$$
$$\sqrt{13.69 - 1.44} = BC$$
$$\sqrt{12.25} = BC$$
$$BC \approx 3.5$$

The rise is 3.5. The run is 1.2.

$\dfrac{3.5}{1.2} \approx 2.9$

The slope is 2.9. No, the ladder is not safe.

8. a) First, convert 0.65 to a fraction.

$0.65 = \dfrac{65}{100}$

$\dfrac{65}{100} = \dfrac{13}{20}$

Use the Pythagorean theorem to find the hypotenuse.

$c^2 = 13^2 + 20^2$

$c \approx 23.9$

Set up equivalent ratios.

$13 \times \dfrac{4000 \text{ m}}{23.9} = \dfrac{\text{rise}}{13} \times 13$

rise $\approx$ 2175.7 m

Their elevation gain is 2175.7 m.

b) Find the elevation at 5 km and subtract the elevation at 4 km.

$\dfrac{5000}{23.9} = \dfrac{\text{rise}}{13}$

rise $\approx$ 2719.7

$2719.7 - 2175.7 = 544$

They will be 544 m higher.

SAMPLE CHAPTER TEST

Name: ______________________ Date: ______________________

Part A: Multiple Choice

1. A line that goes up to the right has a ____________ slope and is ____________.

   a) negative, increasing b) positive, decreasing

   c) positive, increasing d) negative, decreasing

2. A line that goes down to the right has a ____________ slope and is ____________.

   a) negative, increasing b) positive, decreasing

   c) positive, increasing d) negative, decreasing

3. What is the slope of a line with coordinates (–10, 1) and (2, 7)?

   a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) –2 d) 2

4. Which of the following is a slope of 4?

   a) $\frac{-12}{-3}$ b) $\frac{4}{4}$ c) $\frac{1}{4}$ d) $\frac{4}{-1}$

5. Which of the following have the same slope?

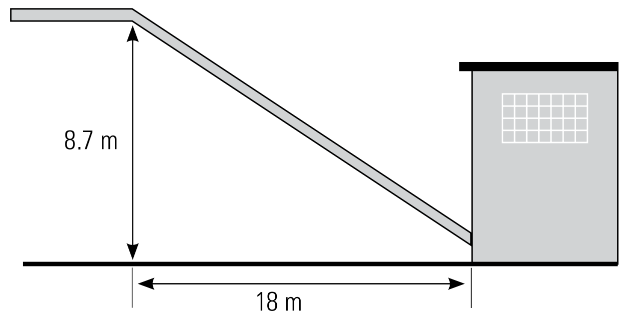
   a) 29° and $\frac{1}{2}$ b) 2.2 and 65° c) $\frac{7}{11}$ and 65% d) 9:8 and 112.5%

Part B: Short Answer

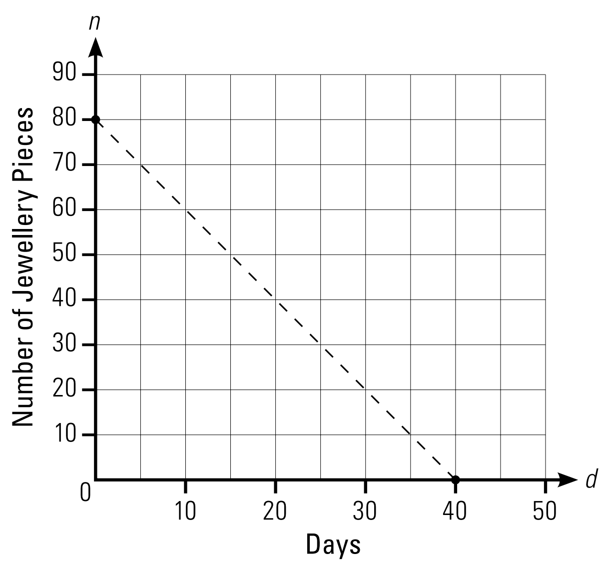
6. What is the slope of a 112% grade?

7. A ramp is 26″ high and has a run of 10′. How long is the ramp? Round to the nearest inch.

8. The high point of a pipeline is 8.7 m above the ground. It has a downward slope of 0.45. At what height will it enter the refinery 18 m away?



9. Jenna makes necklaces and bracelets that she sells during summer vacation. The graph below shows the number of pieces of jewellery she had in stock for the number of days in the summer that she spent selling the jewellery.



a) Calculate the slope. What does the slope of this graph tell you?

b) How many pieces of jewellery did Jenna start with?

c) How many days did it take to sell them all?

Part C: Extended Answer

10. An 8.2 km section of highway increases 870 m in elevation.

    a) Calculate the slope to three decimal places.

    b) Calculate the angle of elevation to one decimal place.

    c) Calculate the percent grade to one decimal place.

11. Justine and Jason are working as pastry chefs. Justine had made 5 cream puffs after 5 minutes and 20 after 55 minutes. Jason had made 10 cream puffs after 5 minutes and 20 after 60 minutes.

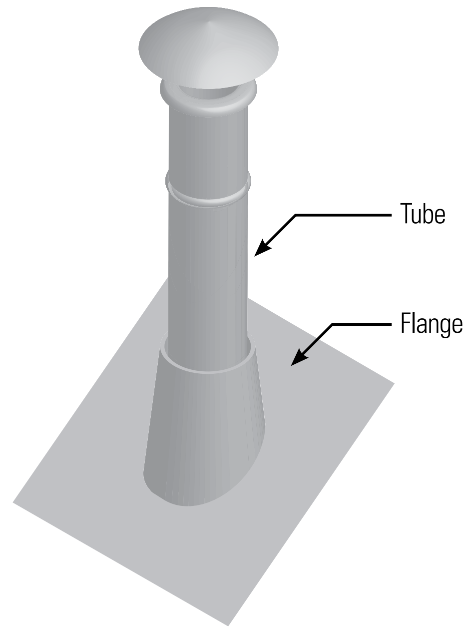
    a) Draw a rate of change graph that show Justine's and Jason's work.

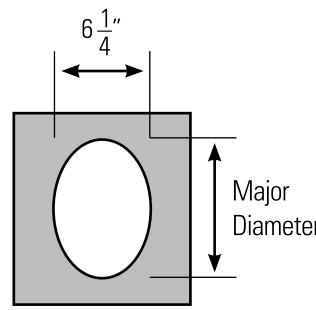
    b) At what rates are Justine and Jason making cream puffs?

    c) After how many minutes have they each made the same number of cream puffs?

    d) Predict how long it will take each person to make 25 cream puffs.

12. Mingmei is a sheet metal worker who is making flashing for a plumbing vent pipe that will be installed on a 5:12 roof. The vent pipe is $6\frac{1}{4}''$ in diameter. She needs to cut an elliptical hole in the pipe flashing to match the angled cut on the bottom of the flange tube. The minor diameter of the hole is $6\frac{1}{4}''$. What is the major diameter of the hole? Express your answer to one decimal place.





SAMPLE CHAPTER TEST: SOLUTIONS

Part A: Multiple Choice

1. c) positive, increasing

2. d) negative, decreasing

3. b) $\frac{1}{2}$

4. a) $\frac{-12}{-3}$

5. d) 9:8 and 112.5%

Part B: Short Answer

6. $\frac{112}{100} = 1.12$

7. $\text{length} = \sqrt{26^2 + 120^2}$

   $\text{length} \approx 123''$

   The ramp is 123″ long.

8. Find the drop (rise) and subtract from the high point.

$$0.45 = \frac{\text{rise}}{18}$$

$$18 \times 0.45 = \frac{\text{rise}}{18} \times 18$$

$$\text{rise} = 8.1$$

$$8.7 - 8.1 = 0.6$$

It enters 0.6 m or 60 cm above the ground.

9. a) Use the coordinates (0, 80) and (40, 0) to calculate the slope.

$$m = \frac{0 - 80}{40 - 0}$$

$$m = \frac{-80}{40}$$

$$m = -2$$

The slope of the graph is –2. The slope tells you that Jenna's stock of jewellery is decreasing at a rate of 2 pieces per day.

b) The y-axis shows that she started with 80 pieces on day 0.

c) The x-axis shows it took 40 days for the number of pieces to reach 0.

Part C: Extended Answer

10. a) First, use the Pythagorean theorem to find the run, then calculate the slope.

$$\text{run}^2 = 8200^2 - 870^2$$

$$\text{run}^2 = 67\,240\,000 - 756\,900$$

$$\text{run} = \sqrt{66\,483\,100}$$

$$\text{run} \approx 8154 \text{ m}$$

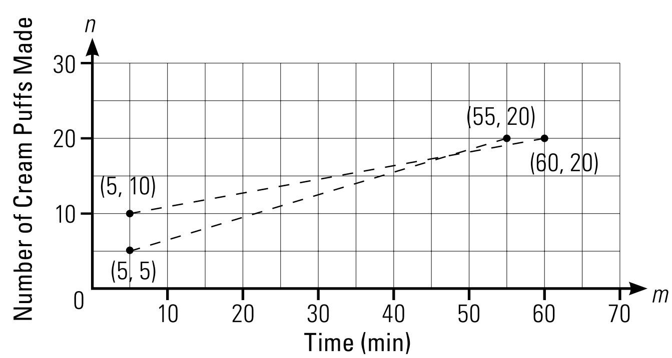
$$\frac{870}{8154} \approx 0.107$$

The slope of the highway is 0.107.

b) $\tan^{-1}(0.107) \approx 6.1°$

c) $0.107 \times 100 = 10.7\%$

11. a)



b) Justine's rate:

$$\frac{20 - 5}{55 - 5} = \frac{15}{50}$$

$$\frac{15}{50} = 0.30$$

Justine makes 0.30 cream puffs a minute.

Jason's rate:

$$\frac{20 - 10}{60 - 5} = \frac{10}{55}$$

$$\frac{10}{55} \approx 0.18$$

Jason makes 0.18 cream puffs a minute.

c) The graph indicates the two lines intersect at about 47 minutes. Therefore, it would take 47 minutes for Justine and Jason to make the same number of cream puffs.

d) Extrapolation from the graph shows that Justine makes 25 cream puffs after about 72 minutes, and Jason makes 25 cream puffs after about 88 minutes.

12. The hole in the flashing is elliptical to match the profile of the cut made on the bottom of the pipe to match the slope of the roof. The major diameter of the ellipse is the length of the angled cut, and the run is the $6\frac{1}{4}''$ minor diameter.

First, use the Pythagorean theorem to calculate the length of the hypotenuse of a 5:12 slope.

$$c = \sqrt{12^2 + 5^2}$$

$$c = \sqrt{169}$$

$$c = 13$$

Create equivalent fractions to find the major diameter of the hole.

$$\frac{13}{12} = \frac{x}{6.25}$$

$$6.77 \approx x$$

The major diameter of the hole is 6.8″.

BLACKLINE MASTER 1.1

CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ In addition to the jumps, what other features will your terrain park include (rails, quarter pipes, etc.)?	
❒ How can the features be modified for different skill levels?	
❒ How will you test your jumps and measure the results?	
❒ How will you modify your jumps to create three different levels of difficulty?	
❒ How will you construct your model jumps? What materials could you use?	
❒ Other notes	

BLACKLINE MASTER 1.2

CHAPTER PROJECT: TEST YOUR DESIGNS AND GATHER DATA

Name: ____________________ Date: ____________________

BEGINNER JUMP						
Rise/run of downhill section	*Rise/run of jump*	*Distance between downhill section and jump*	**DISTANCE MARBLE TRAVELLED**			
			Trial 1	*Trial 2*	*Trial 3*	*Average*

INTERMEDIATE JUMP						
Rise/run of downhill section	*Rise/run of jump*	*Distance between downhill section and jump*	**DISTANCE MARBLE TRAVELLED**			
			Trial 1	*Trial 2*	*Trial 3*	*Average*

EXPERT JUMP						
Rise/run of downhill section	*Rise/run of jump*	*Distance between downhill section and jump*	**DISTANCE MARBLE TRAVELLED**			
			Trial 1	*Trial 2*	*Trial 3*	*Average*

BLACKLINE MASTER 1.3

GRAPH PAPER (0.5 CM X 0.5 CM)

Name: ______________________ Date: ______________________

BLACKLINE MASTER 1.4

DESIGN A TERRAIN PARK: STUDENT SELF-ASSESSMENT

Name: ______________________________ Date: ______________________________

Partner/group member names: ______________________________

To evaluate how well you did on your project, you will want to consider the following:

- the accuracy of your calculations and the completeness of your work;
- the effectiveness of your use of technology for organizing your project and for creating your final presentation;
- the creativity you brought to planning and creating the terrain park;
- your completion of all the assigned tasks on time; and
- your contributions to the group.

How do you feel you have done, given the criteria above? ______________________________

____

____

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

____

____

In what areas did you excel? ______________________________

____

____

Are there areas in which you could improve? ______________________________

____

What strengths did each person in your group bring to the project?

____

____

____

If you had to do the project over again, what would you do differently?

____

____

____

BLACKLINE MASTER 1.5

SLOPE AND TANGENT

Name: ____________________ Date: ____________________

Use the space below to answer questions 1 and 2 in Activity 1.4.

1.

2.

3.

SLOPE AND TANGENT RATIO		
Triangle 1		*Triangle 2*
rise	is the same as	
run	is the same as	
slope	is the same as	

4. slope $\frac{\boxed{}}{\boxed{}}$

tan $\frac{\boxed{}}{\boxed{}}$

BLACKLINE MASTER 1.6

LENGTH OF A PARTICULAR SLOPE

Name: ______________________________ Date: ______________________________

Use the space below to answer questions 1 and 2 in Activity 1.5.

LENGTH OF A PARTICULAR SLOPE		
Triangle 1		*Triangle 2*
a	is the same as	
b	is the same as	
c	is the same as	

Formula for the length of the hypotenuse:

____

Formula for the length of a line representing a particular slope:

____

BLACKLINE MASTER 1.7

CLIMBING AND CALCULATING

Name: ____________________ Date: ____________________

Use the table below to answer question 2 in Part A of Activity 1.6.

MEASURING INCLINES			
Slope description	*Rise (metres)*	*Run (metres)*	*Slope*

Use the table below to answer question 1 in Part B of Activity 1.6.

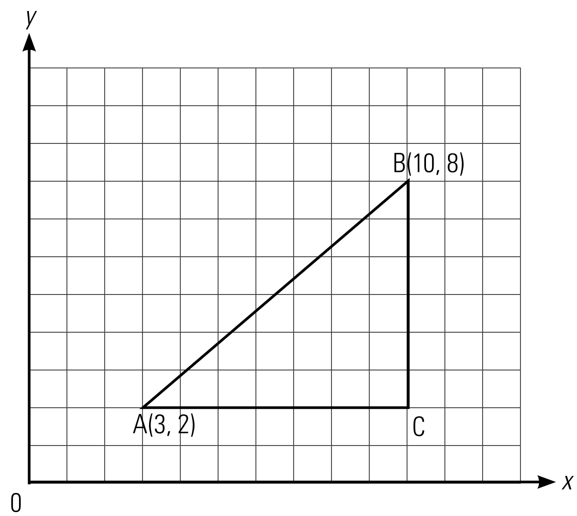
RATE OF CHANGE			
Slope	*Number of steps*	*Time (seconds)*	*Height (metres)*

BLACKLINE MASTER 1.8

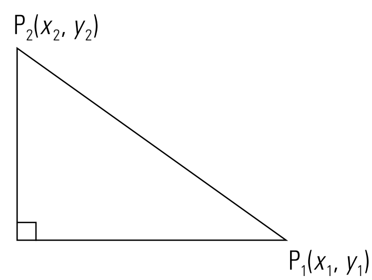
USING COORDINATES TO CALCULATE SLOPE

Name: ______________________ Date: ______________________

Use the space below to answer questions 1–5 and 7–8 in Activity 1.8.



COORDINATES AND SLOPE							
x coordinate of point A	*y coordinate of point A*	*x coordinate of point B*	*y coordinate of point B*	*coordinates of point C*	*Rise*	*Run*	*Slope*



$$m = \frac{\text{difference between } \square \text{ coordinates}}{\text{difference between } \square \text{ coordinates}}$$

$$m = \frac{\square}{\square}$$

ALTERNATIVE CHAPTER PROJECT—WHEELCHAIR ACCESSIBILITY

TEACHER MATERIALS

GOALS: In this project, students will use the concepts of slope and rate of change to build skills and to synthesize learning in this chapter.

OUTCOME: In this project, students will apply what they have learned in this chapter to solve a real-world problem of adapting a building to be wheelchair accessible.

PREREQUISITES: To complete this project, students will need to understand how to calculate slope. They must also be able to think creatively and critically to solve problems within given parameters.

ABOUT THIS PROJECT: This project is divided into four parts. Initially, students will plan their project by sketching out possible layouts and routes for the ramps to follow to satisfy the functional and safety criteria. Once students have explored the different possibilities for solving the problem, they can apply their knowledge of slope to design ramps, including making calculations for each ramp section. After they have created their formal drawings with calculations, they will draw graphs showing the rate of change of elevation over a horizontal distance for the ramp paths. As a final activity, students will present their solution to the class.

T Some students may be able to create their graphs electronically, using software such as spreadsheet programs.

Students should be given a few class periods to work on this project during the time spent on this chapter. This will allow for questions from students and feedback from the teacher, as well as allowing the teacher to observe the quality of work as it is done, rather than at the end of the chapter. This project can be done by small groups or pairs of students.

The building plans (Blackline Master 1.1A, p. 78) and a self-assessment rubric (Blackline Master 1.2A, p. 79) should be handed out to the students early in the project. The self-assessment rubric outlines the criteria for evaluation of their project and suggests some ways for students to reflect on their learning.

1. Start to plan

T Introduce the project to your students as you begin this chapter. As a class, students can discuss what their lives would be like if they used a wheelchair. What changes would they have to make if they were to become disabled? From this, students can brainstorm ideas about how to design wheelchair ramps. Consider what limitations there would be and what safety and functional considerations they would have to make. Then think of solutions to these problems and sketch some of these design solutions. Students can research wheelchair design considerations on the internet.

2. Create scale drawings

Once students have sketched ideas of how to gain elevation through switchbacks or zig-zags, they can begin applying their ideas to working out solutions for the building ramps. Students will need to make scale drawings for all the slopes as well as calculate all the slope values for all the ramps and express them as fractions, decimals, angles, and percents.

3. Draw rate of change graphs

In this section, students will need to calculate the lengths of the actual ramp sections as well as landings used. They must do this to collect data for the rate of change graphs. They can then compare their solution with the solutions of other groups.

4. Make a presentation

In this part of the project, students will present their solutions to the class. Have students set up their charts and graphs around the room so that they can all be seen. Compare the different solutions.

ASSESSING THE PROJECT

1. Start to plan

- Record your observations. Provide students with numeric information on how they will be assessed, using a scheme that meets your reporting needs. Look for completeness and clarity in the sketches.

2. Create scale drawings

- Provide students with numeric information on how they will be assessed.
- Ensure that they have created solutions for both problems.
- Check that students have completed scale drawings for all the ramps as well as included all slope calculations as fractions, decimals, angles, and percents.

3. Draw rate of change graphs

- Ensure that the students make a separate graph for each problem. The graphs should show the path of the wheelchair, including landings, and the top of the slopes ending at 20″ and 9′.

4. Make a presentation

- Use the rubric on p. 75 as a gauge to accompany a numerical grading rubric you have created that meets your needs.
- Ask students to self-assess their projects using Blackline Master 1.2A (p. 79). Projects can be arranged around the room so that students can provide constructive feedback to their peers. If some students have created electronic presentations, arrange to have a projector available.

PROJECT ASSESSMENT RUBRIC: WHEELCHAIR ACCESSIBILITY				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Explanations show understanding of creating and calculating slope	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are often incomplete or somewhat confusing	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and thorough
Procedural Understanding				
Accurately: • sketches designs and plans • draws scale diagrams of slopes • calculates the slope values • creates rate of change graphs • compiles information for a presentation	limited accuracy; major errors or omissions For example: • sketches are incomplete • diagrams not to scale and missing information • many calculation errors • graphs incomplete • project is incomplete	partially accurate; some errors or omissions For example: • sketches missing some information • diagrams not drawn to scale and missing some information • a few calculation errors • graphs missing information and inaccurate • project could use more work to ensure information is complete and accurate	generally accurate; few errors or omissions For example: • sketches missing little information • diagrams drawn to scale but missing some information • very few calculation errors • graphs have a few mistakes • project is complete and meets minimum requirements	accurate and precise; very few or no errors For example: • sketches complete • diagrams drawn to scale and information is complete • no calculation errors • graphs complete and accurate • extra creativity added to project
Problem-Solving Skills				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
Communication				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

ALTERNATIVE CHAPTER PROJECT —WHEELCHAIR ACCESSIBILITY

STUDENT MATERIALS

PROJECT OVERVIEW

In this project, your team has been commissioned to make a two-storey building fully accessible for people in wheelchairs. An elevator cannot be installed in the building, so you must create a safe ramp system inside the building that will enable a person to enter the building on the first floor and also get up to and down from the second floor.

Safety and function are of the utmost concern when designing wheelchair ramps. Some questions to think about when people design and build wheelchair ramps include: "In what ways can a steep ramp affect a user on the way up?" and "What effects does a steep ramp have on the user on the way down?" These considerations make it necessary to have safety standards to which all wheelchair ramps must be constructed.

The safety specifications for wheelchair ramps state that the maximum slope is 1:12.

There is also a maximum rise of 30″ per section of ramp, meaning that the maximum run of a section is 30′. A level landing must be built as a rest area. The landing can also function as a place to change directions at 90° or 180° depending on the space.

Your challenge is to work within the dimensional constraints of the building and the safety standards to create the two ramp systems. The two diagrams on Blackline Master 1.1A (p. 78) show a top view of the building on the property and a front view of the building showing the elevations of the entrance and the second floor.

GET STARTED

For this project, you will need to:
- create sketches to brainstorm possible solutions to the problems;
- create scaled drawings of the top view showing the change in horizontal directions of the ramps and a front view showing the change in vertical directions of the ramps. Include lengths of the rise, run, and slopes as well as the calculations for slope as a number, angle, and percent;
- create graphs showing the rate of change of the vertical distance gained compared to the distance travelled for each ramp system; and
- give a presentation of your solution to the class.

To begin your project, consider the restrictions of the safety specifications and brainstorm different ways in which you can create ramp systems that would allow a person to enter the building and go up to and down from the second floor safely in a wheelchair. Sketch out some different ideas and then narrow them down to the final designs.

CREATE SCALE DRAWINGS

Using the final design sketches that you drew up, work out the exact dimensions of all the ramps and landings. On the top view of your solution, draw a scale diagram of the paths that each ramp system would follow. On the front view, draw scaled diagrams of each of the slopes. Include the calculations for each of these slopes and express the values as fractions, decimals, angles, and percents.

DRAW RATE OF CHANGE GRAPHS

For each ramp system, create a graph that shows the relationship of vertical distance compared to distance travelled. You will need to calculate the lengths of each ramp as well as the lengths of the landings used in your solutions. Each slope and landing should be linked to form a continuous broken line graph.

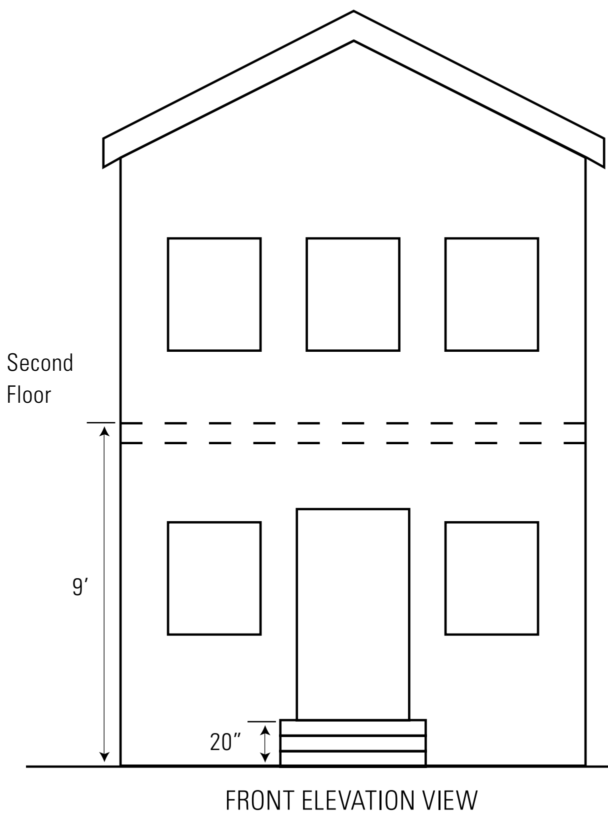
COMPILE YOUR WORK AND PREPARE YOUR PRESENTATION

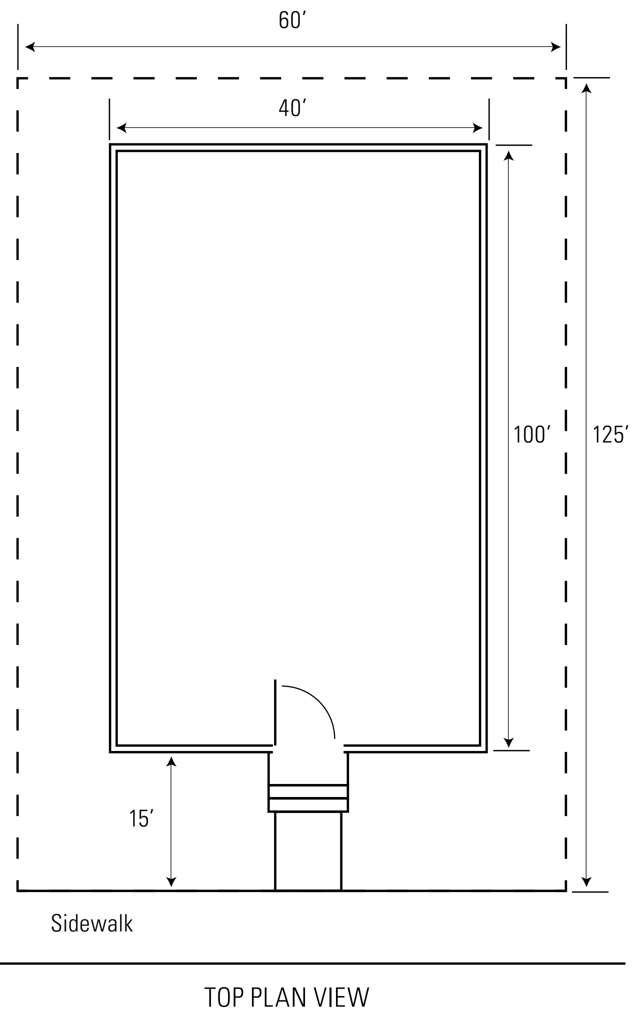
To conclude the project, you will present your solution to the class. Choose a spot in the room to set up your drawings and graphs so that the class can see them. Explain how you decided on your solution and compare your solution with those of the other students in your class.

BLACKLINE MASTER 1.1A

WHEELCHAIR ACCESSIBILITY PLANS

Name: ____________________ Date: ____________________





BLACKLINE MASTER 1.2A

WHEELCHAIR ACCESSIBILITY PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________________ Date: ______________________________

Partner/group member names: ______________________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research;
- the accuracy of your measurements and calculations for all sections of the project;
- the creativity you brought to planning and presenting;
- your contributions to your group; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________________

Were you able to complete all aspects of the project? If not, why? Did you allot your time effectively?

In what areas did you excel? ______________________________

Are there areas in which you could improve? ______________________________

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 1.9

REVIEWING PRIOR CONCEPTS

Name: ______________________ Date: ______________________

Expressing a numerical value as a fraction, ratio, or decimal, and converting between them

1. Fill in the following table.

NUMERICAL VALUE		
Fraction	*Decimal*	*Ratio*
$\frac{25}{1000}$		
	0.04	
		3:5
$\frac{60}{180}$		
	4.0	
		9:8
$\frac{4}{10}$		

Describing and drawing right triangles

2. Draw three right triangles of various sizes, labelling each angle in them.

Using algebra to solve for one unknown variable

3. Solve, giving the value of x.

a) $2x = \frac{-4}{5}$

b) $\frac{-6}{-4} = \frac{-3x}{-2}$

c) $\frac{x}{1.2} = 0.55$

d) $\frac{1}{6} + \frac{1}{3} = \frac{4x}{5}$

e) $\frac{6.3}{x} = 3(-5.05)$

Proportional reasoning

4. A micron is one millionth of a metre. A computer chip is about 4 microns wide. A scale diagram of a computer chip is 10 cm wide. What is the scale of the drawing?

5. The sides of a triangle measure 6 cm, 10 cm, and 12 cm. If one side of a similar triangle (corresponding to 6 cm) measures 15 cm, what are the lengths of the other two sides?

Squares and square roots

6. Estimate and calculate the area of a square with a side length of 2.7 cm.

7. What is the square root of 169? Explain how you arrived at your answer.

8. Estimate and calculate the square root of 0.74. Explain how you arrived at your answer.

BLACKLINE MASTER 1.9: SOLUTIONS

Expressing a numerical value as a fraction, ratio or decimal, and converting between them

1.

NUMERICAL VALUE		
Fraction	*Decimal*	*Ratio*
$\frac{25}{1000}$	0.025	1:40
$\frac{1}{25}$	0.04	1:25
$\frac{3}{5}$	0.6	3:5
$\frac{60}{180}$	$0.\bar{3}$	1:3
$\frac{20}{5}$	4.0	4:1
$\frac{9}{8}$	1.125	9:8
$\frac{4}{10}$	0.4	2:5

Describing and drawing right triangles

2. Answers will vary. Students should provide three drawings, with the right angles drawn at 90° and the other two angles in each triangle drawn so that their sum is also 90°.

Using algebra to solve for one unknown variable

3. a) $2x = \frac{-4}{5}$

$x = \frac{-4}{10}$

$x = \frac{-2}{5}$

b) $\frac{-6}{-4} = \frac{-3x}{-2}$

$\frac{3}{2} = \frac{3}{2}x$

$1 = x$

c) $\frac{x}{1.2} = 0.55$

$x = 0.66$

d) $\frac{1}{6} + \frac{1}{3} = \frac{4x}{5}$

$\frac{1}{6} + \frac{2}{6} = \frac{4x}{5}$

$\frac{3}{6} = \frac{4x}{5}$

$\frac{15}{6} = 4x$

$\frac{15}{24} = x$

$\frac{5}{8} = x$

e) $\frac{6.3}{x} = 3(-5.05)$

$\frac{6.3}{x} = -15.15$

$6.3 = -15.15x$

$\frac{6.3}{-15.15} = x$

$-0.42 \approx x$

Proportional reasoning

4. 10 cm : 4 microns
1 cm : 0.4 microns

5. $6x = 15$

$x = \frac{15}{6}$

$x = 2.5$

$10 \text{ cm} \times 2.5 = 25 \text{ cm}$

$12 \text{ cm} \times 2.5 = 30 \text{ cm}$

Squares and square roots

6. Answers may vary for estimation, but should be between 7 cm^2 and 8 cm^2.

$2.7 \times 2.7 = 7.29 \text{ cm}^2$

7. The square root of 169 is 13. Students could use trial and error, a calculator, or just known from memory. The main point is that they understand that 13 multiplied by itself equals 169 and that is why it is the square root.

8. The answers may vary for the estimate, but the number should be greater than 0.74 but less than 0.9. The actual answer is 0.860.

Chapter 2

Graphical Representations

INTRODUCTION

STUDENT BOOK, pp. 54–113

In this chapter, students will revisit graphing, a topic that they have been exposed to in earlier years. They will consider line graphs, bar graphs, histograms, and circle graphs and will be asked to determine which is the better representation for transmission of information. Computer access for students during class will be beneficial throughout this chapter.

STATISTICS, GRADES 11–12

This chart illustrates the development of the Statistics strand in the Apprenticeship and Workplace Mathematics pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 2 addresses.

Grade 10	*Grade 11*	*Grade 12*
	General Outcome Develop statistical reasoning.	**General Outcome** Develop statistical reasoning.
	Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:
	Solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.	Solve problems that involve measures of central tendency, including mean, median, mode, weighted mean, and trimmed mean.
		Analyze and describe percentiles.

CURRICULUM AND CHAPTER OVERVIEW

General outcome: Statistics
Develop statistical reasoning.

Specific outcome:
Solve probems that involve creating and interpreting bar graphs, histograms, line graphs, and circle graphs.

Section 2.1: Broken Line Graphs

- Math on the Job
- Explore the Math
- Discuss the Ideas: Extrapolating and Interpolating Values
- Activity 2.1: Jobs for Young People
- Discuss the Ideas: Influencing Interpretations
- Activity 2.2: Invest in the Best
- Mental Math and Estimation
- Build Your Skills

Section 2.2: Bar Graphs

- Math on the Job
- Explore the Math
- Discuss the Ideas: Converting a Graph Format
- Activity 2.3: Commuting to School
- Mental Math and Estimation
- Discuss the Ideas: Graphing Condensate Temperatures
- Activity 2.4: Exploring Graphing Software
- Build Your Skills

Section 2.3: Histograms

- Math on the Job
- Explore the Math
- Activity 2.5: Keeping Active
- Mental Math and Estimation
- Discuss the Ideas: Traffic Engineering
- Build Your Skills
- The Roots of Math: Graphing: Inspired by the Bubonic Plague

Section 2.4: Circle Graphs

- Math on the Job
- Explore the Math
- Puzzle It Out: Coin Conundrum
- Discuss the Ideas: Representing Data on a Circle Graph
- Activity 2.6: Graphing Charitable Donations
- Mental Math and Estimation
- Build Your Skills

Chapter Project: Create a Tourism Brochure

- Reflect on Your Learning
- Practise Your New Skills

THE MATHEMATICAL IDEAS

GRAPHICAL REPRESENTATIONS

The chapter focusses on the physical representation and interpretation of data. This concept will not be entirely new to the students since they will have been exposed to it in previous grades, in other subject areas, and in daily life. Through the Math on the Job sections, they will be exposed to real situations in which people use graphs in their daily work. Discuss the Ideas are used to help develop concepts, and the Activities encourage research into different areas of the workplace and employment. The Build Your Skills problems will provide more examples of graphing and graphical interpretations in the workplace.

Before actually beginning graphing, have students check newspapers, magazines, or the internet to find different kinds of graphs and ask them to bring these in for class discussion.

While there are a number of very good online graphing tools, students should be encouraged to explore the graphing of data both manually and by using these technologies.

There are many situations in mathematics where we deal with discrete points. When graphing such points, we do not generally join the points. However, in statistics and in graphing statistical data, we often do join discrete points so that the trend can be more easily seen.

WHY ARE THESE CONCEPTS IMPORTANT?

- In daily life, students will be exposed to graphing and to graphs. It is important that they are able to interpret the data presented to them. They must be able to identify the potential of misleading information as presented in the data or graphically.
- In this chapter, students are exposed to a number of different types of graphs. They will sometimes be asked to represent the same data in different formats so they will be encouraged to consider whether one type of graph is a better means of representing the data than another.

PRIOR SKILLS AND KNOWLEDGE

For this chapter it is expected that students will understand and be able to complete the following items.

1. Concepts
   a) understand how to conduct a brief survey;
   b) understand how to construct a graph with a horizontal and vertical axis.
2. Mathematics Skills
   a) work with decimals and percents;
   b) tally the results of a brief survey;
   c) graph using horizontal and vertical axes.
3. Technology T
   a) use the internet to research information;
   b) use online graphing programs;
   c) use software to design documents.

For students experiencing difficulties with written output, a graph generating computer program will be especially useful. You may wish to complete Activity 2.4, Exploring Graphing Software, with the class earlier in the chapter if this is the case. However, you do want all students to have some experience drawing the graphs themselves. See Blackline Master 2.7 (pp. 139–144) for simple data for a graph of each type. If you assign these, you will want to keep an overhead of the graphs because students will be asked to compare some of them.

Some of the questions require interpretation of data and discussion of social dimensions. Students experiencing difficulties with written output may find it easier to interpret graphs orally through discussion, especially with their peers, rather than in written form.

REVIEWING PRIOR CONCEPTS

There are not many mathematical concepts that students will need to review for this chapter. They will have been exposed to graphing in elementary school and will have seen graphs in daily life.

For Section 2.4: Circle Graphs, however, they will have to be able to do the following:

- calculate percentages;
- measure degrees of a circle; and
- use a protractor to draw angles.

When you get to that point, you may want them to practise using Blackline Master 2.8.

Blackline Master 2.8 contains review questions and solutions. It is found at the end of this chapter of the teacher resource (pp. 154–155).

For remedial students, discuss the Explore the Math sections with them individually or in small groups to help them develop an understanding of the concept(s) considered. Directed questions such as, “What are the data being graphed here?” or “Why is this on the horizontal axis instead of the vertical axis?” will help them to realize the significance of the data.

PLANNING CHAPTER 2

STUDENT BOOK, pp. 54–113

Since students will have had some experience with most of these graph types in elementary school, time should be spent on interpretation rather than drawing. Internet access to graphing programs will be useful and time-saving. You will need approximately 12–14 classes to complete this chapter.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	55	Introduce the Chapter Project: Create a Tourism Brochure	20 minutes for class discussion and review of graphing knowledge	brochures and flyers from some resorts, internet, papers, and magazines advertising resorts Blackline Master 2.2 (p. 133)
2.1	56 56 57 60	Math on the Job: Small Business Owner Explore the Math Examples 1, 2 Discuss the Ideas: Extrapolating and Interpolating Values	60 minutes	
2.1	61 62 63 65 66 66	Mental Math and Estimation Activity 2.1: Jobs for Young People Example 3 Discuss the Ideas: Influencing Interpretation Activity 2.2: Invest in the Best Build Your Skills	1 class	internet access centimetre graph paper Blackline Master 2.1 (p. 132)
2.2	73 74 74 76	Math on the Job: Flight Services Specialist Explore the Math Example 1 Discuss the Ideas: Converting a Graph Format Activity 2.3: Commuting to School	1 class	centimetre graph paper Blackline Master 2.1 (p. 132) internet access
2.2	77 78 79 79 80	Mental Math and Estimation Example 2 Discuss the Ideas: Graphing Condensate Temperatures Activity 2.4: Exploring Graphing Software Build Your Skills	1 class	internet access centimetre graph paper Blackline Master 2.1 (p. 132)
	84	Chapter Project: Write to Promote Your Resort	1 class	internet and photocopier access

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
2.3	85 85 86 88 88	Math on the Job: Real Estate Agent Explore the Math Example 1 Activity 2.5: Keeping Active Mental Math and Estimation	1 class	centimetre graph paper Blackline Master 2.1 (p. 132)
2.3	89 90	Discuss the Ideas: Traffic Engineering Build Your Skills	1 class	internet access
	96	Chapter Project: Research Resort Statistics	1 class	internet
2.4	97 98 99 99 101 102 103	The Roots of Math: Graphing: Inspired by the Bubonic Plague Math on the Job: Food Co-op Explore the Math Example 1 Puzzle It Out: Coin Conundrum Example 2 Discuss the Ideas: Representing Data on a Circle Graph	1 class	calculators, compasses, rulers, protractors
2.4	103 104 104	Activity 2.6: Graphing Charitable Donations Mental Math and Estimation Build Your Skills	1 class	calculators, compasses, rulers, protractors, possible internet access
	108	Chapter Project: Promote Your Resort	2 classes: 1 class for completion and 1 for presentation	internet access
	108 109	Reflect on Your Learning Practise Your New Skills	1 class	
		Chapter Test (p. 125 of this resource)	1 class	

Embedded in each section will be the interpretation of results and the need to create graphs manually and electronically.

PLANNING FOR ASSESSMENT		
Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Chapter launch and project discussions • Math on the Job scenarios • Examples • Discuss the Ideas • Explore the Math	• Check students' blackline masters throughout to see if they have been working on the project so as to determine time needed in class. • Observe how students participate and share work when in groups. • Discuss ideas regularly to check conceptual understanding. • Watch for errors in thinking. • Incorporate quick quizzes into lessons by having the students graph small sets of data including only four or five items. • Present students with simple graphs, either handmade or from the media, and give them opportunities to work with fellow students to interpret the data and share their understanding of it. • Have students bring in graphs that they find in newspapers, advertisements, or magazines. They can present them as a project or they can be viewed by the class and discussed as a whole.
Assessment as Learning	• Explore the Math • Activities • Project	• Make lists on the board of the ideas the students come up with in discussion to reinforce their learning. • Provide students with opportunities to represent data in as many ways as they can, even if it is not graphical. • Discussion and feedback will encourage thinking beyond the data. • Encourage group work and discussion about activities. • Encourage students to always think of what other ways they can represent the data. • Challenge students to research their resort thoroughly.
Assessment of Learning	• Activities • Projects • Quizzes • Chapter Test	• Final presentation of activities can be used as assessment of learning if discussion has taken place. • A final chapter test should include some hand-drawn graphs. Since the circle graph is more complex to do, you will want to consider simple data for it, or use it as an interpretive aspect rather than the graphing.
Learning Skills/ Mathematical Disposition	• Classroom interactions and use of technology	• Observe use of technology outside the classroom and for cooperation on projects and activities. • Check what programs students are using and check on their differences in interpretations of data; take into consideration whether graphs were drawn manually or whether technology was used.

PROJECT—CREATE A TOURISM BROCHURE

GOALS: To use different types of graphs to represent information effectively; to determine how different types of graphs more effectively represent certain types of information; to build graphing, research, and public speaking skills; and to synthesize learning in this chapter.

OUTCOME: In this project, students will use their new knowledge of graphing and graph types to produce a promotional brochure using technology (a word processing program and graphing software). They will also develop their research skills, work with technology, and strengthen their presentation skills.

T **PREREQUISITES:** To complete this project, students should be able to research statistics and images online. They should also be familiar with placing text and images in a word processing program.

ABOUT THIS PROJECT: This project is divided into four parts. To begin the project, students will choose an existing Canadian resort they would like to promote, as well as identify the audience for their brochure, the features it should include, and ways to interest tourists in their resort, such as photographs and information.

After students have learned about the four different types of graphs covered by the chapter, they will apply this knowledge to their project by making data tables and graphing this data using appropriate graph types.

Students should be given a few class periods to perform the necessary internet and photo research needed for their brochures. It is important to give them class time, since students may not all have access to a computer at home, and the project relies on their ability to find information online. Remind students that it is important to record and acknowledge all their sources of information and images, including internet sources.

Providing class time for students to locate data will allow you to help them with their research techniques if they have difficulty finding the information they need. You can use this time to offer feedback on the work students accomplish, answer questions, or make observations that you will use to grade the projects.

Do a brief survey to determine whether or not students are familiar with using online graphing programs or placing images and text in word processing documents. If the majority of students need help with this, you can give them brief demonstrations on these skills at the beginning of the classes devoted to chapter projects. A self-assessment rubric, Blackline Master 2.4 (p. 135), should be distributed to students at the beginning of the project. It describes the criteria on which their projects will be evaluated.

This project can be completed by individuals, pairs, or in small groups.

An alternative project, “Become an Environmental Monitor,” is included on pp. 145–153.

1. Start to plan

STUDENT BOOK, p. 55

During this section of the project, students will need access to computers to research which resort they would like to choose for their brochure. The goal of this section of the project is to have students decide which Canadian resort they will promote in their brochure and which components they will include in their brochure, such as photos, lists, graphs, and text.

Before you introduce this project, you might want to prepare by finding the names and locations of some well-known Canadian resorts. You can offer these as suggestions if students have difficulty thinking of a resort they would like to promote. Examples include Whistler Blackcomb, British

Columbia; Lake Louise, Alberta; Asessippi Ski Area, Manitoba; and Frances Lake Wilderness Lodge, Yukon. You might also bring in promotional materials published by resorts.

To introduce this project to students, begin by asking the class if they have visited a resort in Canada. Ask them to describe what they did there, what they enjoyed about the place, and whether or not they would like to return. You can brainstorm the names of different resorts and write them on the board.

Next, help students decide on the components they will include in the brochure. If you brought in examples of resort publications, pass them around the class and ask students which one(s) they prefer and why. This will help students to identify the components that comprise a successful brochure. As a class, discuss the following questions:

- Who will read this brochure?
- What will they want to know?
- What are the features of my resort that will attract tourists?
- How can I make my resort look exciting and appealing?
- How can I use graphs, photos, and text to interest tourists in my resort?

Have students list the components they will include in their brochure. The components students choose can vary, but they must include two graphs. Ask them to create a chapter project file and include this list.

To finish this section of the project, provide students with class time in the computer lab so that they can perform some research to decide which resort they would like to promote. They can add a note of the resort name and its website URL to a chapter project file. You can check over these notes to make sure that students are making progress with the project.

2. Write to promote your resort

STUDENT BOOK, p. 84

T During this section of the project, students will need access to computers and the internet. Students will locate photographs for their brochure and begin laying it out in a word processing document. At the beginning of the class, you may ask students if they know how to find images online, and if they know how to place photographs and text in a word processing document.

If students are unfamiliar with how to do this, give a quick demonstration to the class. Explain that images can be found by doing an online images search with the search engine of their choice, or direct them to photo-sharing websites such as:

- Flickr: www.flickr.com
- Photobucket: http://photobucket.com
- Wikimedia Commons: http://commons.wikimedia.org/wiki/Main_Page

You can demonstrate how images can be copied and pasted into a word processing document. Next, you can demonstrate how to place text in the word processing program your students are using. Students may need some instruction on setting up text and image boxes and sizing them to hold their materials.

The goal for this section of the project is for students to find images for their brochure, write the text for it, and design their brochure. They must also begin writing a short talk that promotes their resort. Tell students that the first step is to find the images they will use. The next step is to research their resort and write the brochure's text. Let students know that, by the end of class, they should have their text and photos placed, and have a rough draft of their brochure.

Assign the writing of the short promotional talk (30–60 seconds) as homework. If students have not finished laying out their brochure by the end of class, assign this as homework as well.

3. Research resort statistics

STUDENT BOOK, p. 96

T During this step of the project, students will again need access to computers. They will use their research and graphing skills to find the data to make the two graphs they are to include in their brochure. Students should choose two items or features of their resort that they think will be of interest to tourists and that can be represented through graphs.

Tell students that they are to use class time to collect the data from which they will make their graphs. Remind students that they will create two graphs with the data they collect and that they can create their graphs by hand, or use an online graphing program or graphing software to create their graphs.

The first step you can take is to introduce the class to websites where students can find statistics. These include the following.

- For general statistics—Statistics Canada: www.statcan.gc.ca/start-debut-eng.html
- For weather statistics—Environment Canada: www.climate.weatheroffice.gc.ca/
- For tourism statistics—Canadian Tourism Commission: http://en-corporate.canada.travel/Corporate/Flyout.page?id=369&fid=376

As a second step, introduce students to online graphing programs. These can be found at the following links.

- This website can be used to make bar, line, area, and circle graphs—http://nces.ed.gov/nceskids/createAgraph/default.aspx
- This website can be used to make bar and line graphs—www.graphtools.com/
- This website demonstrates how histograms can be made using a spreadsheet program— www.ncsu.edu/labwrite/res/gt/gt-bar-home.html#ith

Histograms are more complicated to produce using an automatic program, so students may prefer to draw their histograms by hand.

After students find their data, they must organize it in two tables, remembering to record their source. They can also print off the web pages on which they find their statistics, for future reference, and add them to their chapter project file. You may need to devote two classes to this component of the project. After students have found their data, they can use it to produce two graphs. Students will likely need the class time to gather data, so you may consider assigning the graphing component as homework.

At the end of this class, you can hand out the chapter project checklist to students (Blackline Master 2.3, p. 134). They can use it to make sure that they have all of the material required for their presentation.

4. Promote your resort!

STUDENT BOOK, p. 108

For the last segment of the project, students will demonstrate the outcome of their graphing, research, and writing skills in a presentation. Before the presentation, you can brainstorm, as a class, the types of questions tourists might ask a marketing representative about a resort, and write these on the blackboard.

For the presentation, or mock marketing fair, have half of the class sit at their desks, acting as marketing representatives, and present their brochures. The other half of the class can act as tourists, circulating among the marketing representatives, looking at brochures, and asking questions. After the marketers have had time to present their brochures, students will switch roles, and the other half of the class will present their brochures and give their talks.

At the end of class, ask students which resorts they would like to visit, based on the brochures they have seen.

ASSESSING THE PROJECT

1. Start to plan

- Provide students with the information on how their project will be assessed.

2. Write to promote your resort

- At the end of this class, ask students to make two lists—one to record the parts of the project they have completed, and the other to record the parts of the project they still need to finish.

3. Research resort statistics

- You can do a preliminary assessment of the data tables and/or graphs students have produced. This is an opportunity to assist students who are having difficulty finding data for their graphs.

4. Promote your resort!

- Ask students to self-assess their projects using Blackline Master 2.4 (p. 135) to be sure they are ready to present.
- The rubric included in this section (p. 95) can be used to help you assess how well the students completed their project. This rubric should be used alongside a numerical grading rubric you have drawn up to evaluate the projects.
- After the presentation and after the projects are graded, you can make a wall display using the brochures.

PROJECT ASSESSMENT RUBRIC

	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Project shows understanding of how to collect data and use it to create line, bar, and circle graphs, as well as histograms; how to use writing, photos, and text to promote a resort to tourists	shows very limited understanding; graphs are incorrect, missing, or use an inappropriate format; writing, photographs, data tables, or talk are omitted or contain mistakes	shows partial understanding; graphs are correct; writing, data tables, photos, or talk are incomplete or somewhat confusing	shows understanding; graphs are correct; writing, data tables, photos, and talk are appropriate	shows thorough understanding; graphs are correct; writing, data tables, talk, and photos are effective and persuasive
Procedural Knowledge				
• Accurately: • creates data tables to record data • records data sources • constructs graphs using data • writes descriptions and a talk that promote the resort • chooses photographs that promote the resort • places text, graphs, and photographs in a tidy, easy-to-read format	limited accuracy; major errors or omissions For example: • data tables are missing or incorrect • data sources are missing or incorrect • graphs are constructed incorrectly • written descriptions and talks are missing or inappropriate • photographs are missing or inappropriate • brochure layout is messy and difficult to read • project is incomplete	partially accurate; some errors or omissions For example: • data tables are correct, but graphs are constructed incorrectly • some data sources are included • some written descriptions are included, but contain mistakes or are too short; talk is adequate • some photographs are included, but not all are appropriate • brochure layout is readable, but messy • project needs more work to ensure graphs are made correctly, data sources cited, photos and writing are appropriate	generally accurate; few errors or omissions For example: • data tables are correct • data sources are recorded • graphs are constructed correctly • some written descriptions are missing or contain mistakes; talk is adequate • photographs are included, and appropriate • brochure layout is easy to read • project is complete, but work does not go beyond what was assigned	accurate and precise; very few or no errors For example: • data tables are correct • data sources are recorded • graphs are constructed correctly • written descriptions are free of mistakes and persuasive; talk is persuasive • photographs are included and appropriate • brochure is easy to read and looks appealing • project is complete, creative, and persuasive
Problem-Solving Skills				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	consistently uses appropriate strategies to solve problems; can explain solutions clearly using mathematical terms
Communication				
• Presents brochure and talk clearly, using appropriate mathematical terminology	does not present brochure and talk clearly, using few appropriate mathematical terms	presents brochure and talk with some clarity, using some mathematical terms	presents brochure and talk clearly, using some appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

2.1 Broken Line Graphs

TIME REQUIRED FOR THIS SECTION: 2–3 CLASSES

STUDENT BOOK, pp. 56–72

MATH *ON THE JOB*

STUDENT BOOK, p. 56

The first graph the students are exposed to in the Math on the Job is a simple one for interpretation. Students are asked to think about what the points represent and why there might be differences over time. This will be a recurring theme throughout the chapter. Not only are students asked to draw graphs, they are asked to interpret them and to provide potential reasons for the fluctuations in numbers.

SOLUTIONS

1. Trevor's income was best during November and December. People may have been buying Christmas gifts, or his business may have become more well-known and popular.
2. Trevor's profits from March to August were probably low because he was just starting his business and people did not know about his store, or he may have had incorrect stock for the time of the year or the clientele.

EXPLORE THE MATH

STUDENT BOOK, p. 56

Begin this section by reminding students that they have been exposed to most of the concepts in this chapter previously and that now you want them to think about them in greater depth. When they hear the term "line graph," they may think of a straight line graph because they will have studied these in the previous chapter. Indicate that the type of graph they are considering in this section is sometimes referred to as a "broken line graph," while the others are often referred to as "linear graphs" to distinguish them from each other.

Read through this or discuss the use of broken line graphs with the class, or have them work in groups to read through and analyze the information in this section and the examples. In your discussion with the class, consider the following: After having briefly looked at the single line graph in Math on the Job, students are presented with the idea of the use of a multiple line graph as a means of comparing data. The use of multiple graphs is a theme throughout the chapter as they are often used for the purpose of comparison.

The first example has the students explore a multiple line graph to compare bread sales. The resulting graph in this case has two points that could be considered outliers. This distracts one from the otherwise fairly smooth data. Consider asking the students to determine the average number of each type of bread sold per month (German rye, 87; Volkenbrot, 90; Expedition Bread, 67). Was this expected from simply looking at the graph?

The second example has students reasoning about climate conditions and exposes them to extrapolating data from given information.

DISCUSS THE IDEAS

EXTRAPOLATING AND INTERPOLATING VALUES

STUDENT BOOK, p. 60

In a discussion with a partner, students will consider what it means to extrapolate and interpolate from known data.

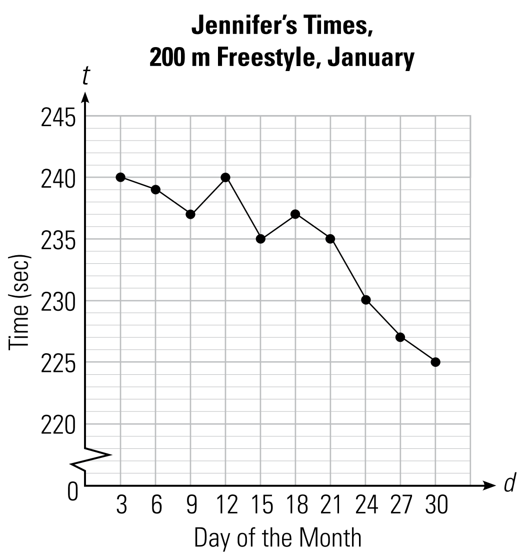
SOLUTIONS

1. The graph shows that the temperature of the cooler is declining at a fairly steady rate of 0.1° C per week. Students should be able to interpolate that the temperature in the third week of January will therefore be 3.8° C.
2. Given the steady decrease of temperature of 0.1° C per week, the temperature will measure 3.4° C in the third week of February.
3. Students can use extrapolation to predict that since the temperature is declining by 0.1° C per week, the temperature will measure 3.1° C in the second week of March and 2.9° C in the fourth week of March.
4. Interpolating is likely to be more accurate since the predicted value is bracketed by two known values. Extrapolating, on the other hand, is less reliable because a new trend could occur.

Mental Math and Estimation

STUDENT BOOK, p. 61

SOLUTION



Students will have to extrapolate from the graphed data to answer this question. The time it takes Jennifer to complete the 200 m freestyle has been decreasing steadily. From day 27 to day 30, her time decreased by 10 seconds. If students assume her time decreases at the same rate, Jennifer's time will be 215 seconds on day 33 and 200 seconds on day 36.

However, students should be made aware that there is a limit to how fast she can swim and that the decrease in time will ebb.

ACTIVITY 2.1
JOBS FOR YOUNG PEOPLE

STUDENT BOOK, p. 62 T

In this activity, students are asked to consider rates of employment of youth compared to the general population. Computer access will be necessary, either during class or assigned as homework. You may want to let the students explore on their own to find a website, or you may direct them to Statistics Canada or Human Resources and Development Canada (HRDC). Students, at this point, will draw all graphs manually, since this will give them more of a feel for what the graph represents.

In this activity, have students work with a partner. They should use the internet to do the research for this activity. The Statistics Canada website (www.statcan.gc.ca) provides the information students need to locate. If students need some assistance locating the information, suggest that they follow this link:

www.statcan.gc.ca/start-debut-eng.html

SOLUTIONS

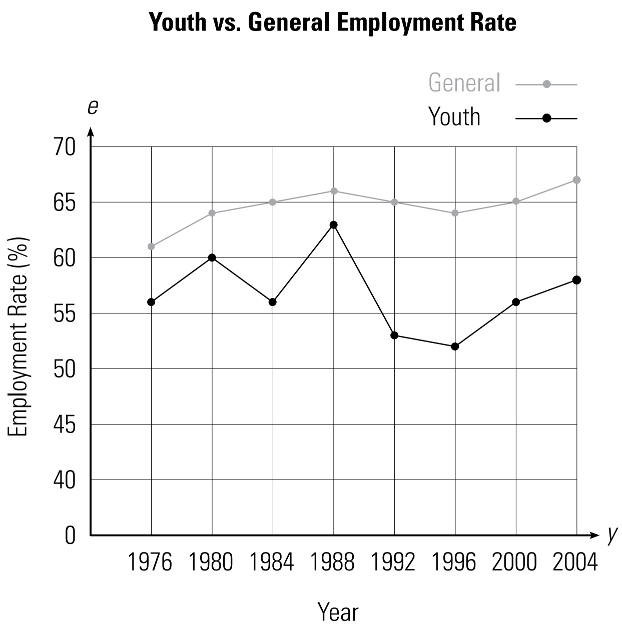
1. To find youth employment rates, students can click on the "search" tab on the Statistics Canada website and enter a keyword search term such as "youth employment rates." This will take students to a page full of articles on this topic. Articles that have the words "youth employment rates" highlighted in their descriptions will be useful. This article has relevant information:

   www.statcan.gc.ca/pub/75-001-x/11105/8840-eng.pdf

2. To find information on general employment rates, students can do a search on the Statistics Canada website for “employment rates.” This will take them to a page full of related articles. The following URL leads to a useful article:

   www.statcan.gc.ca/daily-quotidien/060601/dq060601a-eng.htm

3. Graphs will vary depending on the data sources students use, but should convey the general trend that the general employment rate is higher than the youth employment rate. A sample graph is shown below.



4. a) The youth employment rate data are based on young people across Canada 15 to 24 years old. The general employment rate data are based on working-aged people. Working-aged people are defined as those aged 18 to 65.

   b) Answers will vary depending on the sources students use, but based on the data supplied in the links above, it appears that the youth employment rate is somewhat lower than the general employment rate. The youth employment rate could be lower because many young people do not have jobs while they are going to school, or because when the economy is performing poorly, the entry-level jobs most young people hold are the first to be cut.

   c) Based on the Statistics Canada data referred to above, the youth employment rate increased from around 1997 to 2004. This implies that a larger proportion of young people are now employed and that young people looking for work have a better chance of finding it than they had in the past.

   d) Answers will vary depending on the results of student research. Based on the Statistics Canada information referenced above, the general employment rate began to increase in 1996.

   e) It seems that someone from the general population has a better chance of finding a job.

Example 3 continues with the theme of employment rates. Work through this question with the class. It leads into Discuss the Ideas.

DISCUSS THE IDEAS

INFLUENCING INTERPRETATIONS

STUDENT BOOK, p. 65

Graphs can influence the way that people read them in many ways. For example, the scale a graph uses will affect how data is perceived.

In this section, students are asked to consider, as in the previous example, how the change in scale and setting of a starting point can affect their interpretation of data. As this follows directly from the questions in Example 3, it should also be read with the class. You could then discuss with them when they have seen graphs that have been misleading, or you could ask them to look for some for homework.

SOLUTIONS

1. Graph 1 uses much larger units. This makes it look like there are more dishes missing

and also that there are greater differences by month.

2. Answers will vary. Sample answers are below.

   Both graphs are made with the same data, so they are both accurate.

   Using a vertical scale of 10 makes it easier for the viewer to determine the number of lost/broken dishes, so graph 1 is more accurate.

3. Student responses will differ. Here are some sample responses.

   If a person wanted to make the number of lost/broken dishes look smaller than it was, he or she might choose to use graph 2.

   If a person was more interested in graphing a general trend, not specific numbers, she or he might choose to use graph 2.

ACTIVITY 2.2

INVEST IN THE BEST

STUDENT BOOK, p. 66

T Read through the activity with your students. It encourages them to think about the stock market, commodities, and investments.

At this point, you might also explore with the students the use of spreadsheet programs for graphing.

SOLUTIONS

1. Graphs will vary depending on the companies students choose to follow. Assume that one student chooses Microsoft Corporation to research. She or he would first visit the NASDAQ website:

   www.nasdaq.com.

   Next, she or he would click the "Home" tab and select "Company list":

   www.nasdaq.com/asp/symbols.asp?exchange=Q

   She or he would then click on the "M" link in the alphabetical links section at the top of the page and locate Microsoft:

   www.nasdaq.com/asp/symbols.asp?exchange=Q&start=M&page=2&sort=name&Type=0

   Then, she or he would click on the MSFT box in the middle of the page and select "Real-time quote":

   www.nasdaq.com/aspx/nasdaqlastsale.aspx?symbol=MSFT&selected=MSFT

   The student would obtain her or his data from under the "Close price/date" heading. After she or he had done this for two companies for one week, she or he would have the data needed to draw her or his double broken line graph.

2. Answers will vary, but based on the graph they draw, it should be clear to students which company's stock rose or fell the most during the week.

3. If the worth of one company's stock rises consistently and is worth more money than the other, students may say that it would be worthwhile to invest their money in the most expensive one. Others may consider buying the less expensive one because it has a better chance of increasing in price. Others might say that they would keep track of how a company performed for longer than one week before they invested in it.

4. Depending on how their companies performed, students' answers will vary but the likely trend is an increase.

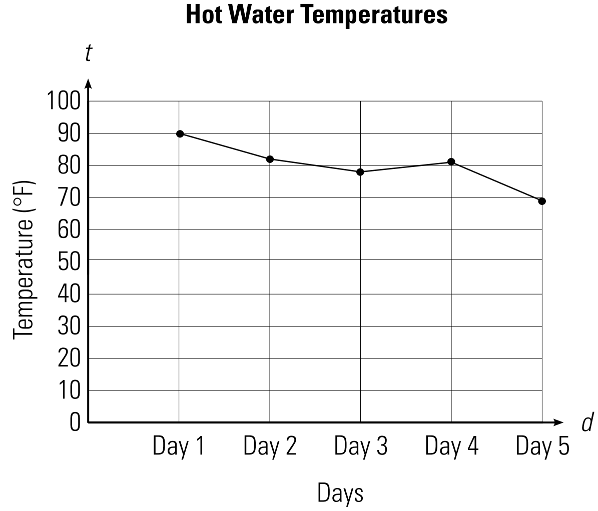
Extension

Since this is an extended time activity, it could be extended even further to become an alternative chapter project by having the students work in groups and consider two commodities from two different categories, possibly following them for a longer period of time. They would then present the data in graphic form using two different types of graphs, much as outlined for the chapter project on pp. 91–95 of this resource.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 66

1.



2. a) The most people, fourteen, were placed in March.

   b) The fewest people, two, were placed in April.

   c) When she began, not as many people were placed in housing. Then during the summer and fall, placement numbers increased, but over the winter season they dropped. After the holiday season, the number of people placed in community housing increased greatly. There is no general trend.

   d) Answers will vary. First, use Julie's data to find the average number of people placed in community housing. The average is

$$\frac{2 + 5 + 4 + 8 + 11 + 10 + 12 + 7 + 5 + 3 + 11 + 14}{12}$$

$$\approx 7.7$$

Because there are more months that are below average than above, this year's placement numbers are below average. Reasons for placement numbers being below average will vary but could include the following.

Brandon's population decreased, so there were fewer people, in general, looking for housing.

Families can live in community housing. If the size of the families placed in community housing decreased, this would result in fewer people being placed in community housing.

People with moderate or low incomes can live in community housing. If Brandon was experiencing economic prosperity, and there were more high-paying jobs available, fewer people might be placed in community housing.

Seniors can live in community housing. If the number of seniors in Brandon declined, the number of people placed in community housing might decline.

3. a) It displays the average weekly earnings of people in Canada from 1997–2008.

   b) The average weekly earnings in 2003 were approximately $730.00.

   c) In general, income has increased, with a slight drop in 2003. The fact that the cost of living continues to increase and people get pay increases explains this.

   d) The graph begins at $600.00 with $40.00 increments. This may make it look like there is a greater discrepancy between the 1997 and the 2008 incomes.

4. a) Tradespeople, such as carpenters, plumbers, and electricians, earn higher hourly wages than the people working in the other professions shown on the graph. If Chi is concerned with income, he should consider a trade.

   b) According to the graph, an electrician earns an average hourly wage of $25.00, while a food services employee earns about $12.00 an hour. There is a difference in earnings of $13.00 an hour between these two careers. Reasons for the difference in wage could include the following:
   - Wages are generally based on the amount of training and education an employee brings to the job. Most jobs

in the food service industry do not require the same amount of training or education as do electricians and are, therefore, paid a lower hourly wage.

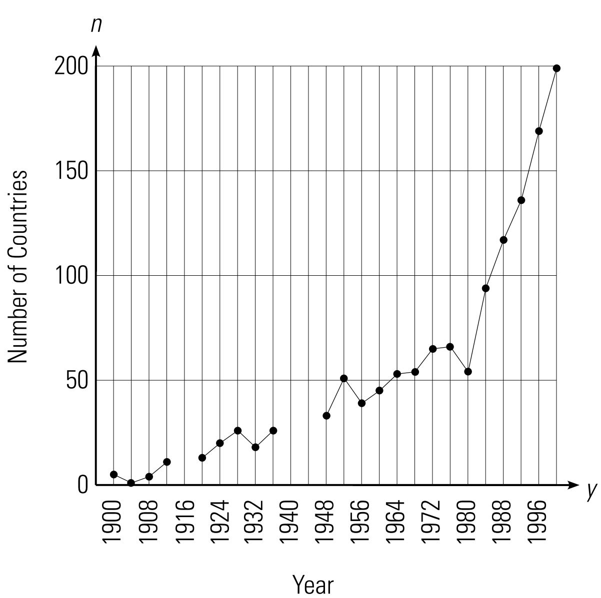
- There are fewer certified electricians than food service industry workers. Therefore, employers will pay a higher hourly wage to attract these trained workers.

c) The wages on this graph are averages, taken from across Canada. For this reason, they might not accurately reflect the high, low, or average wages for that profession in a specific province. For example, in Calgary, the lowest wage for a painter/decorator might be $14.00 an hour, and the highest wage $32.00 an hour. The lowest wage for the same profession in Winnipeg could be $9.00 an hour and the highest wage $25.00 an hour.

d) Answers will vary. A sample answer follows. Before Chi chooses one of these careers, he will have to discover whether or not he needs post-secondary education to find a job. He will have to determine the type of education, its duration, cost, and the type of certification he will achieve.

5. a)

Number of Countries with Female Olympians in the Summer Olympics



Discuss with the class the breaks in the graph and that they should not make the graph without these.

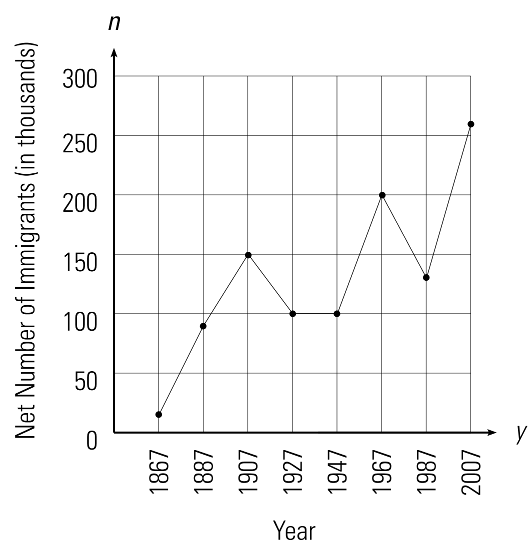
b) There is a general increase in the number of countries from which women athletes originated.

c) The Olympic Games did not occur during the World Wars.

d) The number of countries is increasing dramatically. Predictions will vary—possibly 215 and 240 but there will be an upper limit. Through their internet search, students should find that women athletes represent 196 countries in 2004 and 203 countries in 2008.

6. a)

Canadian Immigration Rates



At the time of Confederation, net immigration was low. It steadily increased for the next 40 years and dropped again for 40 years. Twenty years later there was then an increase, followed by a decrease, and by 2007 there was a sharp increase.

b) At Confederation, Canada was a small country and needed immigrants to increase the population. There was then a drop in immigration during World War I, and during the Depression it remained fairly constant. After World War II, there was once again an increase in immigration.

Since then, a decline in growth rate has caused Canada to encourage immigration.

c) Answers will vary, but generally, students should realize that it is not very accurate because it shows the net immigration only every 20 years and not the average of each individual year's net immigration over the 20-year period.

Extension

To enrich this question about Canadian immigration, you can use this question as a springboard to discuss the history of different cultures in your town or city. You can also discuss the contributions these cultures have made to the place you live, or Canada as a country.

As a class, you can brainstorm and list the different cultures living in your community or province. Have each student choose one culture that they feel they know little about. As homework, you can assign each student to research and record five facts about this culture, or the history of this cultural group in Canada, or your town. Students can write a small summary or essay based on these facts.

The Multicultural Canada website describes the economic, community, religious, and cultural life of people from many different cultures living within Canada.

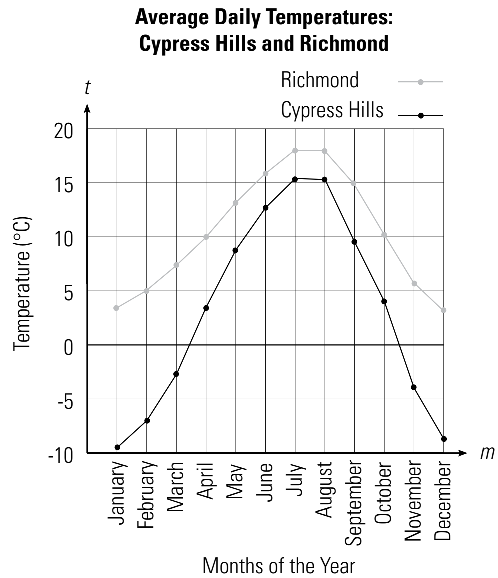
www.multiculturalcanada.ca/Encyclopedia/A-Z/m9

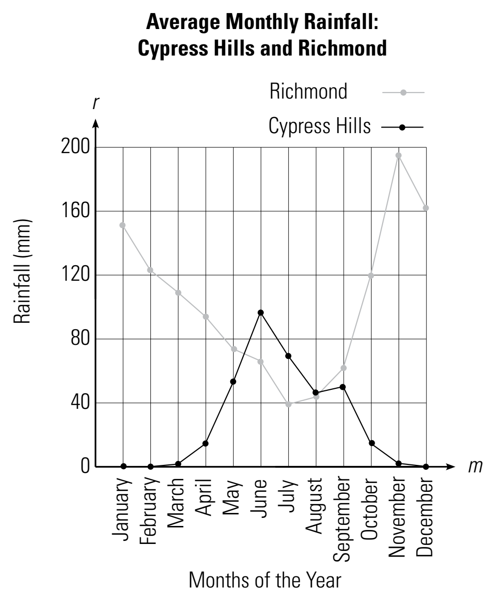
If students are interested in learning more about Vancouver's modern-day Chinatown, or the history of the area, you can direct them to the Chinatown Vancouver website, found at the following link.

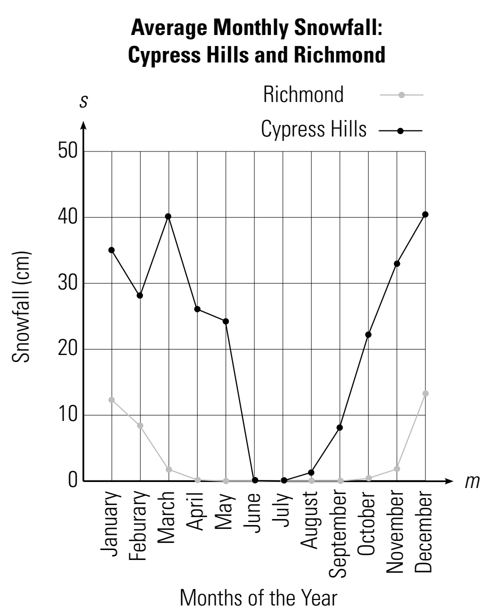
www.vancouverchinatown.ca/index.html

Extend Your Thinking

7. You may wish to use this question as an opportunity for students to discuss graphing below zero.







Discussions will vary but should include some of the following:

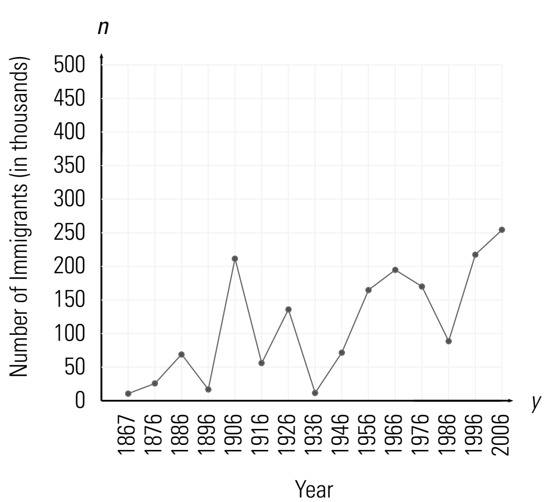
- The temperature in Richmond is consistently higher than it is in the Cypress Hills.
- Richmond's average temperature is never below zero while for the Cypress Hills it is for five months of the year.
- The temperature variation between the two is less in the summer than in the winter.
- The Cypress Hills have more extreme temperatures.
- In the winter, Richmond has more rain than the Cypress Hills, but the Cypress Hills has more snow than Richmond.
- In the summer, the Cypress Hills tend to have more rain than Richmond, but neither tends to have snow.
- Richmond has comparatively little snow compared to the Cypress Hills.

Extension

As an extension, distribute copies of Blackline Master 2.5 (p. 136). Students can explore a misleading graph's construction.

8.

Annual Number of Immigrants Since Confederation, 1867–2006 (thousands)



a) The Statistics Canada graph is much more detailed, showing more rises and falls. It is more accurate because it shows the net immigration every ten years rather than just on the twentieth year.

b) In 1905, Alberta and Saskatchewan joined Canada and immigrants came and moved to the west to farm. In 1949, Newfoundland and Labrador joined. Also it was after World War II and there were more immigrants.

c) This was during the Great Depression and World War II.

2.2 Bar Graphs

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 73–83

MATH *ON THE JOB*

STUDENT BOOK, p. 73

Read through this Math on the Job with students. Mention that James likely works with weather observers. Weather observers often work at local airports to record climate conditions, such as temperature and rainfall levels. This is a job that requires a high school education, so if students are interested in this type of work, they might consider this job.

Some students may need help in reading the graphs and estimating the number of millimetres of precipitation. Show them how they can mark off more units on the vertical scale and then use a ruler to help interpolate the results.

SOLUTIONS

1. The title is: Monthly Precipitation Levels in Yellowknife, January–December, 2006. This implies that the graph is indicating how much precipitation fell in Yellowknife during the 2006 calendar year.
2. 14 + 16 + 18+ 34 + 31 + 42 + 53 + 20 + 10 + 46 = 304

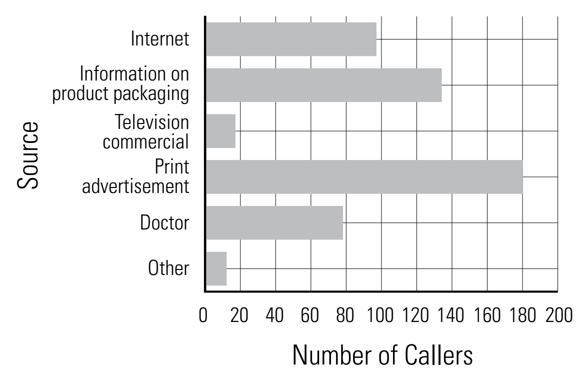
   The total amount of rainfall that fell in 2006 was 304 mm.
3. $\frac{42}{304} \times 100 \approx 13.8\%$
4. It is possible that large amounts of precipitation fell during these months because it was spring and the weather was unsettled. In general, most precipitation falls on Yellowknife from June to October, with August receiving the most precipitation.

EXPLORE THE MATH

STUDENT BOOK, p. 74

When teaching this section, emphasize that it is important to choose an appropriate scale for a graph. Students can choose an appropriate scale by considering the highest number that will be plotted on the graph, as well as the differences that they want the graph to emphasize.

Be sure that students know what discrete data refers to and be sure that they understand the difference between the words “vertical” and “horizontal.” Discuss the difference between a vertical and horizontal bar graph.



SOLUTIONS

1. The scale for the vertical axis should be divided by source of contact information. There are six sources, so the vertical axis should have six divisions.
2. Answers will vary, but a scale of 20 would be suitable for the horizontal axis since there is quite a wide range of numbers and they differ by a significant amount.
3. Student's answers may differ. Some possible examples of titles are below.

   Source data: May
   Different sources of contact information: May

   Number of people who obtained contact information from different sources: May

Using Example 1 as a model, ask the students why they think a horizontal bar graph was used for this data.

Example 1

This example provides an opportunity for students to learn more about Inuit culture. If students are interested in learning more about traditional Inuit tools, information can be found on Canada's First Peoples website.

http://firstpeoplesofcanada.com/fp_groups/fp_inuit3.html

Inuit Tapiriit Kanatami is Canada's national Inuit organization. The group's website is given below.

http://www.itk.ca/

The SILA website provides lesson plans and other resources for teachers that focus on teaching Inuit culture.

http://www.sila.nu/home?l=en

Extension

As an extension to Example 1, ask students to create a bar graph that represents information about Inuit people within Canada. For example, students could display the number of people fluent in Inuktitut in different areas of Canada. If you choose to have students graph this information, you can mention that there are five main dialects that compose the language known as Inuktitut. Statistics Canada provides information suitable for making this graph.

www12.statcan.ca/census-recensement/2006/as-sa/97-558/p9-eng.cfm

DISCUSS THE IDEAS

CONVERTING A GRAPH FORMAT

STUDENT BOOK, p. 76

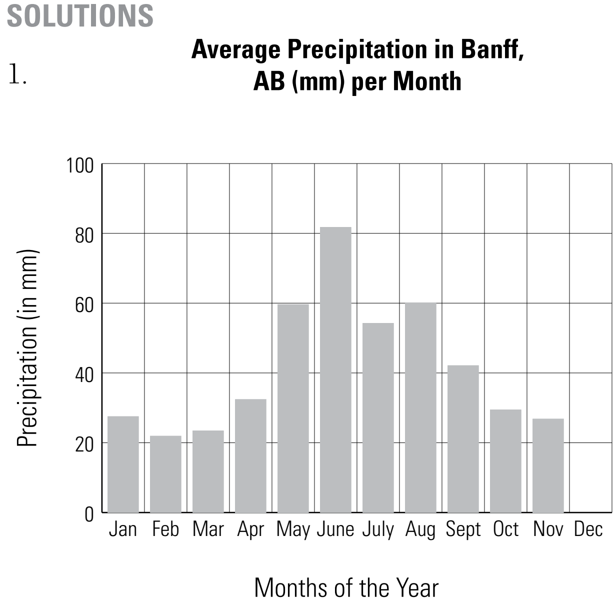
In this Discuss the Ideas, students are asked to consider the same data as in Example 2 on p. 59, but this time they are asked to present the data on a bar graph instead of a line graph. Have them compare the two graphs and discuss which they think better represents the data. Although they will have different ideas, it is probable that the bar graph will be considered the better representation as the data are discrete and each month can be considered independently.

At any point throughout the rest of the chapter, you may want the students to explore the potential of other graph forms than the one asked for in the specific question.

Make overheads showing the broken line graph and the bar graph, using the same scale on each, so that they can be displayed one on top of the other. This will provide visual learners with a clearer picture that they are the same.

SOLUTIONS

1.



2. Students were asked to present the material on a vertical bar graph because the data are "height"-related rather than "length"-related, so although either a vertical or a horizontal bar graph could be used, the vertical is likely considered the better representation.
3. Both seem equally effective, although the line graph seems to show a continuum whereas the bar graph indicates discrete amounts.
4. Because of the continuum, it might seem easier to predict for December from the line graph than the bar graph.

ACTIVITY 2.3
COMMUTING TO SCHOOL

STUDENT BOOK, p. 76

In this activity, students are asked to generate their own data and present it on a graph. Read through the activity with them. Because of the added layer (collecting the data), and the need to work outside of class, it is advised that they work in pairs or small groups.

Use this as an opportunity to review some polling techniques with your students.

If your school is small and surveying fifty individuals seems too much, you could reduce the number.

To save time, you may want the students to work as two groups of two, each group polling only twenty-five students. Assign one person as the questioner and one as the recorder. The group of four could then amalgamate their data, and to ensure that all are involved and understand the process, randomly select one individual to explain.

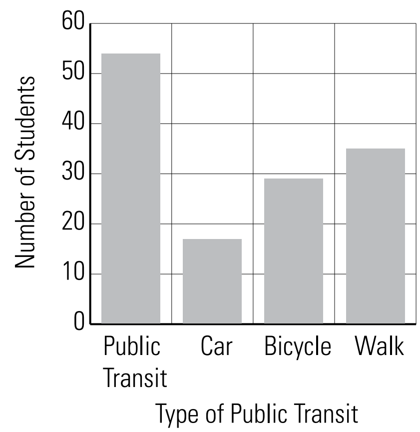
It is likely that the graphs will be similar between groups, but there will be some differences. Reasons for these differences could then be discussed. There are many possible reasons for them, including: students interviewed people who shared their own interests; the time of the day they did their interview (before school, lunch time, after school, randomly); their means of interviewing (Did they stop people in the hall? Did they go up to them in the cafeteria at lunch? Did they do a phone survey?); and other things would affect the results.

SOLUTIONS

Graphs will vary. An example is provided which is used to answer the questions.

1. Public transit had approximately 54 votes, car had about 17, bicycle had 29, and walk, 35. Public transit was the most common because it had the most votes.

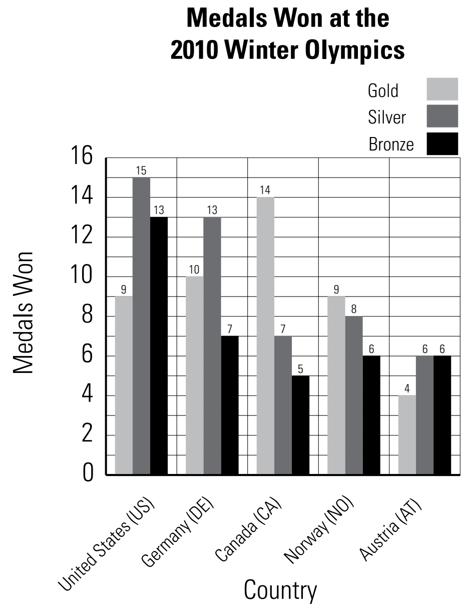
Number of Students Using Different Types of Transportation



2. A vertical graph was chosen but a horizontal graph could have been used as effectively. Some students may prefer horizontal because of the number line.
3. The spread in the numbers is more obvious from a bar graph than just looking at numbers.
4. There will likely not be much difference because it is the same school.
5. Graph results could be affected by such things as the quality of public transportation in the area, population density around the school, and if the area is urban or rural.

Mental Math and Estimation

STUDENT BOOK, p. 77



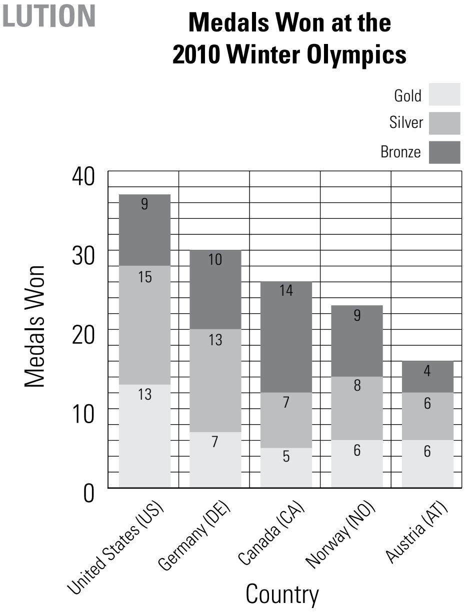
SOLUTION

From looking at the graph, students should be able to see that the US and Germany won more medals than Canada, and the US, Germany, and Norway won more silver medals than Canada.

Extension

For students who are able to look at data from different vantage points, you may want to introduce the idea of a stacked bar graph. Use the data from the Mental Math and Estimation on page 77 of the student book, which is reproduced on Blackline Master 2.6 (p. 138). Ask students to try the graph.

SOLUTION



Ask the students if a stacked bar graph would be suitable for the information in Discuss the Ideas. The fact is, it would not be because stacked graphs only apply for multiple sets of data such as gold, silver, and bronze medals. The temperature or precipitation levels per month could not be graphed in a stacked graph.

At this point, it is a good idea to again discuss with the students how to choose a scale for their graphs. Most have been fairly simple to this point, but students are now starting to use decimals and will be solving problems containing larger numbers.

DISCUSS THE IDEAS

GRAPHING CONDENSATE TEMPERATURES

STUDENT BOOK, p. 79

Once again, this Discuss the Ideas brings up the possibility of using graphs to compare data by doing a double bar graph. To answer the questions, students have to interpolate data.

SOLUTIONS

1. The actual temperatures tend to be higher than the temperatures provided by the manufacturer.
2. The steam traps that were one year old had the most accurate temperatures.
3. Ashok used the double bar graph so that he could compare the actual temperature of the steam trap condensate to the manufacturer's

suggested temperature. The graph also allows him to see the age of the steam trap. This allows him to keep track of the age of the steam trap and to determine whether or not age is affecting the steam trap's performance. If Ashok sees that an older steam trap is emitting condensate that has a higher temperature than the one provided by the manufacturer, he might consider replacing or fixing the trap.

4. Student responses will differ. Example responses are listed below.

   Nurses have to compare the different levels of medication a person is receiving.

   Business owners compare the prices of different brands of the same supply.

   Employees in industrial laundries compare the different levels of soap, bleach, and fabric softener that are added to the loads of laundry.

5. A line graph would not be as effective as a double bar graph because the double bar graph allows the viewer to see the different values as separate entities. The difference between the values is more noticeable with the bars than it would be with lines.

ACTIVITY 2.4
EXPLORING GRAPHING SOFTWARE

STUDENT BOOK, p. 79

T

Until now, students have been asked to draw graphs manually. Ask the students to experiment with a graphing tool online by redrawing several of the graphs they have already drawn, including the temperature variation one from the example above and the stacked bar graph. Discuss the benefits of graphing manually over technologically and vice versa. If you have done graphs using a spreadsheet program, discuss its advantages compared to the other tools the students have found.

It is to be expected that most students will prefer using the graphing program to manually drawing the graphs, but it is important that they have the experience of graphing manually. You may want to indicate from here on when you want a manual graph and when you will accept a technologically produced one.

SOLUTIONS

1. Two links to internet sites that provide graphing programs are listed below.

   http://nces.ed.gov/nceskids/createAgraph/default.aspx

   http://graphtools.com/

2. Answers will vary depending on the program used, but the graph should look similar to the one provided in the student resource.
3. Answers will vary, depending on the program used.
4. Advantages to drawing a graph manually include not needing a computer or internet access to complete your work, and having more control over the graph's appearance. Advantages of using a software program to make a graph include producing a graph that is more professional looking and being able to quickly make different types of graphs from the same data.

Extension

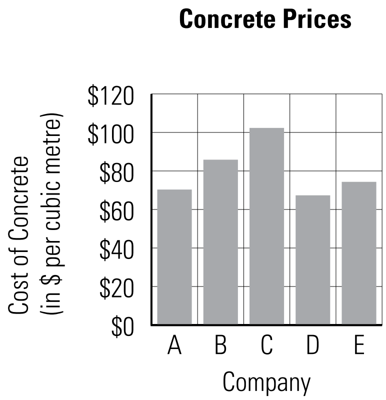
Some students may find one form of graph easier to draw conclusions from than another. Have students use the information from the graph on p. 58 of the student resource, Number of Loaves Sold by Type, to make a triple bar graph, either manually or using technology, and ask them which gives them a better idea of the bakery's selling rates.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 80

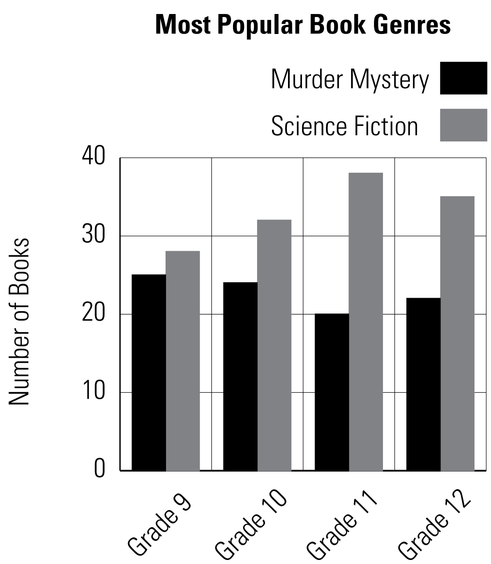
1. a) It tells you how many students earned how many points out of 10 on a quiz.
   b) 1
   c) 7
   d) No students earned a 0.
   e) 1 + 1 + 4 + 6 +12 + 2 + 3 + 1 = 30

2.



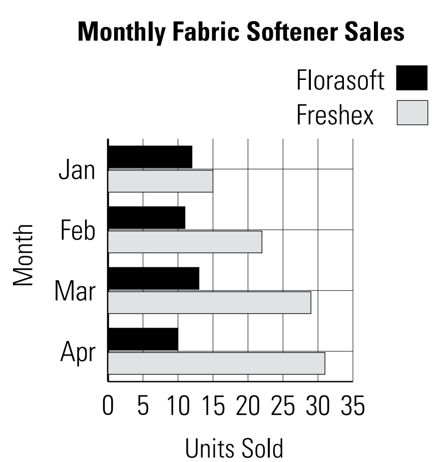
Jessica will need to consider if there is a delivery fee, if there is a minimum or a maximum amount of concrete that the companies can deliver, and if the composition of the concrete is right for the job.

3.



There seems to be a tendency for the number of science fiction books to go up with the grade, and the number of murder mysteries to stay stable.

4.



There seems to be a tendency for the number of buckets of Freshex sold to go up as the months pass, while the numbers for Florasoft stay stable.

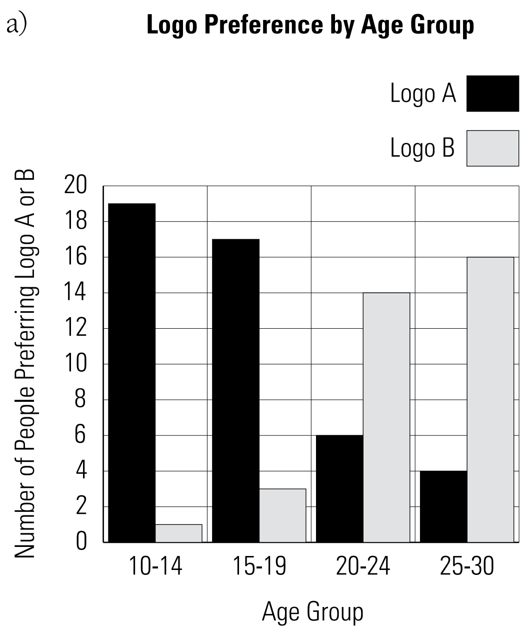
5. a) A cursory glance of the graph seems to indicate that Kay is correct. However, on looking more closely, there is an indication that the lower part of the graph has been omitted and therefore the differences are exaggerated. The difference in graduation rates between boys and girls differs by approximately 1% in each school. School B does have a better rate of graduation by about 6%, but without looking at the numbers, the difference looks greater.

   b) School A: girls approx. 68%, boys 67%

   School B: girls approx. 75%, boys 74%

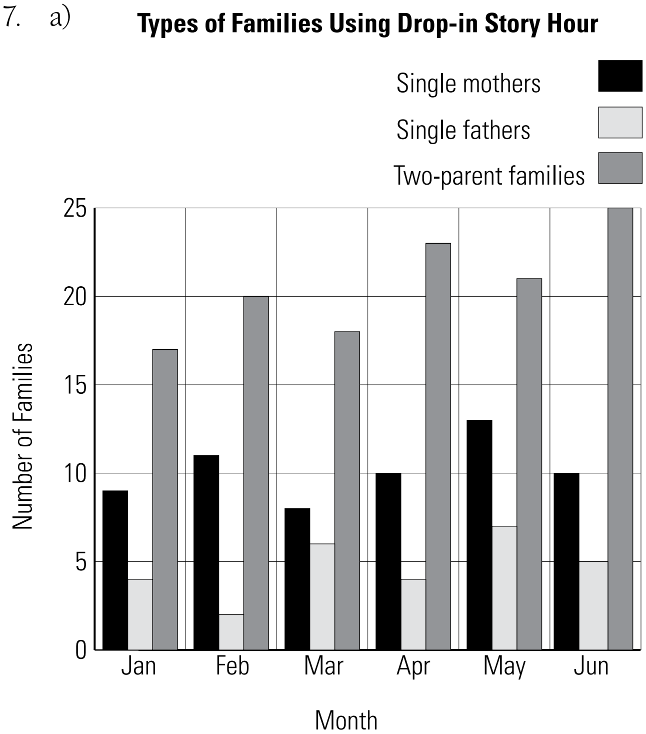
   School C: girls approx. 72%, boys 71%

6. a)



   b) Answers will vary to include such statements as: the older age groups have more buying power so she will want to cater to their taste; she prefers the logo herself.

7. a)



b) Based on the graph, the community centre should develop more programs for two-parent families, since they are the largest group using the drop-in story hour. However, this may not be an accurate way to assess need because single-parent families might need more support. Also, two-parent families, because they can share responsibilities, have more time to access facilities. Also there may be lower attendance from single parents because there are no programs that fill single parents' needs.

c) Answers will vary. However, a triple bar graph might be a better way to convey the information, since it can be difficult or confusing to read the values of a triple line graph if the values of the different lines are close together.

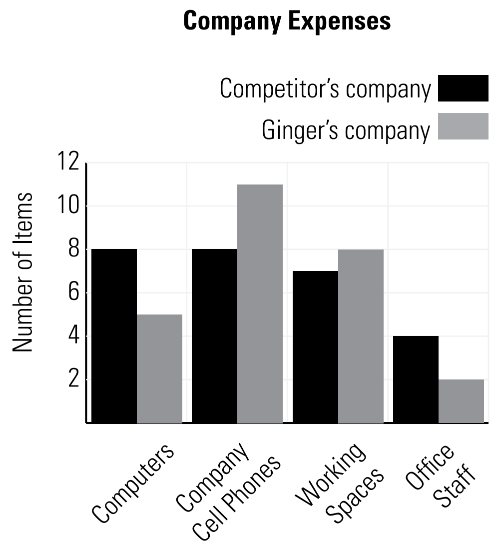
Extend Your Thinking

8. a) Approximately 114.

b) Approximately 595.

c) Yes, this is a fair statement. The difference between grade 10 responses is much greater than the difference between grade 11 students' responses.

9.



a) The competitor has 8 computers and Ginger's company has 5.

b) The competitor has 7 working spaces and Ginger has 8.

c) The implication is that we have working spaces without computers and cannot therefore do our work efficiently.

d) Answers will vary. A possibility is that the competitor's company has 4 half-time staff, while Ginger's company has 2 full-time staff.

2.3 Histograms

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

MATH *ON THE JOB*

STUDENT BOOK, p. 85

Read through this section with your students. It is possible that some of the students have parents in the real estate industry and they may be able to talk about commission, selling features, and other aspects that must be considered in the purchase of a home. They may be able to bring to class other graphs that are used in the industry.

Discuss whether or not students hope to own their own home, and where this home would be located. How will they save enough to own this home? How long do they think that would take?

Discuss with the students how the graph could be drawn as a bar graph, but the lack of breaks between the bars of the histogram creates a smoothness that better shows the trends in apartment prices than if there were discrete breaks.

Extension

Ask students to research the average house or apartment prices where they live and in one or two other areas. Ask them to discuss why the prices might vary and what factors affect housing prices. Using an average wage, have the students calculate how many years' wages it would cost to buy a house in the different areas of the country they have researched.

SOLUTION

1. The histogram seems to indicate that most apartments sold in Vancouver are worth between $200 000.00 and $500 000.00.
2. Answers will vary. Sample answers are below.
   - Most people don't want to spend more than $500 000.00 on an apartment. The greatest number of apartments sold cost between $300 000.00 and $400 000.00 and as prices go up or down, fewer are sold.
   - There are no apartments worth less than $100 000.00 and few worth less than $200 000.00. There are also fewer in the higher price ranges available.
   - People seem to have stable jobs at this point in time since they can buy apartments.

EXPLORE THE MATH

STUDENT BOOK, p. 85

Explain to students that histograms are used to represent trends, or ranges of information. While histograms have bars like a bar graph, the bars in a histogram must be flush to better convey the trend the data conveys. The data are continuous.

Example 1

You can use Example 1 as an opportunity for students to learn about Manitoba's francophone community. This link profiles the community.

www.fcfa.ca/profils/documents/manitoba_en.pdf

Information in English on Manitoba's Collège universitaire de Saint-Boniface can be found at this website.

www.ustboniface.mb.ca/cusb/information/college/about-cusb.php

Extension

Ask students to find out the francophone population in each province and territory within Canada. Next, ask them to display this information using a suitable graph. Students may display this information using French instead of English, if they prefer.

ACTIVITY 2.5

KEEPING ACTIVE

STUDENT BOOK, p. 88

The purpose of this activity is to have students collect data by conducting a survey, and to have them take the data they have collected, display it graphically, and determine questions it can be used to answer. Again, it is advised that they work in partnership as this will lead to better interpretation. Discuss with your students suitable units of time to measure daily exercise with.

SOLUTION

Answers will vary. Possible questions:
- How many students exercise between 0 and 30 minutes per day? What is the trend?
- What is the greatest amount that the students surveyed exercised?
- How many students were interviewed?
- Is this an accurate account of hours students exercise per day?

Mental Math and Estimation

STUDENT BOOK, p. 88

From looking at the graph, students can see that there are approximately ten more employees making \$20 000.00 or more than employees making less than \$20 000.00.

DISCUSS THE IDEAS

TRAFFIC ENGINEERING

STUDENT BOOK, p. 89

Read through this section with the students. Discuss the scale used. While they have done a few graphs where the units were not "one," but were "thousands," this may not have been explicitly dealt with. You can now discuss with them how the numbers might affect the way that they look at the data. Do they see the 1 as 1 car or as 100 cars? Have them look back at question 8, page 72, to reconsider how the numbers affected their interpretation.

SOLUTIONS

1. The heaviest usage was between 4:30 and 5:00.
2. Slightly over 450 vehicles, or approximately 460 vehicles.
3. Approximately 3200 vehicles.

   100 + 270 + 310 + 600 + 510 + 460 + 480 + 300 + 170 = 3200
4. Answers will vary, but should include such things as:
   - the number of lanes in each direction; and
   - the number of vehicles in each direction;
   - types of vehicles.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 90

1. a) 4 people worked less than 16 hours.

   b) The most common number of hours worked was in the 24–28 hours range.

   c) Add the number from each time slot.

   1 + 1 + 1 + 1 + 3 + 3 + 6 + 2 + 3 + 1 = 22
2. a) No, it is not apparent. Although the groupings between 10 and 60 are relatively even, the others are quite different, although declining with age as expected. Also, there are considerably fewer children less than 10 years of age than in the middle groups.

   b) The greatest population is in the 40–50 age group with approximately 5 200 000 people.

   c) There were approximately 200 000 Canadians in their 90s.

d) Approximately 4 200 000 teenagers lived in Canada.

e) There were approximately 2 000 000 children under 10. This implies that there are not as many babies being born as in the past.

f) People under the age of 15, and those 65 and over, are often not part of the work force, although now many Canadians work well past the age of 65.

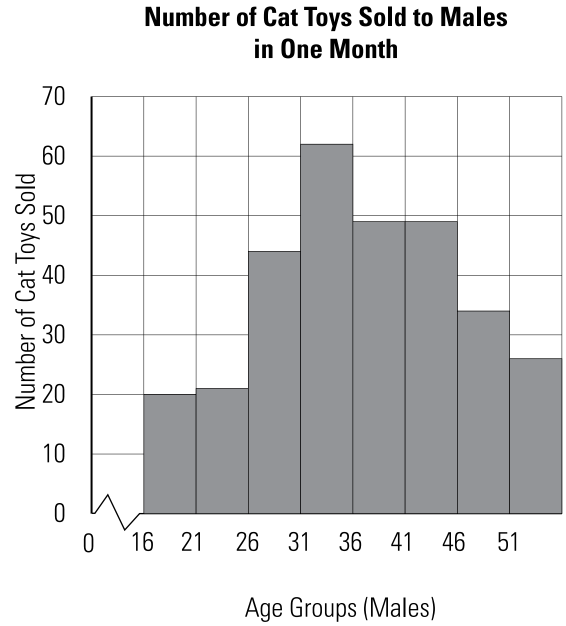
g) These are the actual rounded numbers. From the histogram, you may not get them as accurate, but your numbers should be within one or two hundred thousand.

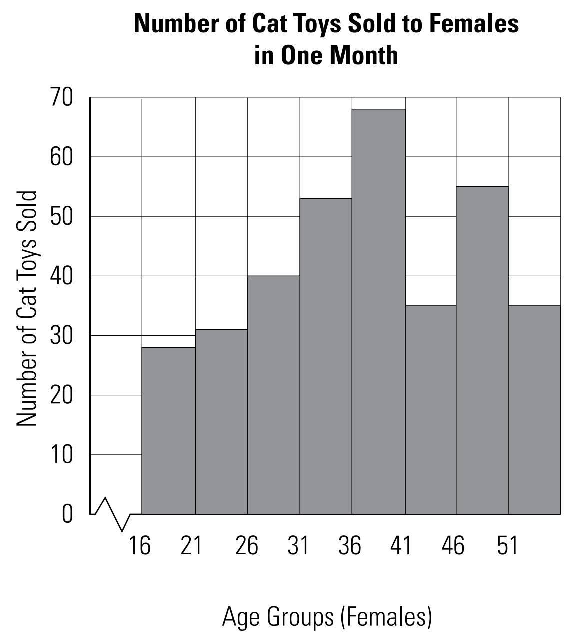
Canadian Population Distribution by Age Groupings	
0–9	2 000 000
10–19	4 200 000
20–29	4 100 000
30–39	4 200 000
40–49	5 200 000
50–59	4 500 000
60–69	2 800 000
70–79	2 900 000
80–89	900 000
90–99	200 000

h) According to Statistics Canada data, there are almost 5000 Canadians over 100 years old.

i) As of May 2010, Elizabeth Buhler of Winkler, Manitoba was thought to be the oldest living Canadian. Buhler was born in 1899. In 2010, she was 111 years old.

3.

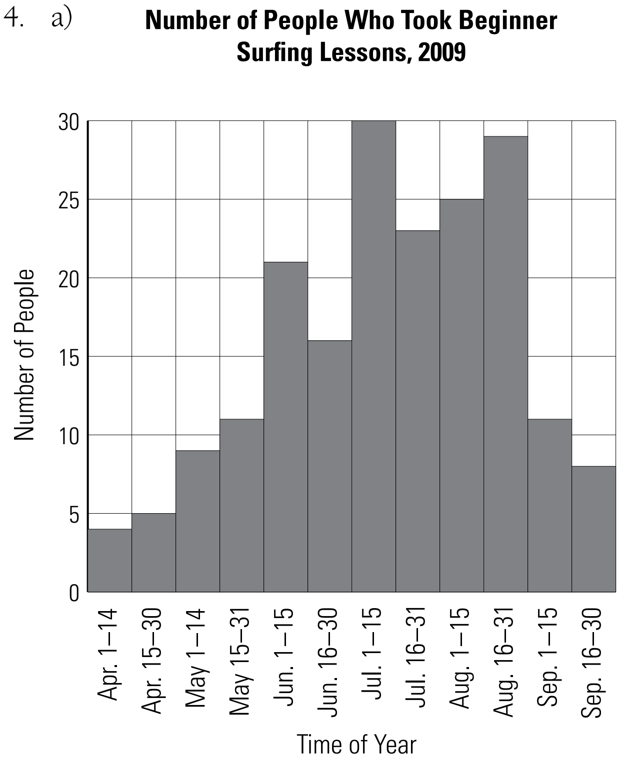




Answers will vary but may include:

- I like the bar graph because you can see the total sales.
- I like the histogram because you can see that for men, there is a fairly smooth change across age groups, but for the women it is more jagged.
- Each give a different type of image and it would depend what you wanted to use the graph for.

4. a)



b) Answers will vary, but students should see that the trend is for the number of people who took beginner surfing lessons to increase in the spring and summer, and to begin to decline in fall. Students might speculate that this is because more people are willing to go in the ocean when the weather is warmer and the seas are calmer, or because more tourists visit Tofino and take surfing lessons during the summer.

c) In August, the highest number of people took beginner surfing lessons. Because each month has two sets of figures, students need to add them to determine the total number of people taking beginner surfing lessons each month.

d) Answers will vary, but students should state that the number of people taking beginner surfing lessons in October will probably decline. Answers should reflect a decline in numbers and might range from 5 to 10 people taking lessons in the first week of October, and fewer than that in the second week.

e) A histogram may not be the best because there are no distinct breaks in the data. A bar graph may be more appropriate.

5. a) 17%

b) 19%

c) There is a slight general tendency for a greater percentage of women to have disabilities.

d) Carrie has added the two percentages together and you cannot do this to determine the percentage of people in the population with disabilities. You must know what percentage of the population is male and what percentage is female to determine the overall percentage. Since 32% of men and 33% of women in this age group have disabilities, about the same percentage in the general population have disabilities.

e) The age groupings are not equivalent. The last category has no upper limit. We might assume it is 85 or that the age group goes on indefinitely. Also, the "All ages" category is on the end and may be misread as another age group.

6. a) It displays the distribution of salaries for employees in a company.

b) 1 + 5 + 4 = 10

Ten employees earn less that $33 000.00.

c) The highest salary appears to be just under $77 000.00.

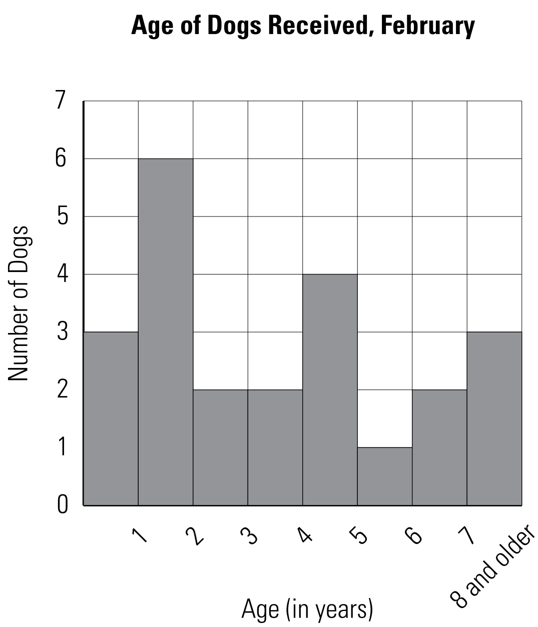
7.

8. a) Answers will vary, but the age or weight of the animals that were dropped off might be the most suitable data to display on a histogram, since these can be displayed as a range.

   b) Answers will vary, depending on the information the student chooses to graph. If students choose to graph weight, they could use kilograms as a unit of measurement, as well as number of animals. If students choose to graph age, they could use years as a unit of measurement, along with number of animals.

   c) Answers will differ. Because Lise compiles this information on a monthly basis, students should name a month or include the word "monthly" in their graph's title. Lise compiles this data by species, so the graph's title should refer to the species it is about (cat, dog, bird, etc.). Possible vertical axis labels for the graph could include "Number of animals," "Number of dogs," or "Number of birds." Horizontal axis labels could include "Age in years" or "Weight in kilograms."

   d) Answers will vary. A sample graph is given below.

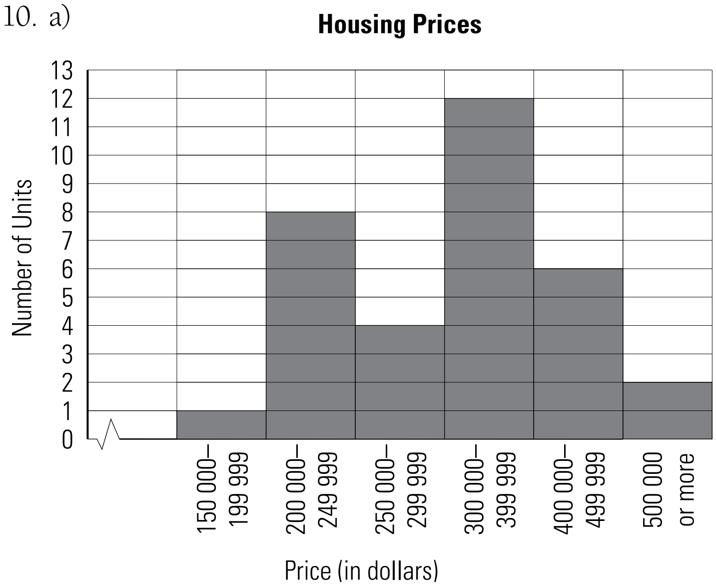


Extend Your Thinking

9. a) Answers will vary but may include questions such as the following: How many people were surveyed? What is the greatest amount of time spent on the internet? What is the most common amount of time spent on the internet? What two time spans have the same number of people?

   b) Who does the histogram represent? Is this on one particular day, or an average over time? Do all people have access to the internet equivalently?

10. a)

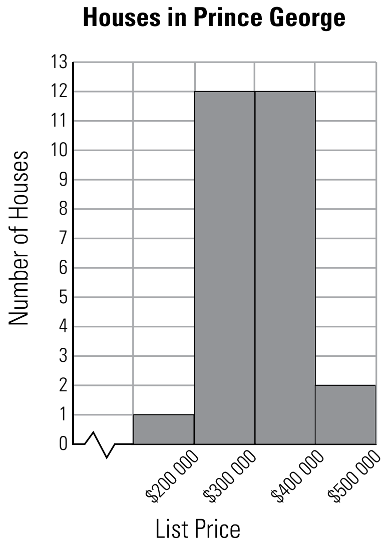


   b) The dollar values do not vary by the same amounts.

   c) Answers will vary but students may suggest that the cost of housing is divided into different price categories.

   d) Answers will vary. Some will think it is not misleading because they would notice the differences while others would look only at the number sold and not realize that the groupings were different. Point out to students that if the price ranges are not the same, people sometimes change the widths of the graphs to reflect this.

e) Answers will vary although the second graph does not seem to discriminate between the pricing as much as the first.



THE ROOTS OF MATH

GRAPHING: INSPIRED BY THE BUBONIC PLAGUE

STUDENT BOOK, p. 97

This is a teacher-led activity with the entire class. Your students may need you to provide some background information about plague epidemics that swept through Europe periodically before the advent of modern medical practices. They may also be unfamiliar with the lack of statistical data collection in that era.

SOLUTIONS

1. The data were used to predict the potential outbreak of new epidemics and to determine areas most hardly hit.
2. It is likely that he improved the techniques and set rules for their presentation.
3. The link, www.wiley.com/legacy/wileychi/eosbs/pdfs/bsa261.pdf, presents a good article “Graphical Presentation of Longitudinal Data” that has illustrations of the various graphs mentioned.
4. Answers will vary depending on who the class chooses.

2.4 Circle Graphs

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

MATH *ON THE JOB*

STUDENT BOOK, p. 98

Have a student read the Math on the Job aloud. Ask students if they are familiar with any of the tasks that Trina does at her job. Some students might be employed and responsible for ordering merchandise, as Trina is. If any students perform this task at work, ask them to describe how they take inventory and/or place orders.

Neechi Food Co-op Ltd. is a fair trade Aboriginal specialty store with a focus on healthy foods and local products. You can use the Math on the Job as a way to discuss food choices. Ask students if they have tried traditional ethnocultural foods; you may specify local First Nations food or other ethnoculturally identifiable groups in your area.

If you are interested in finding out more about Neechi Foods Co-op Ltd., the links below will provide more information.

- www.arch.umanitoba.ca/greenmap/pages/GrnMapPl_msNeechi/
- www.winnipegfreepress.com/local/neechis-expansion-will-bring-another-food-store-to-main-street-59442842.html

SOLUTION

1. Wild Products accounts for 24% of jam sales.
2. Shannon Foods is the least popular brand of jam. It accounts for 2% of sales.
3. Answers will vary. Sample answers are provided.
   - Displaying data on a circle graph makes comparing the data easy.
   - When data are displayed using a circle graph, you can easily see which product is most popular and which product is least popular.
   - Displaying data on a circle graph allows you to see all the data at once. This can give you an overview of the data.

EXPLORE THE MATH

STUDENT BOOK, p. 99

If your class needs practice with percentages, refer to Blackline Master 2.8, on p. 145 of this resource.

While a circle graph is relatively time-consuming to draw manually, the students should have the experience of drawing one or two on their own. Read through this section with the students and work through Example 1 or a similar type of question with them. You may have to review briefly that there are 360° in a circle, and how to use a protractor, especially how to turn it after the initial sector is drawn. Thus, you will probably want to work through this example on the board.

It will be important for the students to consider rounding to the nearest degree, and you will need to explain that, because of this, at times the angles will not add to exactly 360° but should not be off by more than a degree and that the errors caused by rounding will not make much difference in the overall graph.

PUZZLE IT OUT

COIN CONUNDRUM

STUDENT BOOK, p. 101

SOLUTIONS

To solve the problem, divide the coins into one pile of seven and one pile of five. Flip over all of the coins in the smaller pile of five. Now, each pile will have an equal number of coins with the heads side facing up. This problem can be solved using simple algebra and statistical reasoning.

When the coins are separated into two piles, x number of heads will be in the pile made of seven coins.

Number of heads in the five-coin pile:

$5 - x$ heads

Number of tails in the five-coin pile:

$5 - (5 - x) = x$ tails

When students flip all the coins in the pile of five, the $5 - x$ heads becomes tails and the x tails becomes heads. As a result, each pile contains x heads.

DISCUSS THE IDEAS

REPRESENTING DATA ON A CIRCLE GRAPH

STUDENT BOOK, p. 103

1. Answers will vary, but can include that using a circle graph will make it easy to see what Max spends the most money on.
2. To make a circle graph, Max needs to first add up all of his expenses for the month. Then he needs to find out what percentage of his total monthly expenses each category of expenses represents. He then needs to calculate the number of degrees each expense represents. Finally, Max needs to use a protractor to draw his circle graph.
3. Answers will vary. Some possible answers include:
   - A store manager might use a circle graph to show sales of different products.
   - A contractor might use a circle graph to show expenses on a building project.

ACTIVITY 2.6

GRAPHING CHARITABLE DONATIONS

STUDENT BOOK, p. 103

You can introduce this activity by asking students if they donate to any organizations, or if their families do. You can ask them to tell you a bit about the organization and why they choose to support it.

SOLUTIONS

1. To draw the graph, ask students to calculate the percentage of donations that come from the different sources.

Add the total amount of money from donations.

$3540 + 800 + 1278 + 4421 + 630 + 367$

$= 11\ 036$

Determine the percentages.

Individuals:

$(3540 \div 11\ 036) \times 100 \approx 32\%$

Provincial government:

$(800 \div 11\ 036) \times 100 \approx 7\%$

Local businesses:

$(1278 \div 11\ 036) \times 100 \approx 12\%$

Corporations:

$(4421 \div 11\ 036) \times 100 \approx 40\%$

Religious organizations:

$(630 \div 11\ 036) \times 100 \approx 6\%$

Educational organizations:

$(367 \div 11\ 036) \times 100 \approx 3\%$

Students then have to work out how many degrees of the circle are included in each percentage.

Individuals 32%

$0.32 \times 360° \approx 115°$

Provincial government 7%

$0.07 \times 360° \approx 25°$

Local businesses 12%

$0.12 \times 360° \approx 43°$

Corporations 40%

$0.4 \times 360° \approx 144°$

Religious organizations 6%

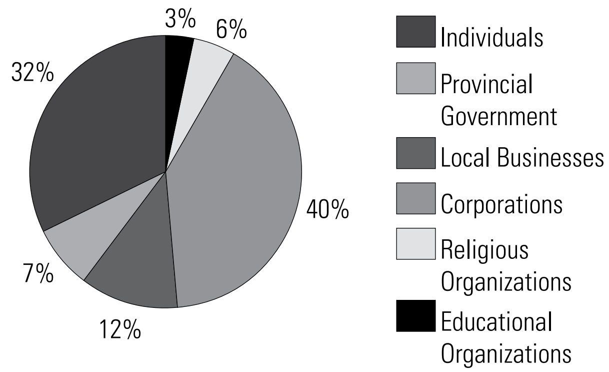
$0.06 \times 360° \approx 22°$

Educational organizations 3%

$0.03 \times 360° \approx 11°$

Students then need to use a protractor to draw the circle graph.

Source of Charitable Donations



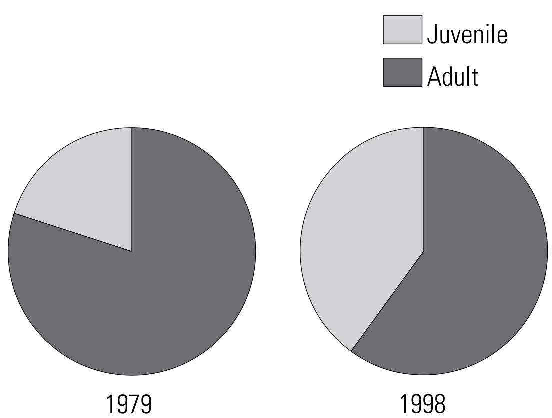
2. Camp Tamarack should try to increase donations from the provincial government, religious organizations, and educational organizations, since these three sources donate the least.

3. Individuals and corporations donate the most. Students might justify this response by saying that corporations are likely to have more money, so they can afford to donate more. Students might also reason that wealthy individuals or individuals who want to increase educational opportunities for children with learning disabilities are more likely to donate to Camp Tamarack.

Mental Math and Estimation

STUDENT BOOK, p. 104

Vancouver Island Marmot Population, Adult and Juvenile Composition



By looking at the first graph, students should be able to estimate that in 1979, about 80 percent of the Vancouver Island marmot population was composed of adults. By looking at the second graph, students should be able to estimate that 60 percent of the marmot population was composed of adults. Students can calculate how much the percentage of adults decreased by subtracting 60 from 80. The percentage of adults decreased by about 20 percent.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 104

1. a) Mexico is the most popular country to visit, but students will likely say that Mexico and the U.K. are equally popular.

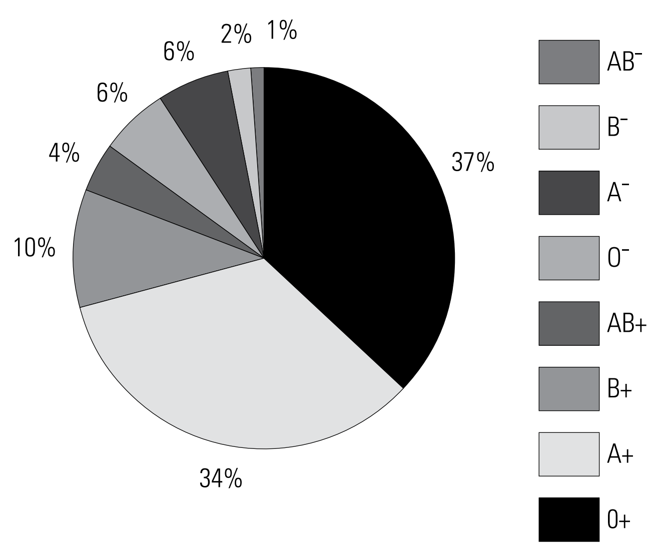
   b) Approximately 20% of the people visited Cuba.

   c) Answers will vary but should include things like: You cannot tell the actual number of people who visited each country, the time spent in each country, the amount of money spent, if people travelled alone or in family groups or on tours.

2. First find the number of degrees of the circle that will be included for each blood type.

$$
\begin{aligned}
0.37 \times 360 &= 133^\circ \\
0.34 \times 360 &= 122^\circ \\
0.10 \times 360 &= 36^\circ \\
0.04 \times 360 &= 14^\circ \\
0.06 \times 360 &= 22^\circ \\
0.02 \times 360 &= 7^\circ \\
0.01 \times 360 &= 4^\circ
\end{aligned}
$$

Blood Type Distribution



3. If the average Canadian's food expenses were to increase substantially, expenses on other items would need to decrease— some of the other slices of the circle graph would have to be smaller. People would need to consider ways to save money in other areas of their lives, perhaps on entertainment and recreation or on clothing expenses.

4. a) $33 \times 0.06 = 1.98$

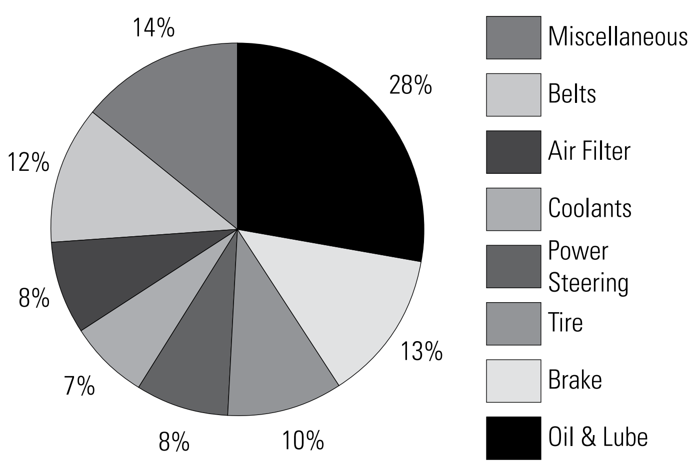
   Therefore, it is likely that 2 of them were electricians.

   b) 3% of the injured workers were painters.

   c) Since 27% of the injured workers were plumbers, it looks like the most dangerous job. However, you cannot be certain because you don't know how many workers of each type were at the different sites. There may have been many more plumbers than other workers.

5. First determine what percentage is left for the miscellaneous category. It is 14% because the other categories add up to 86%.

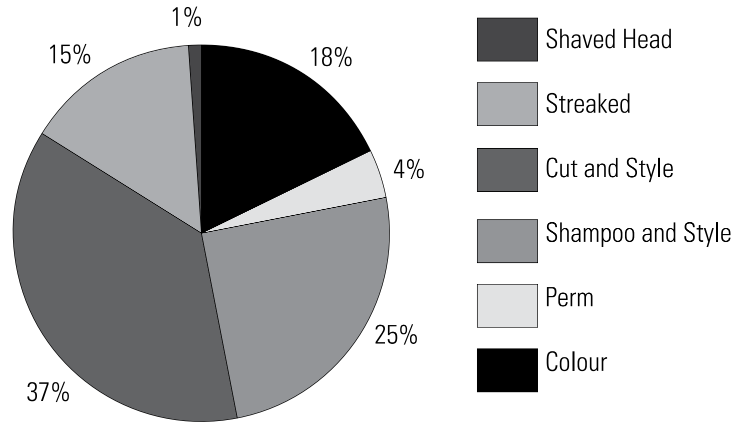
Percent of Customers Receiving Each Type of Car Maintenance



Extend Your Thinking

6. a) Add up the numbers of clients. Calculate what percentage has each treatment. Then calculate how many degrees this represents. Alternatively, if students are using graphing programs, they can import numbers and output a circle graph.

Number of Each Type of Hair Treatment



   b) Answers will vary but may include the following.

   The circle graph because the proportions are more obvious.

   The bar graph because you can see that cut and style is the most popular and shaved head the least popular.

REFLECT ON YOUR LEARNING

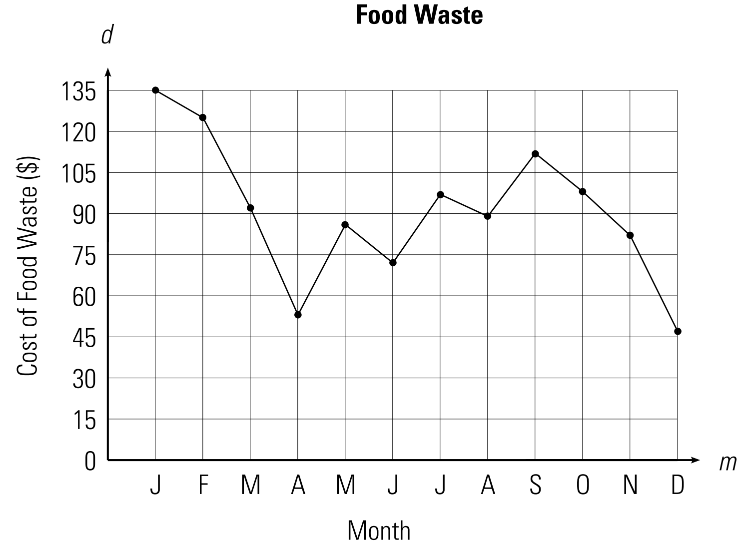
Ask students to review and reflect on the list of new skills and knowledge they have encountered in this chapter.

PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 109

1. A broken line graph is the best choice here, although a bar graph could be used.

Food Waste



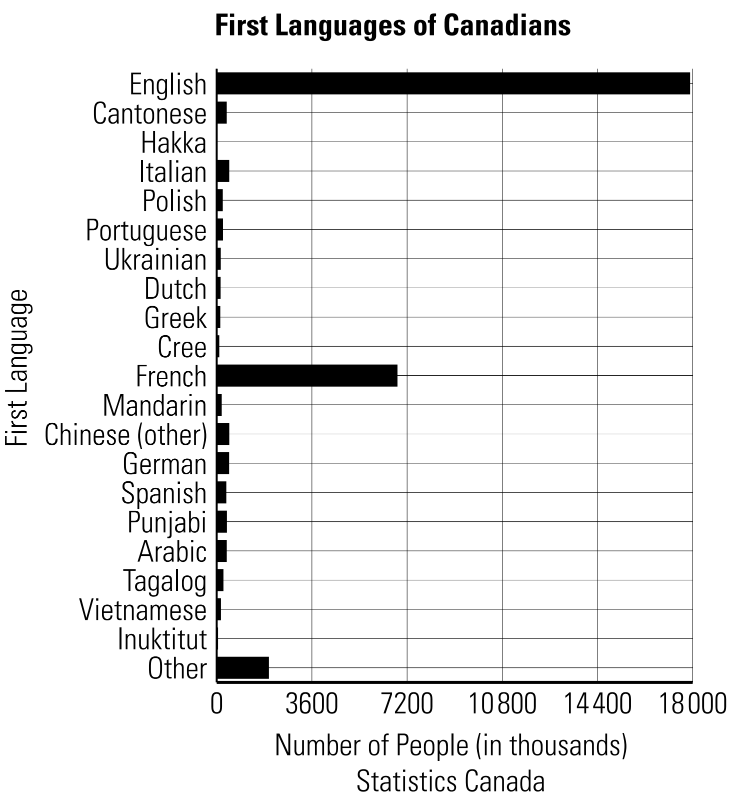
2. a) The graph seems to indicate the trend in electricians' salaries every three months from October 2007 to October 2009.

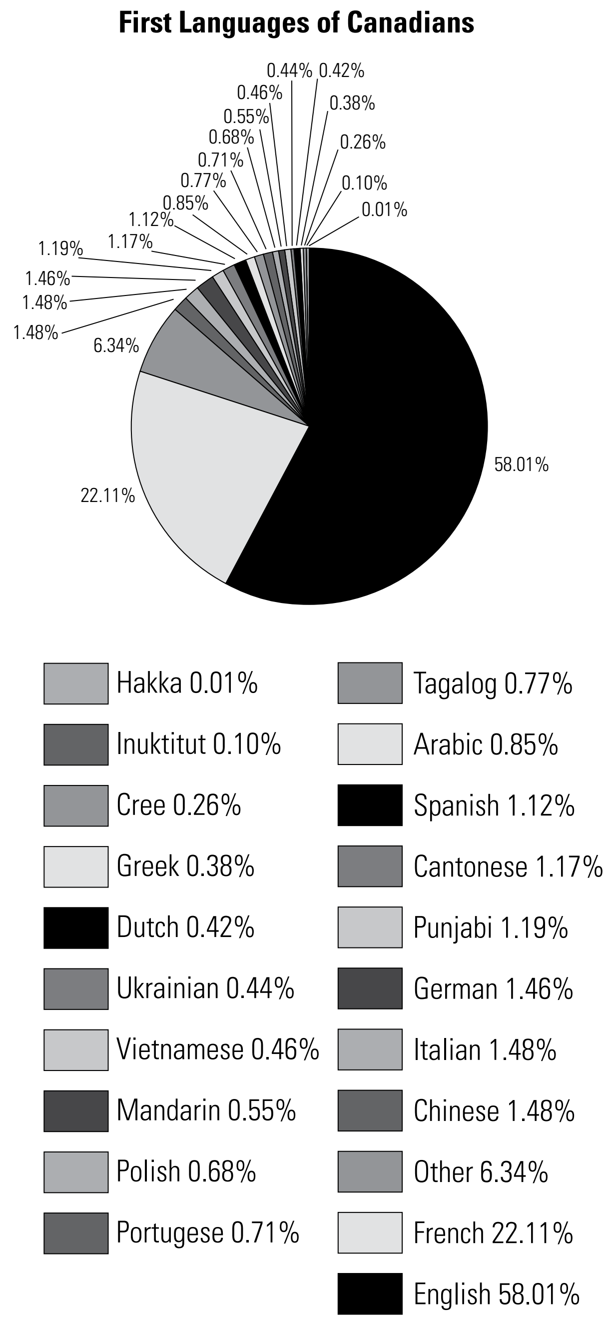
   b) It is misleading in that the vertical axis does not have continuous data from zero, so that it looks like the salary dropped to almost zero in July 2009. Also, it does not indicate what it is comparing the salaries to.

3. a) This was not the most suitable display because it looks like a continuum.

   b) A more suitable display would have been a bar graph or a circle graph. The bar graph shows the discrete numbers, and the circle graph shows proportions.

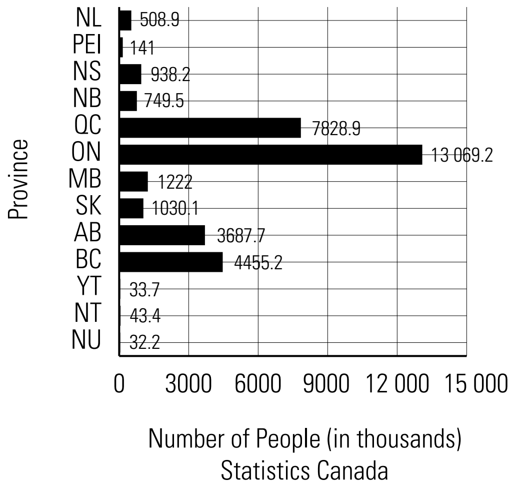
4. a) Either a bar graph or a circle graph will work.



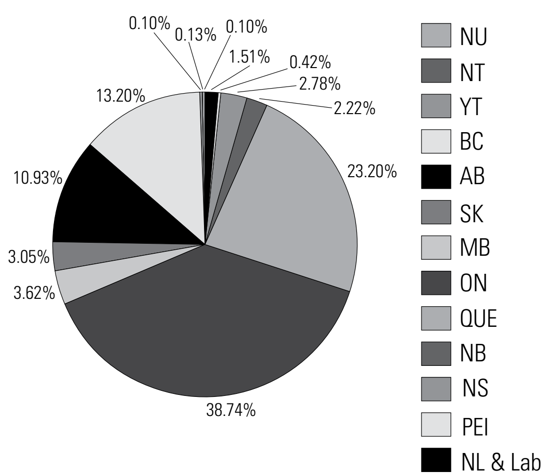


   b) Answers will vary but may include the following.
      - One can see the differences among the smaller numbers more easily in the circle graph.
      - The numbers spread out more on the bar graph.

5. a) **Canadian Population by Province and Territory**



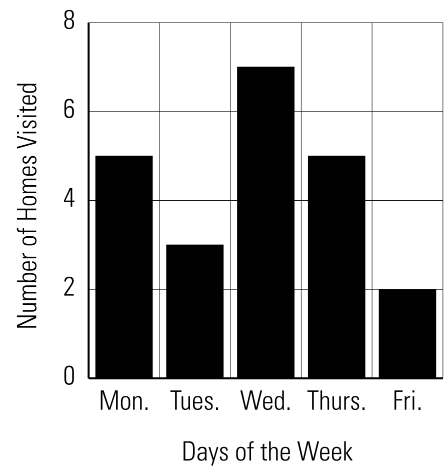
b) **Canadian Population by Province and Territory**



c) The circle graph seems to give a better picture of the overall distribution. On the bar graph, the numbers indicate differences, but the lengths of the bars on the northern territories is hard to distinguish.

6. a) The best type of graph here is probably a bar graph, either vertical or horizontal.

Harpinder's Plumbing Jobs



b) The data are discrete, and specific for each day.

c) One could have used a broken line graph, but since the data are discrete, this is not as good. One could also have used a circle graph, but there is no advantage to that because Harpinder is not likely interested in the proportion of jobs done each day.

7. a) The power plant electrician earns the most, at approximately $66 000.00.

b) They each earn about $49 000.00 per year.

c) You should not use a broken line graph because there is no continuum.

You could not use a histogram because the data are discrete, not continuous.

You could not use a circle graph because you do not know the total amount that you are working with.

8. a) This is a suitable way to display the information because the trees are of differing heights and Ray would not be able to determine their exact heights, just a range.

b) The tallest tree is between 90 and 100 feet tall.

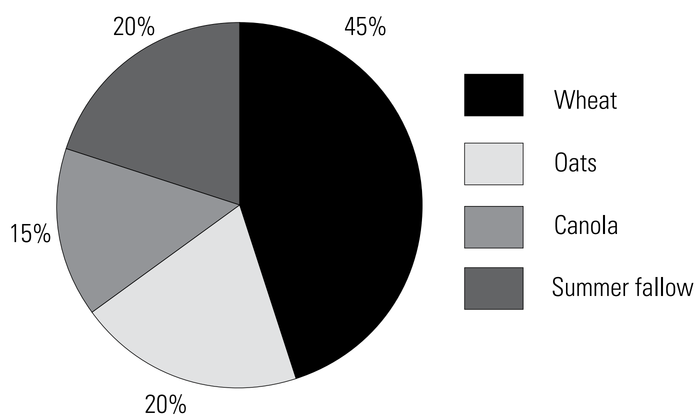
c) There are 32 trees taller than 50 feet.

d) You cannot be sure how many trees are less than 50 feet tall because Ray did not indicate how many trees were less than 10 feet tall. However, there are 45 trees between 10 and 50 feet tall.

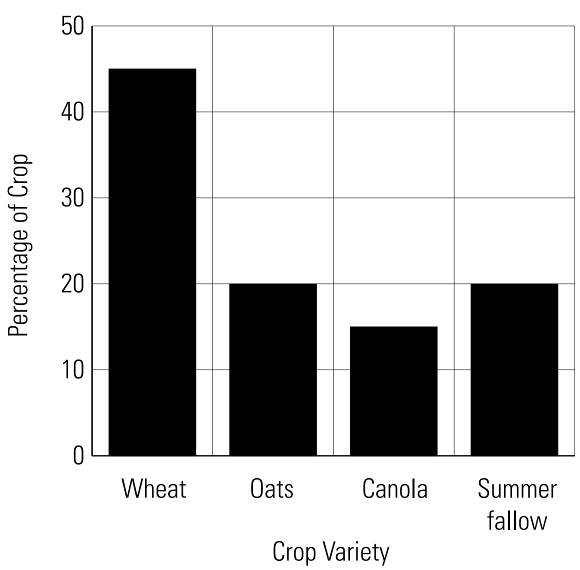
e) The information could have been displayed on a circle graph.

9. A circle graph is a good way to display this data because you have the percentages given and it shows the distribution clearly. A bar graph could also be used.

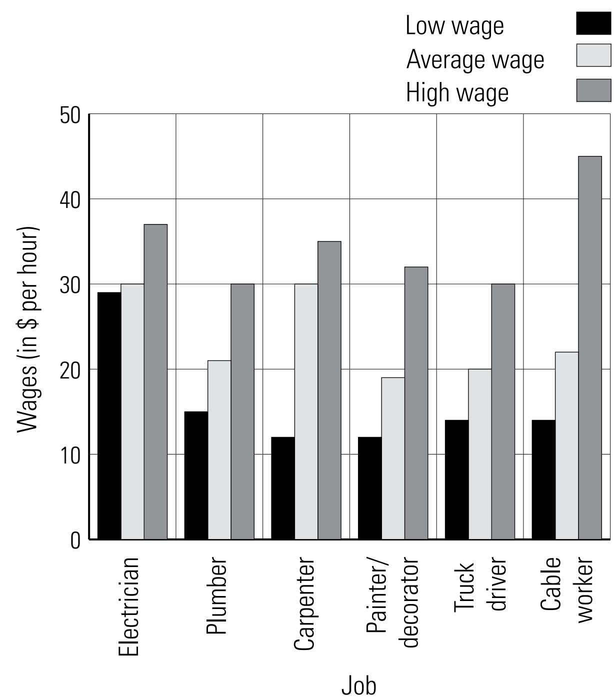
Distribution of Crops



Distribution of Crops



10. a) **Comparison of Salaries in Calgary**

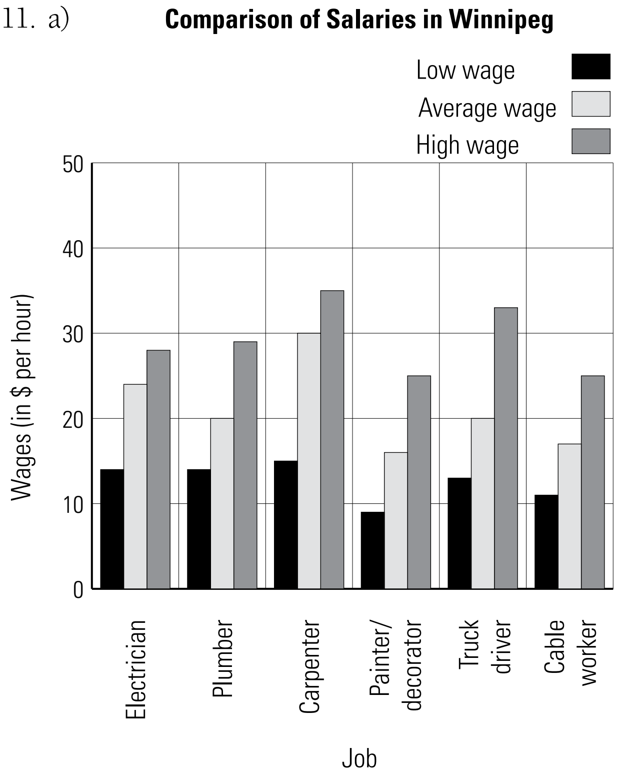


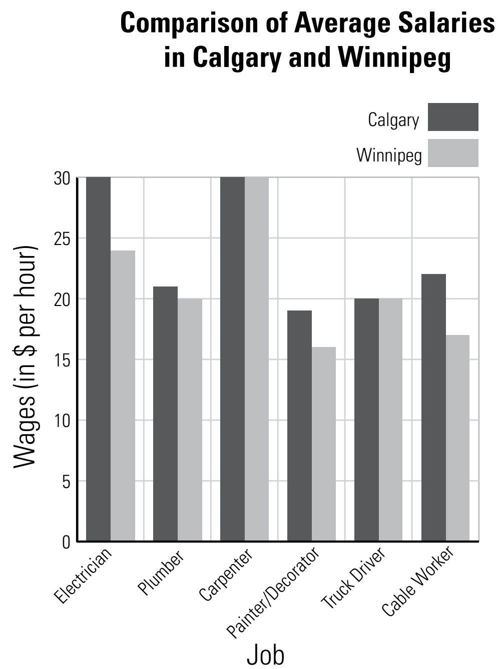
b) Cable workers have the greatest discrepancy.

c) Electricians have the least discrepancy.

d) Answers will vary but should include comments such as: If the average wage is closer to the higher wage, more people tend to be in the upper category and vice versa. Thus, for carpenters, it appears that there are more in the upper category, but for cable workers, more are in the lower category.

11. a)





b) No. In Calgary, the highest paid occupation is the cable worker while in Winnipeg, it is the carpenter. The lowest in both cities is the painter/decorator.

c) Answers will vary but should include comments such as:
- Winnipeg salaries tend to be lower.
- There is not as much discrepancy between the high and the low in Winnipeg.
- While electricians' salaries in Calgary don't vary much, they vary considerably in Winnipeg.
- Cable workers in Calgary earn considerably more than in Winnipeg.

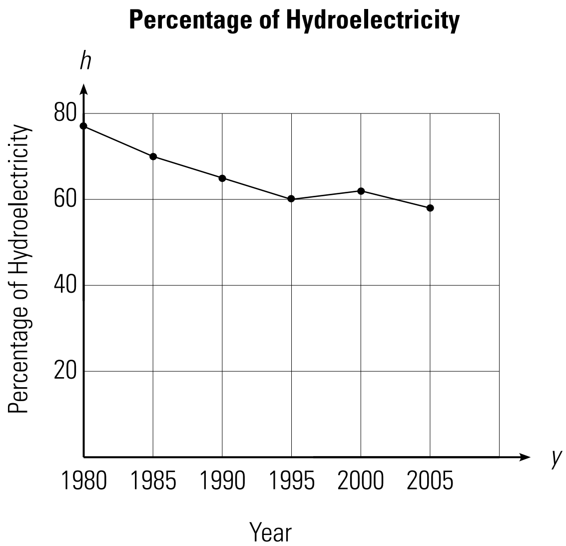
d) The difference is important, but one would also have to consider cost of housing and other living expenses such as travel time and distances, food, etc.

12. The most meaningful graph would be a double bar graph of the average wages.

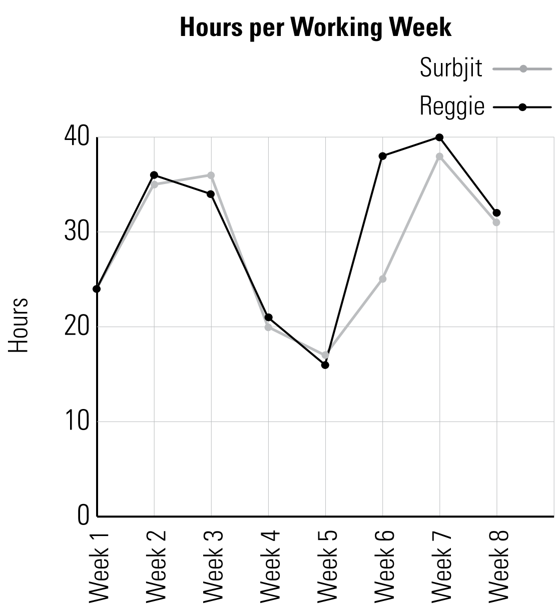
SAMPLE CHAPTER TEST

Name: ______________________________ Date: ______________________________

1. The graph below indicates the approximate percentage of hydroelectricity use in Canada compared to other sources of electricity. Describe the general trend in the use of hydroelectricity and predict what percentage of all electricity use in Canada in 2010 will be hydroelectricity.



2. The graph below shows the number of hours Reggie worked at the loading dock during the past eight weeks and the number of hours Surbjit worked during the same time period.



a) During what weeks did they both work over 30 hours?

b) During what weeks did Surbjit work more hours than Reggie?

c) What was the minimum number of hours Surbjit worked in any one week?

d) In what week did they have the greatest difference in the number of hours worked, and approximately how many hours was that?

3. The table below indicates the number of acres of wheat and oats George seeded over the past five years.

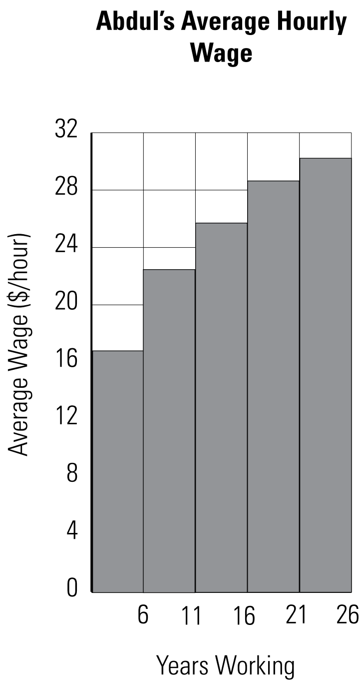
CROPS SEEDED BY YEAR

Year	*2005*	*2006*	*2007*	*2008*	*2009*
Wheat (acres)	520	400	640	520	400
Oats (acres)	340	400	220	280	120

a) Draw a double bar graph to represent this data.

b) Compose and answer two questions that can be determined from the information on the graph that does not involve simply stating a year.

4. Abdul works as an electrician. During the first year he worked as an apprentice. He then worked his way up through to journeyman. The histogram shows Abdul's average hourly income for the 25 years after his apprenticeship.



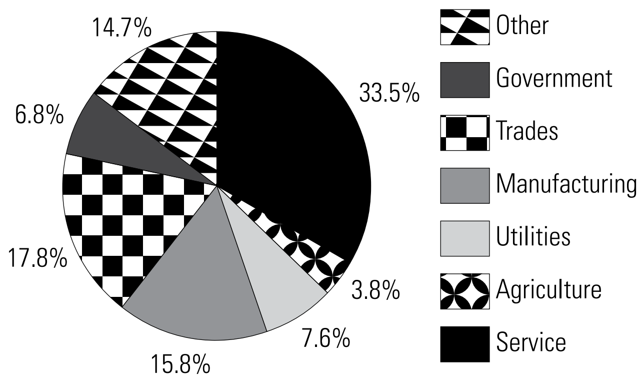
a) Between what two working periods did he have the greatest hourly increase?

b) Approximately what was his wage from the 11th to the 15th year?

c) Assuming a similar growth pattern, what would you expect his average hourly income to be for 26–30 years? Explain.

5. The graph below shows the approximate distribution of Canada's labour force.

Distribution of Canada's Labour Force



a) Approximately what percentage of the labour force works in agriculture?

b) Which industry respresents the greatest percentage of the labour force?

c) Which two industries combined represent approximately the same percentage of the labour force as service?

6. Draw a graphical representation of the data below that displays the percentage of the adult population with a disability. Justify your choice of graph type.

PERCENTAGE OF ADULT POPULATION WITH A DISABILITY										
pain	*mobility*	*agility*	*hearing*	*seeing*	*learning*	*psycho-logical*	*memory*	*speech*	*develop-mental*	*unknown*
11.7	11.5	11.1	5.0	3.2	2.5	2.3	2.0	1.9	0.5	0.5

7. Theresa works as a chambermaid in a tourist resort. She indicates that she spends 35% of her time dusting, 35% vacuuming, 20% cleaning the bathrooms, and the rest making beds. Draw a circle graph to indicate how her work is distributed. Be sure to show all calculations.

SAMPLE CHAPTER TEST: SOLUTIONS

1. There is a general trend towards using less hydroelectricity. The predicted amount should be about 55%.

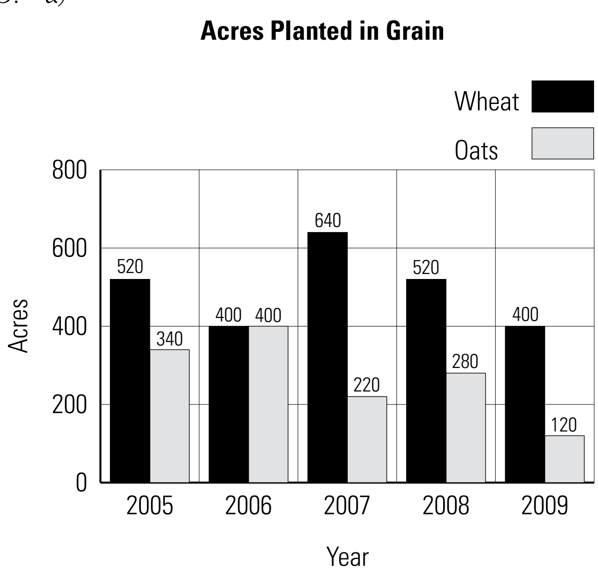
2. a) They both worked over 30 hours during weeks 2, 3, 7, and 8.

   b) Surbjit worked more hours than Reggie in weeks 3 and 5.

   c) The fewest hours Surbjit worked was 17.

   d) The greatest difference in the number of hours worked was during week 6. Surbjit worked approximately 25 hours and Reggie approximately 38 hours, so the difference was approximately 13 hours.

3. a)



   b) There are many possible questions, including:

   i) During what year did George plant the same number of acres of wheat and oats, and how much was it?
   Solution: He planted 400 acres of both grains in 2006.

   ii) In what year did he have the greatest difference in the number of acres of each grain, and by how much?
   Solution: The greatest difference was in 2007 and was about 420 acres.

   iii) What is the general trend in George's planting pattern?
   Solution: The only general trend is that he tends to plant fewer acres in oats than in wheat, the exception being 2007 when he planted the same amount.

4. a) The greatest increase occurred between 1–5 and 6–10 years.

   b) Answers many vary slightly but should be between \$25.00 and \$26.00.

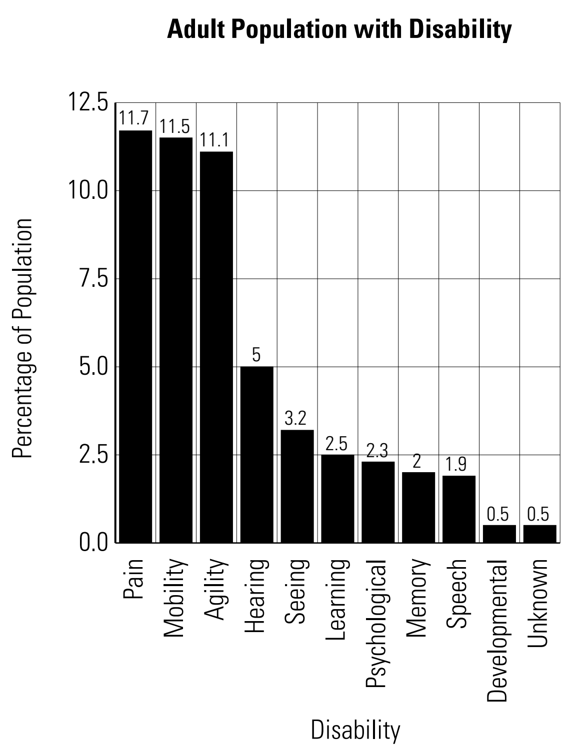
   c) Answers may vary slightly but should be around \$31.00. The increase seems to be decreasing slightly at this level so the increase in wages will not be as much as over the previous five years.

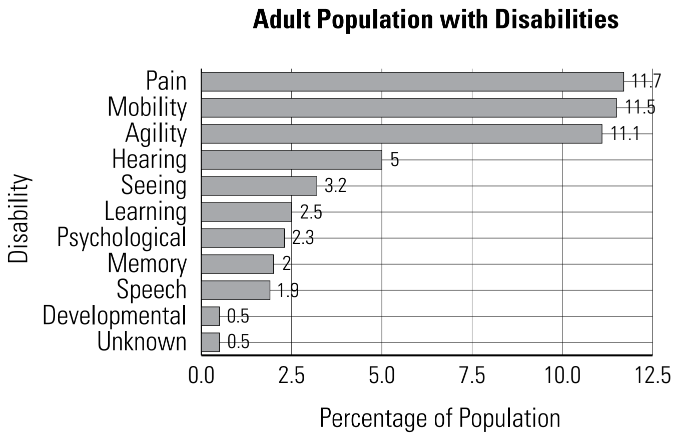
5. a) Approximately 3.8% of the labour force works in agriculture.

   b) Service represents the greatest percentage of the labour force.

   c) Manufacturing and trades add up to about the same percentage as service.

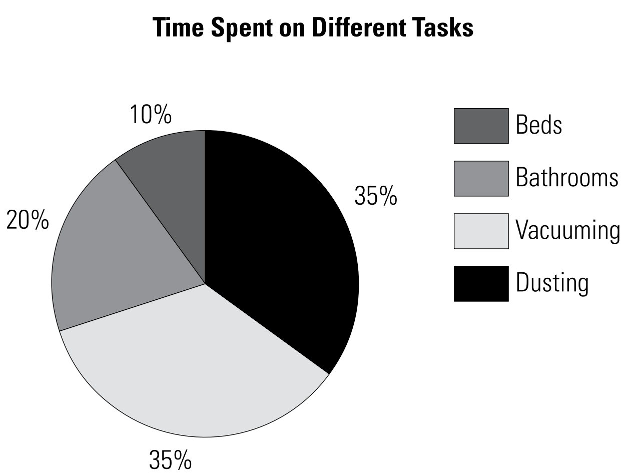
6. A bar graph, either vertical or horizontal, is the best choice here.





The bar graph indicates the discrete items. A circle graph could be used, but the "whole" is then the population with disabilities and it might be a bit misleading. Percentages would have to be changed.

7. Change each percent to a decimal and multiply by 360° to find the portion of the circle for each type of work.



Dusting:	$0.35 \times 360° = 126°$
Vacuuming:	$0.35 \times 360° = 126°$
Bathrooms:	$0.20 \times 360° = 72°$
Beds:	$0.10 \times 360° = 36°$

BLACKLINE MASTER 2.1

GRAPH PAPER (0.5 CM × 0.5 CM)

Name: ______________________________ Date: ______________________________

BLACKLINE MASTER 2.2

CHAPTER PROJECT BROCHURE DESIGN CHECKLIST

Name: ______________________________ Date: ______________________________

This page will give you some ideas to think about during your design process. Write down your ideas as you think about them. Keep all ongoing ideas, pictures, and other information on either a computer file or in a file folder.

BROCHURE DESIGN CHECKLIST	
❒ Where can I go or who can I ask to get information about this resort?	
❒ What attractions does my resort offer?	
❒ Who will be my intended audience?	
❒ What types of pictures will entice these people to visit the resort?	
❒ How can I promote both summer and winter activities?	
❒ How will I design my brochure?	
❒ What type of information will I display on my graphs?	
❒ What graphs best display this information?	
❒ Other considerations	

BLACKLINE MASTER 2.3

CHAPTER PROJECT BROCHURE AND PRESENTATION CHECKLIST

Name: ______________________ Date: ______________________

BROCHURE AND PRESENTATION CHECKLIST	
❐ Does your brochure include two graphs, photographs, and written text?	
❐ Is your brochure two pages long?	
❐ Do you have your two data tables with the data filled in and the name of your data sources included?	
❐ Have you written a short promotional talk (30–60 seconds) about your resort?	
❐ Are you prepared to explain why you chose to use the two types of graphs you included in your brochure?	
❐ Do you have your four graphs ready to hand in?	
❐ Are you ready to answer questions your classmates might have about your resort?	

BLACKLINE MASTER 2.4

SELF-ASSESSMENT RUBRIC

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following things.

- the thoroughness of your research;
- the accuracy of your graphs;
- the effectiveness of your use of technology for researching, making graphs, and laying out a brochure;
- the effectiveness and persuasiveness of your finished brochure and talk; and
- your completion of the assigned tasks on time.

How do you feel you have done overall, given the criteria above?

____

____

Were you able to complete all aspects of the project? If not, why? Did you allot your time effectively?

____

____

In what areas did you excel?

____

____

Are there areas in which you could improve?

____

____

If you collaborated with a partner or small group, what strengths did each person bring to your project?

____

____

Do you think your brochure and talk would effectively persuade a tourist to visit your resort? Why or why not?

____

____

BLACKLINE MASTER 2.5

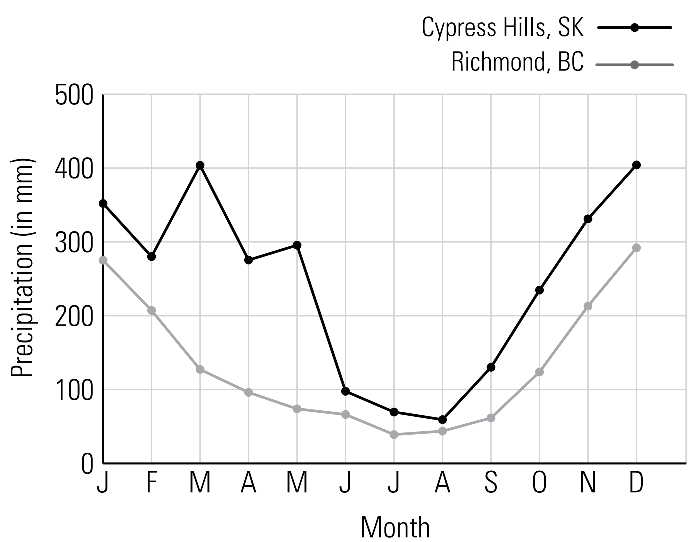
INVESTIGATING A MISLEADING GRAPH

Name: ______________________________ Date: ______________________________

Betsy wanted to compare the overall average monthly precipitation between Cypress Hills, Saskatchewan and Richmond, British Columbia, recorded on p. 71 of *MathWorks 11*. She realized that she had to use the same units of measurement, so she converted the average monthly snowfall for each area into millimetres. She then added these amounts to the average monthly rainfall. The table and graph she created as a result are shown below.

AVERAGE MONTHLY PRECIPITATION IN CYPRESS HILLS, SK AND RICHMOND, BC												
	Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
Cypress Hills	351	280.1	403.7	275.4	295.5	97.7	69.6	59.3	130.2	234.7	331.2	404.2
Richmond	275.2	207.2	127.3	96.2	73.9	66.3	39.1	43.7	61.6	123.9	213.1	292.1

Proof that Cypress Hills, SK is Wetter than Richmond, BC.



A friend of Betsy's looking at the chart and graph informed Betsy that she had made the graph using incorrect data.

1. Why did Betsy's friend feel that the data Betsy used to create the graph was incorrect?
2. What assumptions did Betsy make when she calculated the total precipitation for each month?
3. Calculate the correct amount of precipitation for each month in each area and construct a multiple line graph of the data.

BLACKLINE MASTER 2.5: SOLUTIONS

1. Answers will vary. Sample answers may include the following. BC is known to receive large amounts of rain, so it seems unlikely that the average amount of precipitation in Cypress Hills would be greater than that in Richmond. Snow is less dense than rain, so it contains less water.
2. Betsy assumed that 1 cm of snow is equivalent to 1 cm of rain. Thus, changing the snow from centimetres to millimetres gives the same amount of snow, but not the equivalent amount of rain.
3. The water equivalent of snow varies, depending on the density of snow. However, a ratio of 10 to 1 is commonly used when converting snow to its water equivalent. This would mean that 10 cm of snow would equal 1 cm of water, or rain. To convert snow to water, divide the values on the Average Monthly Snowfall (cm) data table by 10. Multiplty these values by 10 to convert to millimetres. Students should realize the numbers on the chart below will stay the same, though they are now measured in millimetres.

AVERAGE MONTHLY SNOWFALL MEASURED AS WATER (MM)

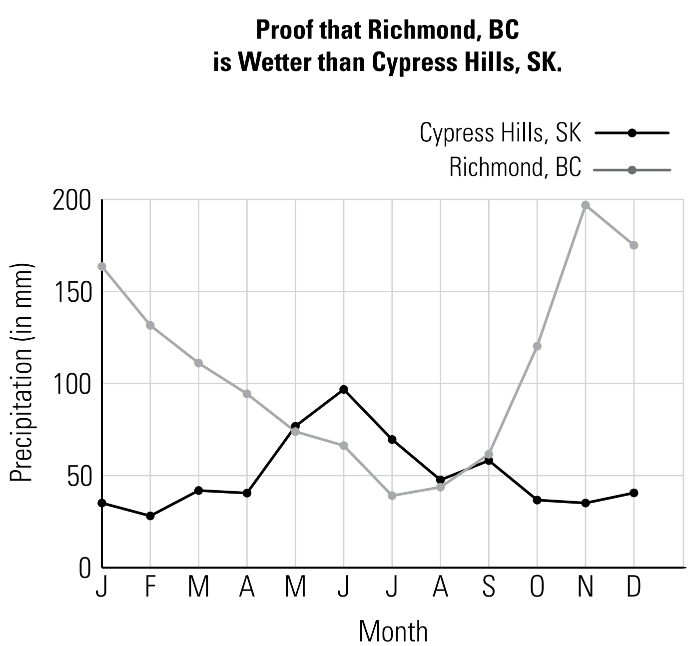
	Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
Cypress Hills	35.1	28	40.2	26.1	24.3	0.1	0	1.3	8	22	32.9	40.4
Richmond	12.4	8.4	1.8	0.2	0	0	0	0	0	0.4	1.8	13

Next, add the snowfall measured as water values to the rainfall values from the student book.

AVERAGE MONTHLY PRECIPITATION (MM)

	Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sept*	*Oct*	*Nov*	*Dec*
Cypress Hills	35.1	28.1	41.9	40.5	76.8	96.8	69.6	47.6	58.2	36.7	35.1	40.6
Richmond	163.6	131.6	111.1	94.4	73.9	66.3	39.1	43.7	61.6	120.3	196.9	175.1

Use this data to construct a multiple broken line graph.

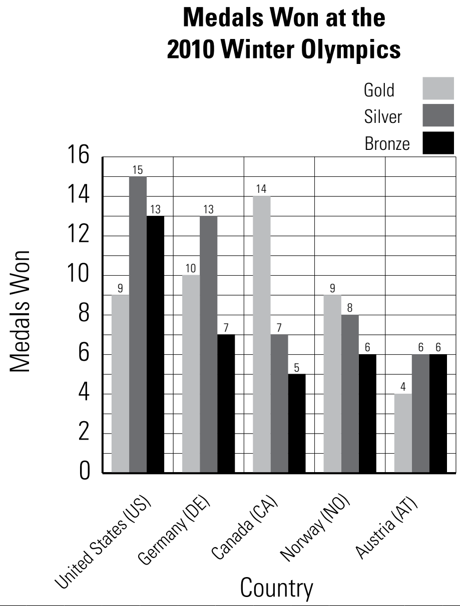


BLACKLINE MASTER 2.6

STACKED BAR GRAPH

Name: ______________________________ Date: ______________________________

Use the data from the following graph to create a stacked bar graph.



What information does this graph provide that the triple bar graph does not?

____

____

BLACKLINE MASTER 2.7

PRACTISE DRAWING GRAPHS — A. BROKEN LINE GRAPH

Name: ______________________ Date: ______________________

Use the following information to draw a broken line graph on the grid provided.

NUMBER OF STUDENTS PARTICIPATING IN EXTRACURRICULAR ACTIVITIES BY DAY				
Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
150	175	200	150	50

1. The data are "discrete." Explain what discrete data are in your own words.

2. Although a line graph is usually used to show continuous data, it can also be used to graph discrete data. Why is it acceptable to use the broken line in a case like the one you have graphed here?

BLACKLINE MASTER 2.7

PRACTISE DRAWING GRAPHS — B. BAR GRAPH

Name: ____________________ Date: ____________________

Use the following information to draw a bar graph on the grid provided.

NUMBER OF STUDENTS PARTICIPATING IN EXTRACURRICULAR ACTIVITIES BY DAY				
Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
150	175	200	150	50

Compare this graph to the graph you drew on Blackline Master 2.7A.

1. Which graph seems to give a truer picture of student involvement in extracurricular activities?

2. What information can you get from one graph but not the other?

BLACKLINE MASTER 2.7

PRACTISE DRAWING GRAPHS — C. HISTOGRAM

Name: ______________________ Date: ______________________

The following table gives the information about the number of used cars sold and the prices paid for them in May at a particular dealership. Use it to construct a histogram.

NUMBER OF CARS SOLD IN MAY, BY PRICE RANGE				
\$1000.00–\$1999.00	*\$2000.00–\$2999.00*	*\$3000.00–\$3999.00*	*\$4000.00–\$4999.00*	*\$5000.00 and over*
10	15	10	25	20

1. How many cars in total did the dealership sell in May?

____

____

2. What percentage of cars sold were sold for \$4000.00 or more?

____

____

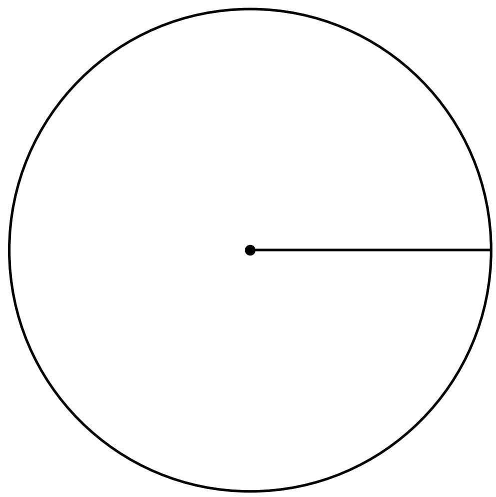
BLACKLINE MASTER 2.7

PRACTISE DRAWING GRAPHS — D. CIRCLE GRAPH

Name: ______________________ Date: ______________________

The following table gives the information about the number of used cars sold and the prices paid for them in May by a particular dealership. Use it to construct a circle graph.

NUMBER OF CARS SOLD IN MAY, BY PRICE RANGE				
$1000.00–$1999.00	*$2000.00–$2999.00*	*$3000.00–$3999.00*	*$4000.00–$4999.00*	*$5000.00 and over*
10	15	10	25	20

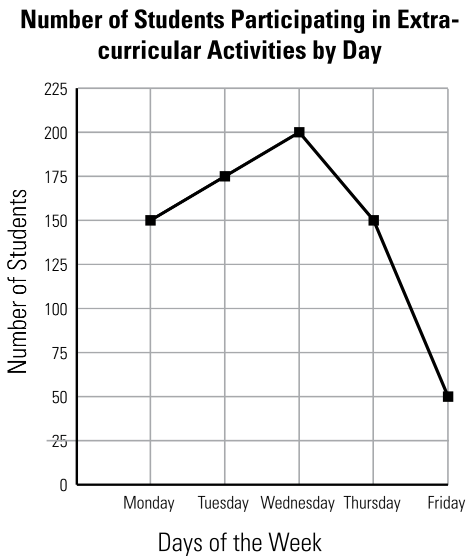


1. What percentage does each range represent?

2. How many degrees do each % represent?

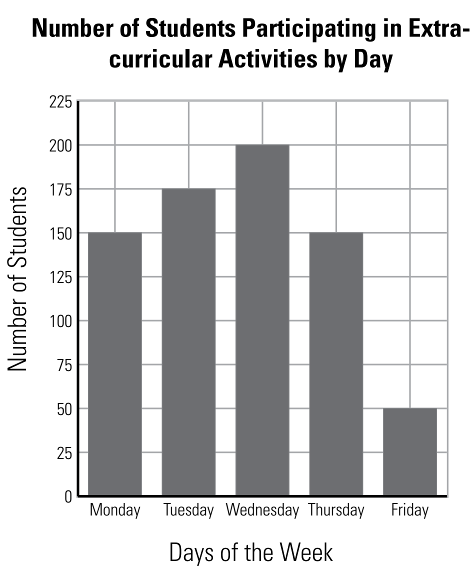
BLACKLINE MASTER 2.7: SOLUTIONS

A. BROKEN LINE GRAPH



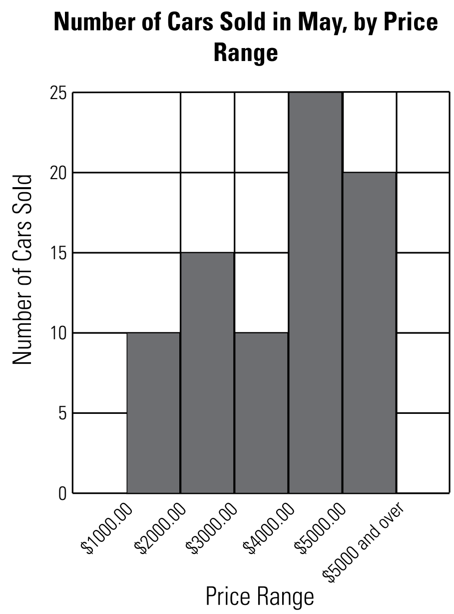
1. Discrete data are data that can include only certain numerical values, often natural numbers.
2. We can use the broken line graph to help show a pattern.

B. BAR GRAPH



1. Answers may vary, but in general, the bar graph should seem to give the better view because of the discrete data.
2. No, you get the same information. However, the first graph makes it more obvious that participation drops off after Wednesday.

C. HISTOGRAM



1. Add all the numbers together:
   10 + 15 + 10 + 25 + 20 = 80

   80 cars were sold in May.
2. 56% of cars sold were $4000.00 or more.

D. CIRCLE GRAPH

First, determine the percentage of each price range by using the following formula, where x is the number of cars in that price range and n is the total number of cars, 80.

$$P = \frac{x}{n} \times 100$$

The percentage of cars in the \$1000.00–\$1999.00 and the \$3000.00–\$3999.00 price range will be as follows.

$$P = \frac{10}{80} \times 100$$

$$P = 12.5\%$$

Since twice as many cars (20) were sold in the \$5000.00 and over range, the percentage will be twice the above, or 25%.

In the \$2000.00–\$2999.00 range, the percentage is as follows.

$$P = \frac{15}{80} \times 100$$

$$P = 18.75\%$$

In the \$4000.00–\$4999.00 range, the percentage is as follows.

$$P = \frac{25}{80} \times 100$$

$$P = 31.25\%$$

Second, find out how many degrees are in each sector by dividing the percent by 100 and multiplying by 360°.

\$1000.00–\$1999.00 and \$3000.00–\$3999.00

$$d = 0.125 \times 360°$$
$$d = 45°$$

\$5000.00 and over

Twice the number of degrees of the above price range

$$d = 2 \times 45°$$
$$d = 90°$$

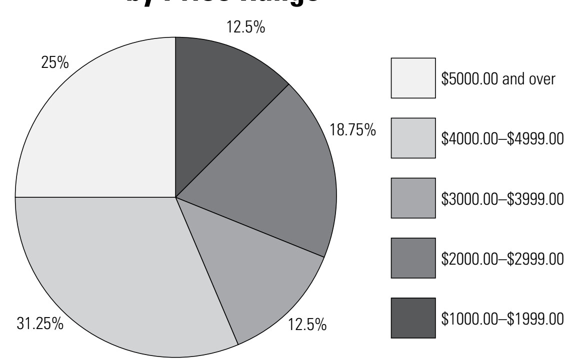
\$2000.00–\$2999.00

$$d = 0.1875 \times 360°$$
$$d = 67.5°$$

\$4000.00–\$4999.00

$$d = 0.3125 \times 360°$$
$$d = 112.5°$$

Number of Cars Sold in May, by Price Range



ALTERNATIVE CHAPTER PROJECT—BECOME AN ENVIRONMENTAL MONITOR

TEACHER MATERIALS

GOALS: To collect and represent data using graphs; to use different types of graphs to effectively represent information; to build graphing, research, and writing skills; to produce a piece of workplace writing; and to synthesize learning in this chapter.

OUTCOME: Students will take their learning on graphical representation from this chapter and apply it to a workplace scenario. By working as environmental monitors, students will record data, represent it graphically, and write a short report based on their findings. Through this project, students will use and build the research, mathematics, and writing skills many employers look for.

PREREQUISITES: Students should understand how to properly label a graph with an appropriate title and horizontal and vertical labels. They should also be able to perform basic calculator functions and be able to calculate percentages.

ABOUT THIS PROJECT: Students will begin this project by researching a piece of land that they will monitor. You will want to decide in advance if you will give students class time to go to their piece of land and collect data, or if you will assign their data collection as homework. If you decide to give students class time to collect data, they will have to choose sites close to the school. If you assign data collection as homework, students should choose sites close to their homes.

For this project, you will have to supply students with litmus paper and a colour pH scale. During this project, you should have one or two field guides to local plants and animals in the classroom for students to use. This will help students to identify the species they encounter. For student reference, you should also place a colour pH scale on the classroom wall. A sample chart can be found at this website: www.ioncleansedetoxfootbath.com/pH%20chart%20generic.pdf

Students will need the materials listed below to complete this project. You will want to let students know in advance that they will need these items. Students should be able to find all materials at home, except for the litmus paper, which the school should be able to supply. The four stakes can be pieces of kindling, sticks, or even drinking straws.

- two clean containers with lids, such as glass jars
- a spoon
- a notebook
- a ruler
- litmus paper
- a colour pH scale
- four stakes
- a pencil
- distilled or bottled water

Check in with students while they are collecting their data. After students have collected their data, they should be given a few class periods to work on their report. This will allow you to answer questions that students might have about which graph will best represent their data, or how to best write their report. This feedback will allow some students to complete the project more confidently.

An assessment rubric for this project follows. Early in the project, hand out Blackline Master 2.1A (p. 152) to students. It contains the criteria that will be used to evaluate the projects.

1. Start to plan

The purpose of this project is to have students collect data from which they can make four graphs, compare it to information on normal or "healthy" environmental conditions, and write a report based on their findings. Remind students that they will collect data on water and soil pH, as well as plant and animal species.

At the beginning of the chapter, introduce this project to students by explaining what an environmental monitor does, and that they will be working as one. Environmental monitors collect air, water, soil, and plant samples to determine whether or not human activities affect the environment. This career requires a high school education, so if any students are interested in working directly after they graduate, this job might appeal to them. Here is a link to a website describing this career.

www.beahr.com/aec/enviro09e.html

Remind students to choose a site that is easy for them to get to and contains water. Tell them they must not choose a body of water that is dangerous in any way. They must not go to fast-flowing rivers, or bodies of water with steep banks.

After choosing a site to monitor, students will use their class time to collect research information. Students need to find and record the normal or "healthy" pH levels of soil and water in the area, as well as identify and list common animal and plant species. Tell students to make four data tables and record this information in the applicable table. Students will compare this information to the data they record during their visits to their site.

At the end of the class, remind students of the materials they will need to complete this project. As homework, tell students to collect these materials and put them in a plastic bag, so they can easily bring them to the site they are monitoring.

2. Collect your data

Before they set out to collect and test samples, you may want to give students a short lesson on the pH scale. Tell them that pH is measured on a scale of 0 to 14. pH is based on the number of positive hydrogen ions and negative hydroxide ions in water. If a substance has a high concentration of positive hydrogen ions, it will be very acidic and measure close to 0. A substance with a high level of negative hydroxide ions will be very basic, or alkaline, and measure close to 14.

Pure water is exactly in the middle, and has a pH of 7. Most natural water has a pH of 6.5 to 8.5. Natural soil can have a pH of 3.5 to 10.

Here are links to websites that describe pH levels in water:

- www.h2ou.com/h2wtrqual.htm#pH
- www.ncsu.edu/sciencejunction/depot/experiments/water/lessons/pH/pHcolor.html

Here are links to websites that describe pH levels in soil:

- http://asssi.asn.au/downloads/educational/factsheets/02_understanding-soil-pH.pdf
- www.life123.com/home-garden/gardening-tips/soil-fertilizers/learn-the-soil-ph-of-different-soil-types.shtml

Students will visit their site three times to collect water and soil samples and to record plant and animal species. Ensure that they did their homework from the last class and assembled all of the materials they will need to record data at their site.

Before students visit their site, give each one litmus paper and a colour pH scale. Tell students that pH is to be measured by the colour that appears on the litmus paper after it is dipped into the sample. Use a colour chart of pH levels to show them that the pH of water is close to 7 and point out the corresponding colour.

Here are links to colour pH scales:

- http://staff.jccc.net/pdecell/chemistry/phscale.html
- http://richardbowles.tripod.com/chemistry/acids/acids.htm

Remind students not to disturb soil and vegetation when they visit their site, and to be careful around bodies of water.

3. Graph your results and prepare your report

Give students two class periods to make their graphs and write their reports. Students will put their graphing and writing skills to use as they choose and construct different types of graphs to best represent their data and write about their findings. They will use their technology skills to place their graphs in their reports and format it. Giving students class time to prepare their report will allow you to offer feedback on their work. Ensure that students have labelled and titled their graphs properly.

Each report should include the following.

- a table of contents;
- a brief introduction explaining the location, body of water, and land of the area you monitored;
- your data displayed using four different and appropriately titled and labelled graphs;
- a brief conclusion describing your findings; this should include an assessment of the health of the soil and water in your area, as well as an assessment of the number of species of plants and animals; and
- the four data tables students used to record their data and make their graphs.

Each student should address the following questions in his or her report. Is the pH of the water you recorded similar to the "healthy" level you researched? Is the pH of your soil similar to the "healthy" level you researched? Why do you think this is? How does the number of species of plants you observed compare to the numbers you learned were common in your area? Were the number and types of animals you observed similar to the types of animals you learned were common in this area? Why do you think this is?

EXTENSION

Environmental monitors often monitor stream bank erosion. Students can do this as well by driving a stake into a stream or river bank on their first visit to their site. The stake should be flush with the bank. On their return visits, students can measure the space between stake and bank to determine if any erosion has occurred. The results can be recorded and displayed on a separate graph. Findings related to stream bank erosion can be addressed in the student report.

PROJECT—BECOME AN ENVIRONMENTAL MONITOR

STUDENT MATERIALS

START TO PLAN

You will learn what it is like to work as an environmental monitor in this project. Environmental monitors study the health of the natural world by testing its air, soil, water, and plants. As an environmental monitor, you will be responsible for testing the pH of soil and water samples, and collecting data on types of plants and animal species. You can choose to complete this project on your own, or in a group of two to three.

You will collect data once a week, for three weeks. After you have collected your data, you will use it to generate graphs and to write a short report on the environmental health of the place you studied.

T The first step of this project involves choosing a park or piece of land to monitor. It should contain a body of water such as a lake or stream. Try to find a piece of land that is easy for you to access. Perform internet or library research, or use your own knowledge of your area. Choose a place where you can obtain water samples safely. Do not choose a place that contains fast-flowing rivers, or water bodies with steep banks. After you have chosen an area to monitor, prepare four data tables. You will use these to record soil pH, water pH, number of plant species, and number and type of animal species.

Next, collect the information that you will measure your data against. Find out and record the natural, or healthy, pH of the soil and water in your area of study. Record this information on your data tables for water and soil. Next, find out and list the names of common animal species that inhabit the same place. Finally, find out and list the names of common plant, tree, and shrub species in the area you will monitor.

If there are no rural areas close to where you live, you can monitor an urban environment. You can take water (rainfall or tap water) and soil samples from different locations at your school. You can also record data on the diversity of surrounding plant and animal species.

COLLECT YOUR DATA

Now that you have chosen land to monitor, it is time to collect your first set of data.

Step One

Assemble your materials for your fieldwork. You will want to put these materials in a plastic bag and keep them ready for use throughout the project. Your teacher will provide you with the litmus paper and a colour pH scale. You will need:

- two clean containers with lids, such as glass jars
- a spoon
- a notebook
- a ruler
- a colour pH scale
- litmus paper
- four stakes
- a pencil
- distilled or bottled water.

Step Two

Go to your site and collect your first water sample by dipping the container into the edge of the water body. (You will collect three water samples in total from the same place.) Dip the container into the water gently so as to stir up sediment or mud. Put the lid on.

Next, collect one soil sample. (In total, you will collect three soil samples from three different areas for this project.) Choose areas that you think contain different types of soil, such as beside a water body, beside a path, or under a tree. Scoop about half a cup (125 mL) of soil into your second jar and put the lid on.

Then, make your observations on plant diversity. Choose an area to observe, put one of your stakes in the ground, and pace out five steps. Put another stake down where you stop. Do this until you have a square. Record the plant species (shrubs, trees, etc.) in your staked-off square. If you do not know the name of a plant, write a brief description of it and identify it later by looking in a field guide from the school library or a website. You do not have to count the number of plants, just their names. For example, you might see alder, cedar, and fir trees, so under trees, you would record these names and a three. Next, stand in one place for five to ten minutes and write down the types and numbers of animals you observe.

At home or at school, test the pH of your water and soil sample. To test the water sample, set it on a flat surface and leave it undisturbed for 10 minutes to allow any sediment to settle. Place the litmus paper in the water for five seconds and record its pH using your colour pH scale. To test the pH of your soil, put two cups of distilled water in with the soil, mix it thoroughly, and let it stand for about 10 minutes to allow the soil to settle to the bottom of the jar. Dip in the litmus paper in this mixture for five seconds and record the pH using your colour pH scale.

Step Three

You will complete the above steps two more times, about one week apart. Return to your site and take one water and soil sample. Remember to take the water sample from the same spot you used last time, but take your soil sample from a different spot. After that, stake off a new area and record the plant species in it. Next, stand in one place for five to ten minutes and observe and record any animal species you see.

GRAPH YOUR RESULTS AND PREPARE YOUR REPORT

After you have collected your data, look at your data tables and decide which type of graph you will use to convey which piece of information. Will you choose a histogram, or a line, circle, or bar graph? You can make your graphs by hand, or with computer software. You will have four graphs in total, for soil pH, water pH, number and type of animal species, and type of plant species. Make and label your graphs. Note the "healthy" soil and water pH levels at the bottom of your graphs for comparison.

Finally, prepare your report. Your report should include the following items:

- a table of contents;
- a brief introduction explaining the location, body of water, and land of the area you monitored;
- your data displayed using four different types of appropriately titled and labelled graphs; and
- a brief conclusion describing your findings; this should include an assessment of the health of the soil and water in your area, as well as an assessment of the number of species of plants and animals.

To prepare your report, write your introduction and conclusion and place your graphs. To write your conclusion, compare your data to the data you found on the natural or healthy pH levels of the water and soil. Address the following questions in your report. Is the pH of the water you recorded similar to the "healthy" level you researched? Is the pH of your soil similar to the "healthy" level you researched? Why do you think this is? How does the number of species of plants you observed compare to the numbers you learned were common in your area? Were the number and types of animals you observed similar to the types of animals you learned were common in this area? Why do you think this is?

PROJECT ASSESSMENT RUBRIC—ENVIRONMENTAL MONITORING				
	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Project shows how to collect data and use it to create line, bar, and circle graphs, as well as histograms; how to draw conclusions based on data	shows very little understanding; graphs are incorrect, missing, or use an inappropriate format; introduction or conclusion contain mistakes or are missing	shows partial understanding; graphs are correct; data tables are correct; introduction or conclusion contain mistakes or are incomplete	shows understanding; graphs are correct; data tables are correct; introduction or conclusion are adequate	shows thorough understanding; graphs are correct; data tables are correct; introduction or conclusion are effective and thorough
Procedural Knowledge				
• Accurately: • creates data tables to record data • constructs graphs using data • researches data • draws conclusions based on data and conveys them clearly in writing	limited accuracy; major errors or omissions For example: • data tables are missing or incorrect • researched data are incorrect • graphs are constructed incorrectly • introduction and conclusion are missing, incomplete, or contain major errors • project is incomplete	partially accurate; some errors or omissions For example: • data tables are correct, but graphs contain one or two labelling errors • researched data are correct • introduction and conclusion are included, but contain mistakes, are too short, or need more conclusions based on data • project needs more work to make graphs correct and writing accurate	generally accurate; few errors or omissions For example: • data tables are correct • researched data are correct • graphs are constructed correctly • introduction and conclusion are clear; conclusion includes logical statements based on data • project is complete	accurate and precise; very few or no errors For example: • data tables are correct • researched data are correct • graphs are constructed correctly • introduction and conclusion are clear, creative, and thoughtful; conclusions are logical, detailed, and based on data • project is complete, detailed, and logical
Problem-Solving Skills				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems	uses appropriate strategies to successfully solve most problems	uses effective and often innovative strategies to solve problems and explain solutions
Communication				
• Presents data, graphs, and conclusions using appropriate mathematical terminology	does not present data, graphs, and conclusions clearly; uses few appropriate mathematical terms	presents data, graphs, and conclusions with some clarity; uses some mathematical terms	presents data, graphs, and conclusions clearly; uses some appropriate mathematical terms	presents data, graphs, and conclusions clearly and in detail; uses appropriate mathematical terms

BLACKLINE MASTER 2.1A

ALTERNATIVE PROJECT SELF-ASSESSMENT RUBRIC

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following things.

- the thoroughness and accuracy of your research;
- the accuracy of your data;
- the accuracy of your graphs;
- the soundness of your conclusions; and
- your completion of the assigned tasks on time.

How do you feel you have done overall, given the criteria above?

____

____

Were you able to complete all aspects of the project? If not, why? Did you allot your time effectively?

____

____

In what areas did you excel?

____

____

Are there areas in which you could improve?

____

____

Do you think your graphs effectively convey the data you collected? Why or why not?

____

____

Do you think your conclusions are accurate? Why or why not?

____

____

BLACKLINE MASTER 2.2A

ALTERNATIVE PROJECT CHECKLIST

Name: ______________________ Date: ______________________

REPORT CHECKLIST	
❒ Have you filled in your four data tables and included them in your report?	
❒ Do your data tables include your researched data for reference?	
❒ Does your report include four graphs?	
❒ Does one of your graphs display water pH and another display soil pH?	
❒ Does your report include an appropriate title, table of contents, introduction, and conclusion?	
❒ Do your graphs have titles that accurately describe what they represent?	
❒ What type of information will you display on your graphs?	
❒ Do your graphs have accurate horizontal and vertical labels?	
❒ Other considerations	

BLACKLINE MASTER 2.8

REVIEWING PRIOR CONCEPTS

Name: ____________________ Date: ____________________

1. Find the following:
   a) 45% of 24
   b) 5% of 32.1
   c) 6.3% of 192
   d) 0.3% of 21

2. If you spend 8 hours a day sleeping, what percentage of the day do you sleep?

3. If you save $100.00 from your paycheque of $520.00, what percentage do you save?

4. Given that a circle has 360°, how many degrees (to the nearest tenth of a degree) are in each of the following?
   a) 5% of a circle
   b) 32.3% of a circle
   c) 78% of a circle
   d) 15.8% of a circle

BLACKLINE MASTER 2.8: SOLUTIONS

1. a) $0.45 \times 24 = 10.8$

   b) $0.05 \times 32.1 \approx 1.6$

   c) $0.063 \times 192 \approx 12.1$

   d) $0.003 \times 21 = 0.063$

2. $$\frac{8}{24} = \frac{x}{100}$$

   $$100 \times \frac{8}{24} = \frac{x}{100} \times 100$$

   $$100 \times \frac{8}{24} = x$$

   $$33.3 \approx x$$

   You sleep approximately 33.3% of the day.

   ALTERNATE SOLUTION

   $$\frac{8}{24} \approx 0.3333$$

   $$0.3333 \times 100 = 33.33$$

3. Use the second method above.

   $$\frac{100}{520} \approx 0.1923$$

   $$0.1923 \times 100 = 19.23$$

   You save approximately 19.23% of your paycheque.

4. a) $0.05 \times 360° = 18°$

   b) $0.323 \times 360° = 116.28°$

   c) $0.78 \times 360° = 280.8°$

   d) $0.158 \times 360° = 56.88°$

Chapter 3

Surface Area, Volume, and Capacity

INTRODUCTION

STUDENT BOOK, pp. 114–163

This chapter delivers the outcomes of the Measurement strand of Apprenticeship and Workplace Mathematics 11. This chapter is also one of three chapters in the student textbook that delivers the outcomes of the Algebra strand of the Apprenticeship and Workplace Mathematics 11.

In this chapter, students will continue to build on their understandings of direct and indirect measurement from grade 10 and characteristics of two-dimensional and three-dimensional shapes from grade 9. The chart below locates this chapter within the curriculum.

MEASUREMENT, GRADES 10–12

This chart illustrates the development of the Measurement strand in the Apprenticeship and Workplace pathway through senior secondary school. The highlighted cell contains the outcome that chapter 3 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop spatial sense through direct and indirect measurement.	**General Outcome** Develop spatial sense through direct and indirect measurement.	**General Outcome** Develop spatial sense through direct and indirect measurement.
Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:	**Specific Outcome** It is expected that students will:
Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass, and temperature; • applying strategies to convert SI units to imperial units.	Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.	Demonstrate an understanding of the limitations of measuring instruments, including: • precision • accuracy • uncertainty • tolerance and solve problems.
Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass, and temperature; • comparing the American and British imperial units for capacity; • applying strategies to convert imperial units to SI units.	Solve problems that involve SI and imperial units in volume and capacity measurements.	

Grade 10	Grade 11	Grade 12
Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.		
Solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.		

ALGEBRA, GRADES 10–12

This chart illustrates the development of the Algebra strand in the Apprenticeship and Workplace pathway through senior secondary school. The highlighted cell contains the outcome that chapter 3 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.
Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:	**Specific Outcome** It is expected that students will:
Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income.	Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges.	Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating.
	Demonstrate an understanding of slope: • as rise over run • as rate of change • by solving problems.	
	Solve problems by applying proportional reasoning and unit analysis.	

CURRICULUM AND CHAPTER OVERVIEW

General Outcome: Measurement
Develop spatial sense through direct and indirect measurement.

General Outcome: Algebra
Develop algebraic reasoning.

Specific Outcome: Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

Specific Outcome: Solve problems that involve SI and imperial units in volume and capacity measurements.

Specific Outcome: Solve problems that require the manipulation and application of formulas related to volume, capacity, and surface area.

Specific Outcome: Solve problems by applying proportional reasoning and unit analysis.

Section 3.1:
Surface Area of Prisms

- Math on the Job
- Explore the Math
- Discuss the Ideas: Calculating Surface Area of a Stained Glass Lantern
- Mental Math and Estimation
- Activity 3.1: Hexominos
- Discuss the Ideas: Surface Area of Cabinets
- Activity 3.2: Building a Shipping Crate
- Build Your Skills

Section 3.2:
Surface Area of Pyramids, Cylinders, Spheres, and Cones

- Math on the Job
- Explore the Math
- Discuss the Ideas: The Muttart Conservatory
- Activity 3.3: Designing Objects Using Pyramids and Cylinders
- Discuss the Ideas: Surface Area of Circles and Spheres
- Mental Math and Estimation
- Activity 3.4: Mortice and Tenon vs. Dowel Joints
- Build Your Skills
- The Roots of Math: Archimedes' Contributions to Surface Area Calculations

Section 3.3:
Volume and Capacity of Prisms and Cylinders

- Math on the Job
- Explore the Math
- Discuss the Ideas: Calculating the Volume of Paving Stones
- Activity 3.5: Volume of an Oblique Prism
- Discuss the Ideas: Investigating Volume
- Build Your Skills
- Puzzle It Out: Cube

Section 3.4:
Volume and Capacity of Spheres, Cones, and Pyramids

- Math on the Job
- Explore the Math
- Discuss the Ideas: Mathematical Formulas for Cylinders and Spheres
- Activity 3.6: Using Micrometers and Calipers
- Activity 3.7: Using Displacement to Measure Volume
- Build Your Skills

Chapter Project: Designing Packages
Reflect on Your Learning
Practise Your New Skills

THE MATHEMATICAL IDEAS

SURFACE AREA, VOLUME, AND CAPACITY

In this unit, students will investigate the surface area of prisms, pyramids, cylinders, spheres, and cones. Students will apply their understanding of area of two-dimensional shapes to determine the surface area of three-dimensional shapes. To make this connection clearer for students, students start by finding the surface area by using a three-dimensional shape's net. This helps students understand that the surface area is found by finding the area of each face of the figure. Students apply their understanding of surface area to right and oblique prisms and to cylinders, pyramids, cones, and spheres.

After investigating the surface area of three-dimensional shapes, students will determine the volumes and capacity of the same shapes. Students will start by understanding volume as the amount of space an object takes up and capacity as the amount of material needed to fill an object. Students start building this understanding by thinking of the number of unit cubes that can fit inside a rectangular prism. They move into identifying the volume of each three-dimensional shape using its volume formula.

Students will also investigate changing a shape's dimension(s) and the effect that change has on the shape's volume. Students will use micrometers and calipers to measure objects. And lastly, students will investigate the effect of metric and imperial measurement conversions on squared and cubed measurements.

WHY ARE THESE CONCEPTS IMPORTANT?

- Understanding surface area and volume allows students to apply their understanding of two-dimensional measurement to three-dimensional measurement. Students will be able to make connections between the properties of three-dimensional shapes.
- Applying the concepts of surface area and volume to real-world applications helps students see the use of three-dimensional measurement in their everyday life.

PRIOR SKILLS AND KNOWLEDGE

Students' work in this chapter will build on certain WNCP outcomes from earlier grades. They will review these mathematical concepts and skills and apply them in a new context to real-life problems involving surface area and volume of three-dimensional figures. The following is a list of concepts and mathematics skills to which students have been exposed in grades 9 and 10.

1. Concepts
   a) understanding of the Systѐme International (SI);
   b) understanding of the imperial system;
   c) area, volume, capacity, mass, and temperature;
   d) technology: presentation software, basic calculator functions, and internet skills.
2. Mathematics Skills
   a) convert SI units to imperial units;
   b) solve problems that involve SI and imperial area measurements of regular, composite, and irregular two-dimensional shapes and three-dimensional objects;
   c) determine the surface area of composite three-dimensional objects.
3. Technology T
   a) computer-assisted design (CAD) software and/or presentation software;
   b) basic calculator operation;
   c) internet research skills.

REVIEWING PRIOR CONCEPTS

Some students may benefit from reviewing concepts that have been covered in prior years. You may want to give some students specific review exercises in the following concepts and processes.

3.1 Surface Area of Prisms

- order of operations;
- creating nets of objects; and
- finding the area of composite figures.

3.2 Surface Area of Pyramids, Cylinders, Spheres, and Cones

- working with squares;
- using a scientific calculator.

3.3 Volume and Capacity of Prisms and Cylinders

- working with cubes; and
- working with formulas.

3.4 Volume and Capacity of Spheres, Cones, and Pyramids

- converting measurements within and between the SI and Imperial systems; and
- reading a ruler.

Blackline Master 3.9 contains review questions and solutions. It is found at the end of this chapter of the teacher resource (p. 217).

PLANNING CHAPTER 3

This chapter will take 2–3 weeks of class time to complete. Class period estimates are based on a class ranging from 60 to 75 minutes. These estimates may vary depending on individual classroom needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	115	Introduce the chapter project: "Designing Packages"	20 minutes for a class discussion on the opening questions on the project	Chart paper, rulers, pencil crayons/markers Blackline Master 3.1 (p. 202)
3.1	116 116 118 119 119	Math on the Job: Drywall Installer Explore the Math Discuss the Ideas: Calculating Surface Area of a Stained Glass Lantern Mental Math and Estimation Examples 1, 2	45 minutes	
3.1	121 122 122 124 124	Activity 3.1: Hexominos Discuss the Ideas: Surface Area of Cabinets Example 3 Activity 3.2: Building a Shipping Crate Build Your Skills	1 class	Blackline Master 3.7 (p. 208)
3.2	127 127 128 129	Math on the Job: Graphic Designer Explore the Math Discuss the Ideas: The Muttart Conservatory Examples 1, 2	1 class	
3.2	130 131 132 133 134 134	Activity 3.3: Designing Objects Using Pyramids and Cylinders Discuss the Ideas: Surface Area of Circles and Spheres Examples 3, 4 Mental Math and Estimation Activity 3.4: Mortice and Tenon vs. Dowel Joints Build Your Skills	1 class	
	136	Chapter Project: Research Your Ideas	1–2 class periods	Blackline Master 3.2 (p. 203)
	137	The Roots of Math: Archimedes' Contributions to Surface Area Calculations	15 minutes	

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
3.3	138 138 139 140	Math on the Job: Scrapbook Materials Supplier Explore the Math Discuss the Ideas: Calculating the Volume of Paving Stones Examples 1, 2	1 class	
3.3	141 142 143 144 147	Activity 3.5: Volume of an Oblique Prism Discuss the Ideas: Investigating Volume Example 3 Build Your Skills Puzzle It Out: Cube	1 class	
	147	Chapter Project: Design Your Packaging	1 class	Graph paper, chart paper, rulers, pencil crayons/markers Blackline Master 3.2 (p. 203) Blackline Master 3.3 (p. 204)
3.4	148 148 149 150	Math on the Job: Plumber Explore the Math Discuss the Ideas: Mathematical Formulas for Cylinders and Spheres Examples 1, 2	1 class	Cups, graduated cylinders, pans, items to measure (tennis balls, golf balls, etc.)
3.4	152	Activity 3.6: Using Micrometers and Calipers	1 class	Calipers, micrometers, items to measure
3.4	153 154 156	Activity 3.7: Using Displacement to Measure Volume Examples 3, 4 Build Your Skills	1 class	
	159	Chapter Project: Present Your Packaging	1 class to finish projects	Graph paper, chart paper, rulers, pencil crayons/markers
		Presentation of projects	1 class	
	159 160	Reflect on Your Learning Practise Your New Skills	1 class	
		Chapter test (p. 193 of this resource)	1 class	

PLANNING FOR ASSESSMENT

Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Use exit cards with one of the “Build Your Skills” problems. • Create samples of strong and weak responses to an example to guide student work. • Ask students to talk out their solution process to determine any misconceptions. • Poll student understanding using “thumbs up” or “thumbs down.”	• Provide descriptive feedback for students as they hand in assignments. • Have students self-monitor their work by checking solutions with partners. • Model the kind of thinking you want students to engage in when they work through an activity.
Assessment as Learning	• At the end of a lesson, ask students to record what they thought was the objective of the lesson in their journals. • Have students record two things they learned and two things they want to work on. • Have students grade each other’s homework as you review answers.	• Record the objective from each lesson on the board and ask students how each problem relates to the objective. • Ask students to model their solution process visually for the class. • Challenge advanced students to solve a problem in more than one way.
Assessment of Learning	• Record correct and incorrect student responses to your verbal questions during a lesson. • Offer students opportunities for focused revision of their work after providing descriptive feedback. • Have students write a letter to their parents describing what they have learned in the lesson or unit.	• Start each class with a small assessment of the previous day’s objective and/or prior knowledge for today’s lesson. • Incorporate the project as part of the unit summative assessment grade.
Learning Skills/ Mathematical Disposition	• Create anecdotal records of how many times students participate in class discussions. • Have students assess themselves and their peers on their participation in group activities.	• Include a participation grade in each student’s overall grade.

PROJECT — DESIGNING PACKAGES

GOALS: To use three-dimensional measurements to determine best packaging options.

OUTCOME: In this project, students develop different packaging options for an item of their choice. They will practise their understanding of three-dimensional measurement in designing each of the packages.

T

PREREQUISITES: Students will need to have an understanding of area and how nets can be used to represent three-dimensional shapes. Students will need to have access to the internet to research common packaging options used around the world. You can guide students to use available search engines to help them with their research. Students should record their research in a document on the computer, using a word processing program.

ABOUT THE PROJECT: This project is divided in four parts. Initially, students will research common packaging options for their item around the world. They will decide on three options and record these in a document. Partway through the chapter, students will complete the table in Blackline Master 3.2 (p. 203) with their item's actual dimensions and the dimensions for each of the three packages. As a final activity, students will create a drawing of their item and the three packaging options, along with their measurements and costs. When students present their designs to the class, allow 3–5 minutes per student.

Students should be given a few class periods to work on this project. This will allow for questions/feedback from the teacher as well as allow the teacher to observe the quality of work as it is done, rather than at the end of the chapter. Interim guidance can help the students to complete the project more successfully.

This project could be completed by pairs or small groups of students acting as co-workers completing an assignment.

An assessment rubric for this project is included and should be handed out to students early in the project. The rubric outlines the criteria for evaluation of their project and suggests some ways in which they can reflect on their learning.

An alternative project, "Renovate and Redecorate Your Bedroom," is included on pp. 210–216. This project can be done by individual students.

1. Start to plan

STUDENT BOOK, p. 115

Introduce the project to your students as you begin this chapter. This initial part of the project allows for group brainstorming as a class. Start with a discussion of what types of packages they could create. Some packages can include boxes found in a grocery store, such as cereal boxes, or shipping boxes, such as crates used to ship cars or furniture.

Explain to students that they are creating several packaging options for an item of their choice. They will research common packaging options from around the world on the internet and record three examples in a document. They will then complete the table in Blackline Master 3.2 (p. 203) with their item's actual dimensions and each of the package's dimensions. And finally, students will create a drawing of each of their packages and present their drawings and measurements to the class. Students will use a poster, handout, or electronic presentation, based on their designs, to make this presentation.

2. Research your ideas

STUDENT BOOK, p. 136

This segment of the project requires students to practise their research skills. Students will research common packaging options for their item and record their findings in a document. Students will also research the materials commonly used to make this packaging, as well as the customer who buys the object they are designing packaging for. Let students know that they do not need to do extensive research on the customer. Students are simply meant to identify who the customer might be, as well as two or three ways that they could make packaging appeal to them.

Students will then choose three packaging options for their item and create a sketch of each package.

At the end of this segment of the project, discuss progress with your students to ensure that all requirements have been met.

3. Design your packaging

STUDENT BOOK, p. 147

During this section of the project, students will use their research to design three packaging options. They can use Blackline Master 3.2 (p. 203) to draw their designs on. Remind students to label their designs with accurate measurements. Students can fill in the first four sections of the "Packaging Measurements" section of the Blackline Master. The section of the Blackline Master that asks them to calculate the volume can be filled in at the end of the chapter, after they have learned how to calculate the volume of spheres, cones, and cylinders.

4. Present your packaging

STUDENT BOOK, p. 159

In this segment of the project, students will synthesize their planning and research activities and practise their presentations skills. Presentations to a manager or company owner are often with handouts and other tools, including electronic presentations, posters, or folders containing several items. Students are therefore required to use one of these methods to make their presentation. Provide students with a copy of Blackline Master 3.1 (p. 202) to give them an opportunity to reflect on the quality of work that they put into their project.

ASSESSING THE PROJECT

1. Start to plan

- Discuss with students that they will make several packaging options for their chosen item. Let them know that before beginning the project, they must complete research on the type of packaging they choose to make. Students must also research the material commonly used to make this packaging, as well as the people who buy their item, in order to design packaging that will appeal to them.
- Let students know how they will be graded. You can tell them about the criteria on the project assessment rubric, or your own criteria you have chosen to grade the project with.

2. Research your ideas

- After this section of the project has been completed, you can ask students to hand in their research findings on packaging options. These findings should be recorded in a word processing document. To assess the quality of research students have completed, consider the variety of packaging styles they found, the variety and accuracy of the dimensions they recorded, and the quality and accuracy of their drawings.
- You can decide and inform students if this work will be part of their final grade or if it will be used as an assessment for learning. If you choose to use this step as an assessment for learning, you can hand the documents back and give general or individual suggestions on how to improve research findings.

3. Design your packaging

- Give students class time to complete their sketches of their packages. Ensure that they have chosen an appropriate unit to express their measurements in. After students have completed their drawings, ask them to decide what materials they will use for their presentation. They can base posters, handouts, or an electronic presentation on their drawings, the goal being to clearly display the three different packaging options to the class.

4. Present your packaging

- Allow class time for presentations. Encourage students to introduce the object they designed packaging for, display their packaging, and describe its dimensions and cost. Students will make their presentation with a poster, handout, or presentation software.
- Note whether students were prepared to answer questions from classmates regarding why they designed their packages as they did, or who they designed their packages to appeal to.

PROJECT EXTENSION

As an extension, you can ask students to collect feedback from their classmates on their packages. Have each student record which design out of the three he or she liked the most, as well as one feature about it that could be improved.

The student presenting can collect the feedback, determine which design was the most popular, and revise this design based on the peer feedback. The student would then construct a version of the revised design. If the student chose to design packaging for a very large object, he or she can make a scale model of the package. Having researched, sketched, presented, revised, and built an actual version of the most popular design, each student will have completed an exercise that roughly approximates what it is like to create a design in the workplace.

PROJECT ASSESSMENT RUBRIC				
	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
CONCEPTUAL UNDERSTANDING				
• Explanations show an understanding of similar figures and how to apply scale factors	Shows very limited understanding; explanations are omitted or inappropriate	Shows partial understanding; explanations are often incomplete or somewhat confusing	Shows understanding; explanations are appropriate	Shows thorough understanding; explanations are effective and thorough
PROCEDURAL KNOWLEDGE				
• Accurately: ▪ determines dimensions for each package ▪ calculates surface area and volume for each package ▪ records all research sources ▪ calculates the cost to make each package ▪ designs each package to be a different shape, with one design incorporating two shapes	limited accuracy; major errors or omissions For example: • each of the package measurements is incorrect • volume and surface area calculations are incorrect • research sources are not recorded • cost calculations are missing or incorrect • packages are not different shapes • project is incomplete	partially accurate; some errors or omissions For example: • each of the package measurements is correct, but some calculations are missing • some surface area and volume calculations are incomplete • research sources are recorded • some cost calculations are present and complete • packages are different shapes • presentation is adequate • project could use more work to ensure calculations are done correctly	generally accurate; few errors or omissions For example: • package measurements and associated calculations are correct • calculations are complete and correct • research sources are recorded • has an informative, clear presentation handout, poster, or electronic presentation • project is complete and correct, but there is nothing beyond what is required	accurate and precise; very few or no errors For example: • package measurements and associated calculations are correct • calculations are complete and correct • research sources are recorded • has an informative, clear presentation handout, poster, or electronic presentation • adds extra creativity to the project
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

3.1 Surface Area of Prisms

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 116–126

MATH *ON THE JOB*

STUDENT BOOK, p. 116

Start your class with a discussion of Shay's work. You can also use this Math on the Job as an opportunity to discuss workplace safety. Ask each student to think of one job and one safety precaution an employee doing this job should take. Next, have a student read the Math on the Job aloud to the class. Before presenting the mathematical solution, have students sketch out each of the four walls Shay is installing drywall on, then have students calculate their answers. In groups of two or three, have students compare their solutions. If their solutions are different, have students discuss why.

For students who are struggling with question 1, start by asking, "How many sheets of drywall will be needed to cover a wall measuring 8 feet wide by 4 feet high?" Once students see one sheet will be needed, ask them, "How many sheets of drywall will be needed to cover a wall measuring 8 feet wide by 8 feet high?" Have students draw a rectangle to represent the wall and draw the drywall sheets over their rectangular wall. Keep following this line of questioning until students are ready to complete question 1.

SOLUTION

1. To calculate the number of drywall sheets needed, first calculate the surface area of one 4 ft × 8 ft sheet of drywall.

   SA = width × height

   SA = 8 × 4

   SA = 32

   The surface area of one sheet of drywall is 32 ft^2.

   Wall 1
   SA = width × height

   SA = 14 × 12

   SA = 168 ft^2

   Number of sheets needed to cover Wall 1:

   168 ÷ 32 = 5.25

   Wall 2
   SA = width × height

   SA = 16 × 12

   SA = 192 ft^2

   Number of sheets needed to cover Wall 2:

   192 ÷ 32 = 6

   Wall 3
   SA = width × height

   SA = 10 × 12

   SA = 120 ft^2

   Number of sheets needed to cover Wall 3:

   120 ÷ 32 = 3.75

   Wall 4
   SA = width × height

   SA = 20 × 12

   SA = 240 ft^2

   Number of sheets needed to cover Wall 4:

   240 ÷ 32 = 7.5

2. Total sheets:

   5.25 + 6 + 3.75 + 7.5 = 22.5

   Shay will use at least 22.5 sheets of drywall.

EXPLORE THE MATH

STUDENT BOOK, p. 116

Students have worked with three-dimensional shapes, such as prisms, pyramids, cylinders, and spheres, in previous grades. Remind students that a prism shares the same shape at the top and bottom of the prism. The example of a box given in the student book is a prism because it has a rectangle on the top and bottom of the box. Have students point to other prisms around the rooms, identifying the similar shape on the top and bottom that makes the object a prism.

For your visual and kinesthetic learners to better understand the concept of surface area, have students break apart a box in your classroom, such as a photocopy paper box or facial tissue box. Cut the box along the folds. Once students have the net of the box, guide them to see that the surface area of the three-dimensional box is the same as the area of each piece of the net.

DISCUSS THE IDEAS

CALCULATING SURFACE AREA OF A STAINED GLASS LANTERN

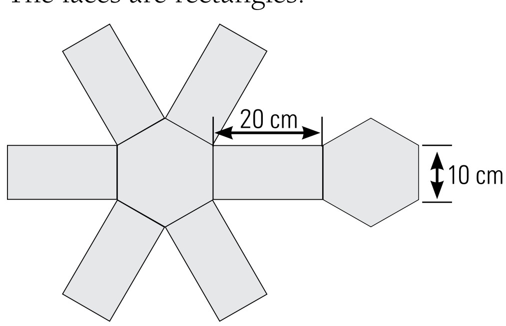
STUDENT BOOK, p. 118

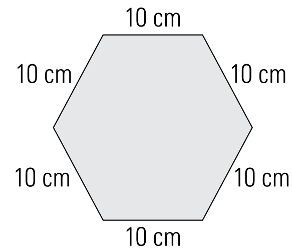
Ask students to identify shapes in your classroom by the proper name. Ensure that students identify each shape using "right" or "oblique" first, and then determine the base of the shape to complete the prism's name. Students may have trouble finding oblique objects, so challenge them to make one of their own.

Nets will help students make the link between area and surface area. Students can find the surface area of a figure by finding the area of each face of the three-dimensional object. By drawing the net of an object, students can make more explicit connections between area and surface area. If students need graph paper to complete their drawings, hand out Blackline Master 3.4 (p. 205).

SOLUTIONS

1. The faces are rectangles.





2. The hexagonal lantern will have six sides. Each side will be rectangular and have a width of 10 cm (the edge length of the hexagonal base) and a length of 20 cm.

   $A = \ell \times w$
   $A = 20 \times 10$
   $A = 200$

   The area of each piece of stained glass will be 200 cm^2.

3. The total area is the sum of areas of all the lateral faces. A hexagonal prism has 6 lateral faces.

   Total area = 6 × 200
   Total area = 1200 cm^2

   Another way to calculate the total surface area is to first determine the total perimeter of the hexagon, and then multiply the total perimeter by the height.

   Total perimeter = 6 × 10

   Total perimeter = 60 cm

   Total area = Total perimeter × height

   Total area = 60 × 20

   Total area = 1200 cm^2

   Shabina will need 1200 cm^2 of glass for one lantern.

Mental Math and Estimation

STUDENT BOOK, p. 119

SOLUTION

a) Students can find the area of the rectangle with the same height and width and then subtract half of a square for each of the four corners.

   8 cm × 8 cm = 64 cm^2

   64 – 2 = 62 cm^2

b) Students can find the area of the rectangular portion of the arrow. Then they can find the approximate area of a rectangle around the triangular portion of the arrow and divide that by 2.

Rectangle
$7 \text{ cm} \times 2 \text{ cm} = 14 \text{ cm}^2$

Triangle
$(4 \text{ cm} \times 2.5 \text{ cm}) \div 2 = 5 \text{ cm}^2$

Total area = 14 + 5
Total area = 19 cm^2

c) Students can find the area of the larger rectangle that would fit around the star. They can then subtract the approximate area of the 5 open spaces, starting with the top left space.

Rectangle
$9 \text{ cm} \times 9 \text{ cm} = 81 \text{ cm}^2$

Total area
$81 \text{ cm}^2 - 12 \text{ cm}^2 - 12 \text{ cm}^2 - 11 \text{ cm}^2 - 9 \text{ cm}^2 - 11 \text{ cm}^2$
$= 26 \text{ cm}^2$

Example 1

To help students visualize the shape of each piece, you may want to supply students with foam that can easily be cut to make three-dimensional pieces that can be put together to represent each piece of the fence.

Example 2

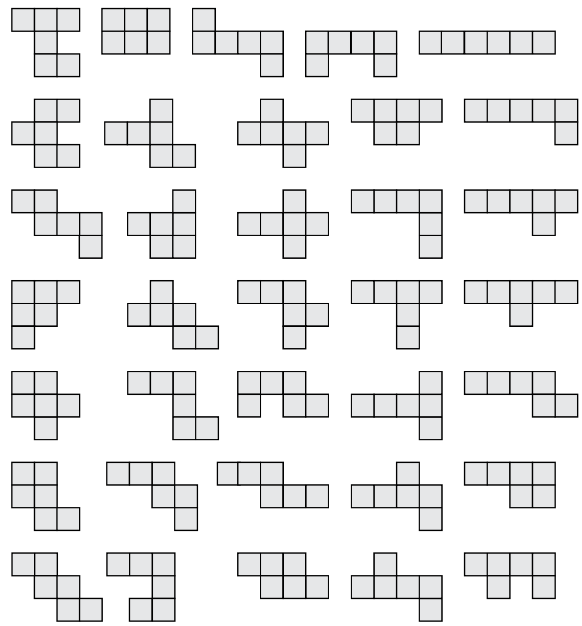
Shipping companies often use these packages. You may be able to pick up a flattened version of this package from a store. This can help students to see the net of the package and physically build the three-dimensional version to determine its surface area.

ACTIVITY 3.1
HEXOMINOS

STUDENT BOOK, p. 121

Students can complete this activity individually or in pairs. Ask students to think about the prefix "hex-" in the word "hexomino." Remind them that the prefix represents six, hence a hexomino is made up of six squares.

For students who quickly grasp this concept, have them research *septominos* and/or *octominos*. Students may see that there is a family of these shapes called *polyominos*, to which *dominos* belong. Students may see that, in addition to the number of squares defining the type of polyomino, there are also free, one-sided, fixed, and "with holes" polyominos. Challenge students to investigate the many relationships in the polyomino families.



SAMPLE SOLUTION

1. To make a cube, the hexomino will need to make four sides with one square on either side to make the fifth and sixth sides. The only hexomino that can be folded to form a closed cube is the third one.

2. Any hexomino that has four squares in a vertical line with one square on either side can be folded to create a cube.

DISCUSS THE IDEAS

SURFACE AREA OF CABINETS

STUDENT BOOK, p. 122

Students may begin by solving this problem individually. After they have completed the questions, divide them into small groups. Ask each student to explain to the group the mathematical processes he or she used to solve the problems. Ask each group to agree on the correct mathematical procedures to solve the problems, the answers to the problems, and their final conclusions.

This activity can be extended by thinking about what happens to the surface area if each dimension is tripled. What if each dimension is halved? What pattern do students see?

Extension

If you feel that students need extra practice with these mathematical concepts, extend this section by assigning students homework. Ask them to find the surface area of two cabinets at home and draw corresponding nets for them.

SOLUTIONS

1. The surface area of a rectangular prism can be expressed by the following equation:

   $SA = 2 \times (\text{length} \times \text{depth}) + 2 \times (\text{length} \times \text{height}) + 2 \times (\text{depth} \times \text{height})$

   The factor of 2 is introduced because the prism has three sets of identical pairs of faces.

   $SA = 2 \times (40 \times 40) + 2 \times (40 \times 70) + 2 \times (40 \times 70)$

   $SA = 14\ 400\ \text{cm}^2$

2. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 70) + 2 \times (40 \times 70)$

   $SA = 23\ 200\ \text{cm}^2$

3. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 140) + 2 \times (40 \times 140)$

   $SA = 40\ 000\ \text{cm}^2$

4. When the length is doubled, the surface area ratio increases by more than half the original ratio. The surface area does not double when one dimension is doubled. When two dimensions are doubled, such as the length and the height, the surface area more than doubles, measuring 40 000 cm^2.

5. $SA = 2 \times (80 \times 80) + 2 \times (80 \times 140) + 2 \times (80 \times 140)$

   $SA = 57\ 600\ \text{cm}^2$

   Ratio of surface areas $= \dfrac{57\ 600}{14\ 400}$

   Ratio of surface areas = 4.0

   Because the total surface area is a sum of several areas, and not all the terms of the sum are affected when one or two dimensions are doubled, the effect on surface area of changing only one or two dimensions is not an integer. When all three dimensions are doubled, all terms of the surface area sum are affected, and the total surface area scales by the square of the multiplication factor.

ACTIVITY 3.2
BUILDING A SHIPPING CRATE

STUDENT BOOK, p. 124

If you can locate 24 cubic objects, such as building blocks, use these items with this activity. Visual and kinesthetic learners will benefit from manipulating these objects to solve this activity.

Students should complete this activity in pairs. Have pairs think about what the 24 represents in this scenario. Students should see that the 24 boxes represent the volume of the larger box. As students will have studied the volume of prisms in previous grades, they may remember that the volume of a prism can be found by multiplying the length by the width by the height. So students will complete the chart by determining different combinations of three numbers that multiply to 24. Students can make and complete the chart in their notebooks.

If students struggle with this concept, you may want to start them off by giving them one set of dimensions, such as 12 by 2 by 1. Invite students to think of other sets of three factors that give a product of 24. Alternatively, you may want to start with a number that has fewer sets of factors, such as 8.

SOLUTION

1.

CRATE DIMENSIONS			
Length	*Width*	*Height*	*Surface Area*
1	1	24	98
1	2	12	76
1	3	8	70
1	4	6	68
2	2	6	56
2	3	4	52

2. The crate with dimensions of 2 by 3 by 4 will require the least amount of material to make.
3. The crate will have dimensions of 20 ft by 30 ft by 40 ft.

   Surface area:
   $1200 + 1600 + 2400 = 5200$ ft^2

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 124

1. a) Calculate the area of siding on each side of the house and add them up.

   $A = (2 \times 28 \times 6) + (2 \times 35 \times 6)$

   $A = 756$ ft^2

   Darcy must paint 756 ft^2.

   b) Total area of two coats:

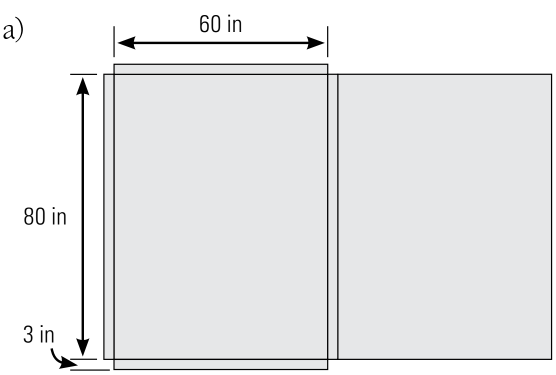
   $2 \times 756 = 1512$

   Divide total area by area per can to get number of cans:

   $\frac{1512}{225} = 6.72$
   Round up to 7 cans.

   Darcy must buy 7 cans of paint.

2. a)



   b) $2 \times 80 \times 60 = 9600$ in^2

   $2 \times 80 \times 3 = 480$ in^2

   $2 \times 60 \times 3 = 360$ in^2

   $SA = 9600 + 480 + 360$
   $SA = 10\ 440$ in^2

   Total material needed = 10 440 in^2

   10 440 in^2 of canvas are needed to cover one divider.

Information about Maillardville's Festival du Bois can be found at the festival's website.

www.festivaldubois.ca/

Information on Vancouver's francophone community can be found at this link.

http://www.discovervancouver.com/GVB/french.asp

Extension

Students may choose and research one of the francophone festivals that occur in their province, or within Canada. (Instead of having students research a francophone festival, you could have them research a festival that celebrates a different culture.) Ask students to research the location of the festival, when it occurs, and the activities that take place. Invite them to design a small three-dimensional, prism-shaped keepsake or souvenir that will be used to promote a specific cultural aspect of the festival (for example, one of the activities that people can enjoy there). This object could be a keychain, fridge magnet, or picture frame. Ask students to create a three-dimensional drawing of the keepsake and calculate its surface area to determine the amount of material that will be needed to manufacture it.

3. a) The area of a trapezoid is the average of the top and bottom edges times the height.

   Area = $\frac{(80 + 40)}{2} \times 200$
   Area = 12 000 cm^2

   The area of one face of the display case is 12 000 cm^2.

   b) The case has hexagonal top and bottom, therefore there are six faces.

   Total area = $6 \times 12\ 000$

   $A = 72\ 000$ cm^2.

   Convert to metres.
   $\frac{72\ 000}{10\ 000} = 7.2$ m^2
   She needs 7.2 m^2 of glass.

4. a) One face:
   $3 \times 3 = 9$

   There are 6 faces.
   $6 \times 9 = 54$

The surface area of the crate is 54 ft^2.

b) She can cut two faces of the crate out of one sheet of plywood, so she needs 3 sheets of plywood.

c) 4 sheets + 1 sheet for the 2 ends = 5 sheets of plywood

She can now get one side face out of one sheet of plywood, but both end pieces out of one sheet, so she needs 5 sheets of plywood.

For students who determine that the second crate will not need twice the area of plywood, have them determine by what factor they could multiply each dimension to require twice the area of plywood.

5. Calculate the area of each face and add them up.

First calculate the missing dimensions x and y.
$y = 24 - 8$
$y = 16$ in

$x = 36 - 8$
$x = 28$ in

Area of face A:
$A = 36 \times 12$
$A = 432$ in^2

Area of faces B and D:
$A = (36 \times 8) + (16 \times 8)$
$A = 416$ in^2

Area of face C:
$A = 12 \times 16$
$A = 192$ in^2

Area of face E:
$A = 28 \times 12$
$A = 336$ in^2

Area of face F:
$A = 12 \times 24$
$A = 288$ in^2

Total area:
A + B + C + D + E + F
432 + 416 + 416 + 192 + 336 + 288 = 2080

Dirk needs 2080 in^2 of sheet metal.

6. No, this is not a reasonable estimate. The bathroom would be around 8 m × 8 m, or the size of a small apartment! Possibly, Zyanya did not check the units of the floor plan and assumed the dimensions were in metres when the plan was actually in feet. A bathroom of 64 ft^2 (8 ft × 8 ft) is not unreasonable.

Extend Your Thinking

7. Because the roof is corrugated, the actual surface area is greater than the area covered by the roof.

To determine the area that he should have used, calculate the length along one corrugation.

Use the Pythagorean theorem to calculate the length, L, of the slope.

$L = \sqrt{(3^2 + 4^2)}$

$L = 5$

Determine the effective length, L_e, of each 6-inch corrugation.

$L_e = 2 \times 5$

$L_e = 10$ in

Each 6-inch corrugation is “stretched out” to 10 inches.

Calculate the number of corrugations, N, in the 25-foot length.

$N = \dfrac{25}{0.5}$

$N = 50$ corrugations

Calculate the total effective length of the roof.

Length = 50 corrugations × 10 inches per corrugation

Length = 500 inches

500 inches = 41 feet 8 inches

Now calculate the actual area that Wolfgang should have used in the area calculation.

Area = 20 × 41.67

Area = 833.4 ft^2

The area to be painted is 66% larger than the area covered by the roof.

3.2 Surface Area of Pyramids, Cylinders, Spheres, & Cones

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

MATH *ON THE JOB*

STUDENT BOOK, p. 127

Start the class by having a student read aloud the scenario describing Marc, a graphic designer in Regina, and his work designing a new label for a fruit smoothie drink. To understand the shape the label must be, you may want to bring in cylindrical containers or bottles with paper labels. Students can pull the labels off and determine the shape needed to create a label.

To help students visualize the container, you can have them take the paper label off a cylindrical can, like a soup can. Have students determine if all cylinders will have a rectangular lateral face.

To help kinesthetic and visual learners, as well as students in general, you could consider assigning a small project, based on the Math on the Job profile, as homework. The goal of the project would be to have students create a label for one product of their choice. The products would have the following shapes: pyramid, cylinder, sphere. Students could design the label, make it out of paper, fabric, plastic, etc., and calculate its surface area.

SOLUTION

The label will be in the shape of a rectangle.

Calculate the circumference.
$C = \pi d$
$C = \pi(6.6)$
$C \approx 20.735$

A = circumference × height
$A = 20.735 \times 12.1$
$A \approx 250.89\text{ cm}^2$

The label's area is 250.89 cm^2.

EXPLORE THE MATH

STUDENT BOOK, p. 127

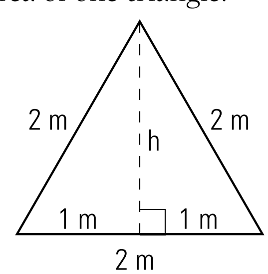
Begin by discussing different types of three-dimensional objects students see around them. Many objects they see will be in the shape of a prism, but help them see that not all three-dimensional objects have the same properties as a prism, such as a pyramid, cylinder, or sphere. Have them contrast and compare these objects with a prism to help them better identify objects around them.

Guide students to discover the shape of the lateral face of a cone. How does it differ from that of a cylinder?

You cannot unroll a sphere onto a flat surface as a net. A geodesic sphere, which has flat sides, can be unrolled. Blackline Master 3.5 (p. 206) is provided for students to find the surface area of a type of geodesic sphere.

SOLUTION

1. The icosahedron is made up of 20 equilateral triangles
2. Find the area of one triangle.



$$2^2 = 1^2 + h^2$$
$$3 = h^2$$
$$1.73 \approx h$$

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(2)(1.73)$$
$$A = 1.73$$

Total surface area:
$SA = 20 \times 1.73$
$SA = 34.6 \text{ m}^2$

DISCUSS THE IDEAS

THE MUTTART CONSERVATORY

STUDENT BOOK, p. 128

Looking at the pyramids of the Muttart Conservatory, discuss what properties make them pyramids. Have them try to name the type of pyramid based on the shape of its base.

SOLUTION

1. Area of each wall:
   $410 \div 4 = 102.5 \text{ m}^2$

   The area of each wall is 102.5 m^2 .

2. Area of each wall:
   $660 \div 4 = 165 \text{ m}^2$

   Side length:

   $$A = \frac{1}{2}bh$$
   $$165 = \frac{1}{2}(b)(16.5)$$
   $$165 = 8.25b$$
   $$20 \text{ m} = b$$

   The side length of each triangle is 20 m.

Example 1

To help students visualize this, take two small cylinders, such as small thread spools, and wrap a thick elastic band around the spools. Have students pull the spools apart and turn them to see how the conveyor belt would work. Guide students to see that the question is asking for the area around the thread spool.

ACTIVITY 3.3

DESIGNING OBJECTS USING PYRAMIDS AND CYLINDERS

STUDENT BOOK, p. 130

Have students work in pairs to complete this activity. Each pair should break the balloon apart into its identifiable pieces, hemisphere, cylinder, and cone. They should work together, using the known formulas and the one given for the cone, to determine the surface area of the balloon.

SOLUTION

$$\text{Surface area of hemisphere} = \frac{1}{2}(4\pi r^2)$$
$$\text{Surface area} = \frac{1}{2}(4)(\pi)(9^2)$$
$$\text{Surface area} \approx 508.94 \text{ m}^2$$
$$\text{Surface area of outside of cylinder} = 2\pi rh$$
$$\text{Surface area} = 2\pi(9)(20)$$
$$\text{Surface area} \approx 1130.97 \text{ m}^2$$
$$\text{Surface area of lateral face of cone} = \pi rs$$
$$\text{Surface area} = \pi(9)(15)$$
$$\text{Surface area} \approx 424.12 \text{ m}^2$$

Add to determine the total surface area.

$$508.94 + 1130.97 + 424.12 = 2064.03 \text{ m}^2$$

The total surface area of the balloon is 2064.03 m^2.

DISCUSS THE IDEAS

SURFACE AREA OF CIRCLES AND SPHERES

STUDENT BOOK, p. 131

Have students explain why a rectangle whose dimensions have been doubled has an area that is now four times as large. Have them think about what the area would be if each dimension was tripled. How does this relationship compare to that of a circle's area when its radius has been doubled or tripled?

Students will use their prior knowledge of the surface area of circles to solve this section. Divide students into small groups. They can begin by sketching the circles and writing out the formula for the area of a circle. Ask students to discuss what they think the formula for the second circle will be. Ask each group to agree upon and write down their solution. As a class, compare solutions and ask each group to provide the mathematical reasoning behind their answer.

SOLUTION

Area = πr^2
Area 2 = $\pi(2r)^2$
Area 2 = π $(2 \times 2\ r^2)$
Area 2 = $4\pi\ r^2$
Area 2 = 4 × Area

The area (of a circle that has twice the radius of another circle) will be four times as large.

Example 3

Students will have seen that doubling the radius of a circle, which is two-dimensional, will increase the circle's area by four. Ask students to explain why doubling the radius of a sphere, which is three-dimensional, also increases its surface area by four.

Example 4

For students who are not familiar with how shingles are placed on a roof, take several books of the same size, such as textbooks. Open them to the middle, flip them over, and place them on top of each other so that they overlap, exposing about two-thirds of the textbook underneath. Each book represents a shingle. Ask students to think about why this type of layout would be useful on a roof.

Mental Math and Estimation

STUDENT BOOK, p. 133

Have students come up with three different approximations of pi, one as an improper fraction, one as a square root, and one as a power with an exponent of 2.

SOLUTION

The approximate value of pi is 3. Using 3 would result in a slight underestimate.

The circumference of a circle with a radius of 8 cm can be calculated using 3 for pi.

$C = 2\pi r$
$C = 2 \times 3 \times 8$
$C = 48$ cm

The circumference of a circle with a radius of 8 cm is about 48 cm.

The surface area of a sphere with a radius of 3 inches can be similarly calculated.

$SA = 4\pi r^2$
$SA = 4 \times 3 \times 3^2$
$SA = 108$ in^2

The surface area of a sphere with radius of 3 inches is about 108 in^2.

ACTIVITY 3.4
MORTICE AND TENON VS. DOWEL JOINTS

STUDENT BOOK, p. 134

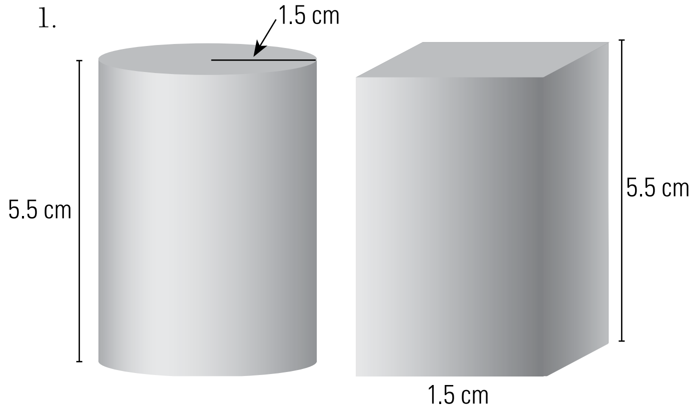
You can introduce this activity by asking if carpentry students have used these joints in any of their projects. If a student has, you can ask him or her to explain the differences between the two joints, and how they are commonly used.

Your visual learners will need to see examples of mortice and tenon joints and dowel joints. Use an internet search engine to find several images online of each of these joints to help students in their calculations.

Next, ask students to discuss which joint they think will have a larger surface area, and why. (One type of joint provides a larger surface area to which glue can be applied. The additional glue can strengthen the joint.) In pairs, have students calculate the surface area of each joint. To check how accurate student calculations are, you can have students compare their answers with another group and discuss the reasons for (possible) differences. After students have compared answers, go over the correct calculation with the entire class.

SOLUTIONS

1.



2. Dowel joint:

   $SA = 2\pi rh + 2\pi rh^2$
   $SA = 2(\pi)(1.5)(5.5) + 2\,(\pi)\,1.5^2$
   $SA = 51.84 + 14.14$
   $SA \approx 65.98\text{ cm}^2$

   Mortice and tenon joint:

   $SA = 4(1.5 \times 5.5) + 2(1.5 \times 1.5)$
   $SA = 37.5\text{ cm}^2$

3. Celeste should use the dowel joint because it has a larger surface area and will therefore be stronger.

ALTERNATIVE ACTIVITY

MEASURING AND CALCULATING THE SURFACE AREA OF A COMPLEX OBJECT

In this activity, assign students a complex object from the classroom, such as a chair, lamp, etc. Working in pairs, ask them to calculate its surface area by breaking it down into simple shapes, measuring and recording the key dimensions, and using the equations for surface area appropriately. They will then convert their calculated total surface area into an equivalent surface area of a square, and make a judgment as to whether their estimate seems reasonable. Students can record their work using Blackline Master 3.8 (p. 209).

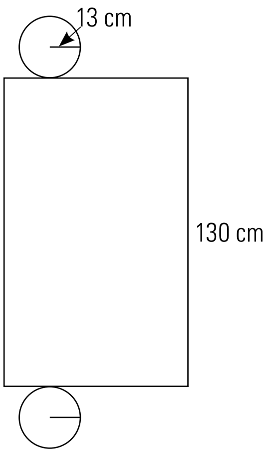
SOLUTION

Answers will vary, depending on the object measured.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 134

1. a) Draw the rectangle to have a width of approximately 3 times the diameter of the circle.



   b) $SA = 2\pi r^2 + 2\pi rh$

   $SA = 2\pi(13)^2 + 2\pi(13)(130)$

   $SA \approx 11\ 680\text{ cm}^2$

   Sasha will need to cover 11 680 cm^2 of space to paint the column.

2. To help students visualize the inside of the tunnel, you can show them an air duct tube, which is easily found at a local hardware store. Have students think about the benefits of using a corrugated surface versus a smooth surface.

   a) Surface area of tunnel:

   $SA = 2\pi r\ell$
   $SA = \pi d\ell$
   $SA = \pi(6)(20)$
   $SA \approx 377\text{ m}^2$

   Area of one steel sheet:

   $A = \ell \times w$
   $A = 1.30 \times 2.00$
   $A = 2.60\text{ m}^2$

Number of steel sheets required:

$\dfrac{377}{2.60} = 145$ sheets

The number of steel sheets used to line the tunnel is about 145.

Note: This method can only be used to estimate. An estimator would have to consider the length and width of the steel sheets and how they would fit onto the surface area of the tunnel.

b) No, it would not be a good idea. Although the steel sheets cover the inside of the tunnel, the actual surface area of the corrugated steel is much greater because of the wavy surface. You would need much more paint to cover the corrugations than to cover a flat surface.

3. The area of the hide needed will be equal to the areas of the circular drum top, plus the extra material needed to cover the hoop.

$r = (45 + 14) \div 2$
$r = 29.5$

$A = \pi r^2$
$A = \pi(29.5)^2$
$A \approx 2734 \text{ cm}^2$

The amount of hide needed is about 2734 cm^2.

The area of the wood needed will be equal to the area of the rectangular lateral face.

$A = 2\pi rh$
$A = 2\pi(22.5)(5)$
$A = 706.9 \text{ cm}^2$

The amount of wood needed is 706.9 cm^2.

If students are interested in learning about the musical ensemble M'Girl, or hearing their music, they can visit the website below.

http://www.myspace.com/mgirlmusic

This link to the Canadian Aboriginal Music Awards will provide information on other First Nations musicians and groups.

http://www.canab.com/mainpages/events/musicawards.html

4. Students may need to see examples of grain stockpile covers to better visualize their shape and solve this question. Although the covers are not usually a perfect cone, have students use a cone to estimate the cover's surface area.

Surface area of the lateral face of a cone:

πrs (where s is the slant height)

Use the Pythagorean theorem to calculate the slant height:

$$s^2 = h^2 + \left(\frac{d}{2}\right)^2$$

$$s = \sqrt{(23)^2 + \left(\frac{96}{2}\right)^2}$$

$s \approx 53$ m

Surface area of the lateral face of a cone:

$SA = \pi rs$

$SA = \pi\,(48)(53)$

$SA \approx 7992 \text{ m}^2$

7992 m^2 of material is needed.

Extend Your Thinking

5. a) $$s^2 = \left(\frac{b}{2}\right)^2 + h^2$$

$$s^2 = 90^2 + 98^2$$

$$s = \sqrt{90^2 + 98^2}$$

$$s \approx 133.1 \text{ ft}$$

The slant height is 133.1 ft.

b) $$SA = \frac{1}{2}bs \times 4$$

$$SA = \frac{1}{2}(180)(133.1)(4)$$

$$SA = 47\,916 \text{ ft}^2$$

The lateral surface area is $47\,916 \text{ ft}^2$.

c) $47\ 916 \div 3 = 15\ 972$

There are about 15 972 blocks on the outside of the pyramid.

6. a) Surface area of a cube $= 6 \times \ell^2$

Use algebraic manipulation; divide both sides by 6 and take the square root to find ℓ.

$1350 = 6 \times \ell^2$

$\ell = \sqrt{\dfrac{1350}{6}}$

$\ell = 15$ cm

The side length of the cube measures 15 cm.

b) Use the surface area, 1350, to solve.

$$SA = 4\pi r^2$$
$$1350 = 4\pi r^2$$
$$\frac{1350}{4\pi} = r^2$$
$$107.43 \approx r^2$$
$$\sqrt{107.43} = r$$
$$10.36 \approx r$$

$d = 2r$
$d = 2(10.36)$
$d \approx 20.7$

The diameter is 20.7 cm.

THE ROOTS OF MATH

ARCHIMEDES' CONTRIBUTIONS TO SURFACE AREA CALCULATIONS

STUDENT BOOK, p. 137

This activity will help students make further connections between the characteristics between three-dimensional objects. In this activity, students will see that the surface area of a sphere is equal to the area of the lateral face of a cylinder with the same height and radius. They will also learn that the surface area of a sphere is two-thirds of the surface area of the same cylinder. In addition, students will make connections between their algebraic formulas.

For students who show interest, you may want to have students investigate other connections between the measurements of three-dimensional objects. You may also decide to leave this part of the activity until the end of the unit so that students can make connections with not only surface areas, but also with volumes.

SOLUTIONS

1. In groups, have students write out and compare the surface area formulas for a sphere and the curved section of a cylinder. Ask them to discuss how the formulas are similar. Next, they can discuss which mathematical operation could be used to make the two formulas the same. You can ask each group which operation they think could be used, write each one up on the board, and have students vote for the answer they think is correct.

$4\pi r^2 = \pi dh$

$4\pi r^2 = \pi(2r)(2r)$

$4\pi r^2 = 4\pi r^2$

2. You can use the same process to find the answer for question 2.

$4\pi r^2 = \dfrac{2}{3}(\pi dh + 2\pi r^2)$

$4\pi r^2 = \dfrac{2}{3}(4\pi r^2 + 2\pi r^2)$

$4\pi r^2 = \dfrac{2}{3}(6\pi r^2)$

$4\pi r^2 = 4\pi r^2$

3.3 Volume and Capacity of Prisms and Cylinders

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

MATH *ON THE JOB*

STUDENT BOOK, p. 138

Before you begin this section, you might want to ask students to individually identify five mathematical operations/concepts that they learned about in sections 1 and 2. Ask students to rate, on a scale of 1 to 5 (with 1 representing poor understanding), how well they understand these operations/concepts and how comfortable they feel doing them. Ask students to identify the operations/concepts they would like to improve their understanding of. Encourage them to set the goal of improving their understanding of these things before the chapter is finished. Encourage students to ask you for clarification of the mathematical concepts or operations that they find challenging.

Ask the class for a volunteer to read the scenario describing Tamye Dunbar, an entrepreneur who runs her own business shipping items to customers all over the country. Have students discuss which box she could use to ship her 10 cm by 10 cm by 10 cm boxes to customers.

Students may use a sketch to work out how the boxes would fit in the packing boxes or they may reach a numerical solution.

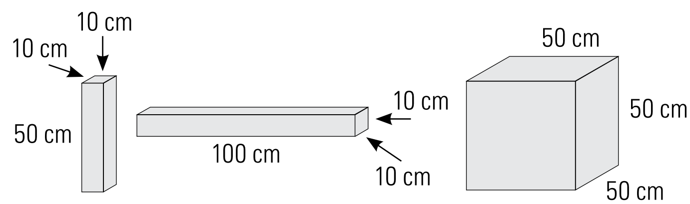
Students may think that Tamye can use any of the boxes, but remind them that she would likely prefer to ship 125 of the smaller boxes in one larger box. You may want to extend the activity by asking students how many of the smaller boxes could be shipped in the other two boxes, or how many of the smaller boxes she would need to use to fill this order.

Challenge students to think about whether a cube will always allow for the largest volume. Challenge them further by asking why a cube allows the largest volume, versus a tall or long rectangular prism.

SOLUTION

To ship 125 of the smaller boxes, Tamye should use the large cube box. She can pack the 10 cm boxes so that they are 5 long, 5 wide, and 5 high, totalling 125 boxes.

The 10 cm × 10 cm × 50 cm box would organize the smaller boxes so that they fit 1 long, 1 wide, and 5 high, or would be 5 boxes. The 10 cm × 10 cm × 100 cm box would organize the smaller boxes so that they fit 1 deep, 10 long, and 1 high, totalling 10 boxes.



EXPLORE THE MATH

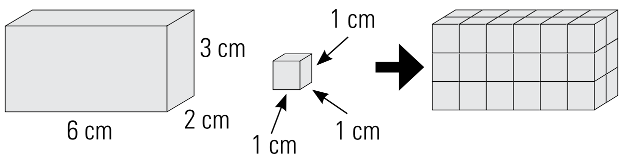
STUDENT BOOK, p. 138

To build on students' thinking from the Math on the Job activity, have students continue to think about the amount of material inside a box. Volume is the measurement of the amount of space a three-dimensional object takes up, so have students try to visualize a larger box in terms of the smaller boxes needed to fill it to determine how much space it takes up.

Students may struggle with the difference between volume and capacity. Ask students to look at the signage the next time they are in an elevator. They will see a safety sign listing a capacity of a certain amount of people. This is because the elevator can only hold a certain amount of people inside. It will not list a volume of people as this would not make sense.

Extension

The volume of a box below is 36 cm^3. You can think of this as a box that takes up the same amount of space as 36 small cubes measuring 1 centimetre by 1 centimetre by 1 centimetre, or 1 cm^3.



a) How many different ways could the 36 small cubes be arranged, without splitting any cubes, to create a box? What would be each box's dimensions?

b) If the length, width, and height of any box were doubled, how would this affect the number of cubes that would now fit inside the box?

c) If the length, width, and height of any box were halved, how would this affect the number of cubes that would now fit inside the box?

d) If the small cubes used in the box measuring 36 cm^3 actually measured 1 mm by 1 mm by 1 mm, how many of these cubes would fit inside the 36 cm^3 box?

SOLUTIONS

a)

BOX	DIMENSIONS
1	36 cm × 1 cm × 1 cm
2	18 cm × 2 cm × 1 cm
3	12 cm × 3 cm × 1 cm
4	9 cm × 4 cm × 1 cm
5	9 cm × 2 cm × 2 cm
6	6 cm × 6 cm × 1 cm
7	6 cm × 3 cm × 2 cm
8	4 cm × 3 cm × 3 cm

b) Doubling each dimension of the box has the effect of multiplying the volume by $2 \times 2 \times 2$, or 2^3. So there will be 8 times more small cubes needed to fill the box.

c) Halving each dimension of the box has the effect of multiplying the volume by $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $\left(\frac{1}{2}\right)^3$. So an $\frac{1}{8}$ of the small cubes will be needed to fill the box.

d) Each dimension of the small cubes was reduced by $\frac{1}{10}$, but the dimensions of the box have stayed the same. So 10 times more small cubes will be needed along each dimension of the box, and 10^3 or 1000 times more small cubes will be needed to fill the box.

DISCUSS THE IDEAS

CALCULATING THE VOLUME OF PAVING STONES

STUDENT BOOK, p. 139

This section demonstrates the connection between mathematical calculations and their application in the workplace. After the questions have been solved, encourage students to participate in a discussion about other jobs that require employees to calculate volume. Examples could include plumbers, gas fitters, and cooks.

Students who struggle with this question should start by making a diagram of a rectangle to represent the first layer. They can label the dimensions and determine how many cubic stones would fit inside the one layer before building more layers.

If students need some help with this formula, suggest that they use visualization to help work out the formula. The volume or capacity of prisms and cylinders can be visualized as a stack of similar shapes, much like a stack of pennies. This leads to understanding the formula for volume as:

Volume = (area of base) × height

SOLUTIONS

1. The number of stones in one layer is the number of stones along the length times the number of stones along the width:

   $n = 10 \times 10$

   $n = 100$

   There are 100 stones in one layer.

2. The number of stones is the number of stones in one layer times the number of layers.

   $n = 100 \times 5$

   $n = 500$ stones

   There are 500 stones on the pallet.

3. The total number of paving stones can be calculated this way:

   number of paving stones = length × width × height

4. Volume of paving stones = length × width × height

   This can also be expressed as:

   Volume of paving stones = area of base × height

   The volume of a prism or cylinder can be calculated by finding the area of the base and multiplying it by the height.

Example 1

Help students to discover the difference between the fish tank's volume and its capacity. Ask, "Why would the volume be represented in cubic centimetres and the capacity be measured in litres?"

Remind students that 1 cm^3 is equivalent to 1 mL. Therefore, 1000 cm^3 is equivalent to 1000 mL or 1 L.

ACTIVITY 3.5
VOLUME OF AN OBLIQUE PRISM

STUDENT BOOK, p. 141

You can use this activity to monitor student mathematical literacy by asking them whether or not you can find the volume of an oblique rectangular prism by turning it into a rectangle. Ask them to explain their conclusions using mathematical formulas.

This activity will help students continue to make connections between area and volume. While the student book provides a pictorial representation of the situation, you can provide students with a concrete situation by having them work with an oblique rectangular prism, such as an eraser or any object that can easily be cut. This will allow students to determine if the theory holds.

SOLUTION

Lead students to discover that the volume of the prism can be found in the same way the area of a rectangular prism can be found. So the volume would still be found by using $V = \ell \times w \times h$. The volume of the prism is 1445 cm^3.

DISCUSS THE IDEAS
INVESTIGATING VOLUME

STUDENT BOOK, p. 142

All the planters in the example are cubes. Ask the students to consider if the same relationship will occur when all of the dimensions of any rectangular prism are increased in the same way

SOLUTIONS

1. Determine the volume of each box.

   1-ft box: $V = (1)^3$
   $V = 1 \text{ ft}^3$

   2-ft box: $V = (2)^3$

   $V = 8 \text{ ft}^3$

   4-ft box: $V = (4)^3$

   $V = 64 \text{ ft}^3$

2. As the side length is increased by 2, the volume is increased by a factor of 8. This is because each of the three side lengths has been doubled, so the volume is increased by 2 times 2 times 2 or 2^3.

   To get the volume of a new box where a scale factor has been applied equally to all sides, multiply the original volume by the cube of the scale factor.

3. Use the volume calculations from question 1 to solve the problem.

   1-ft box:
   $V = 1 \text{ ft}^3$
   $1 \times 5 = 5 \text{ ft}^3$

2-ft box:
$V = 8 \text{ ft}^3$
$8 \times 5 = 40 \text{ ft}^3$

4-ft box:
$V = 64 \text{ ft}^3$
$64 \times 5 = 320 \text{ ft}^3$

Add to find the answer.

$5 + 40 + 320 = 365$

Julia will need 365 ft^3 of soil.

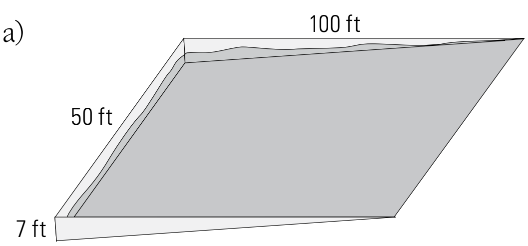
Example 3

Students may have difficulty with the context of this question. If you have an auto repair class in your school, try to bring the instructor into the class to demonstrate the context of the question. If this is not possible, you may want to have students do an internet search to better understand engine displacement.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 144

1. a)



b) Volume of pool:

$\frac{1}{2} \times 100 \times 7 \times 50 = 17\,500$

The capacity of the pool is 17 500 ft^3.

Capacity of pool in US gallons:

$17\,500 \times 7.48 = 130\,900$

The capacity of the pool is 130 900 US gallons.

c) Use the Pythagorean theorem to calculate slope length (ℓ) of the bottom.

$c^2 = a^2 + b^2$
$\ell^2 = 7^2 + 100^2$
$\ell^2 = 10\,049$
$\ell = \sqrt{10\,049}$
$\ell \approx 100.24$

Total surface area:
$7(50) + 2\left(\frac{1}{2}\right)(7)100 + 100.24(50) = 6062$

The surface area of the inside of the pool is 6062 ft^2.

2. a) Calculate the cross-sectional area; multiply by length (converted to inches); convert to cubic feet.

Cross-sectional area:

$(8)(16) + (8)(40) = 448 \text{ in}^2$

Volume of the footing:

$V = \ell \times w \times h$
$V = 16 \times 12 \times 8$

$V = 1536$

Volume of the foundation wall:

$V = \ell \times w \times h$
$V = 8 \times 12 \times 40$

$V = 3840$

Volume of concrete needed for one-foot length of wall:

$V = 1536 + 3840$
$V = 5376 \text{ in}^3$

Convert to cubic feet:

$5376 \div 12^3 \approx 3.11 \text{ ft}^3$

For a one-foot length of wall, 3.11 ft^3 of concrete is needed.

b) Total volume:

$25 \times 3.11 = 77.75 \text{ ft}^3$

Volume in cubic yards:

$\frac{77.75}{3^3} \approx 2.88$

2.88 or about 3 cubic yards are needed.

3. a) Volume $= \pi r^2 h$

$V = \pi\left(\frac{50}{2}\right)^2 \times (70)$

$V \approx 137\,445$

One bin holds 137 445 cm^3 of flour.

b) Volume of sack = length × width × thickness

$V = 46 \times 80 \times 15$
$V = 55\ 200\ \text{cm}^3$

The number of sacks of flour in bin equals $\frac{\text{volume of bin}}{\text{volume of sacks}}$

The number of sacks of flour in bin equals $\frac{137\ 445}{55\ 200}$

$\frac{137\ 445}{55\ 200} \approx 2.49$

2.49 or about 2.5 sacks of flour fit in the bin.

c) Kilograms of flour = number of sacks × kilograms per sack

Kilograms of flour = 2.49 × 20

Kilograms of flour = 49.8 kg

The bin holds 49.8 kg or about 50 kg of flour.

d) Each of the three dimensions of the salt bin is one half the dimension of the flour bin (the radius is used twice in the volume calculation).

$$V_{\text{salt}} = \left(\frac{1}{2}\right)^3 \times V_{\text{flour}}$$

$$V_{\text{salt}} = \frac{1}{8} \times V_{\text{flour}}$$

$$V_{\text{salt}} = \frac{1}{8} \times 137\ 445$$

$$V_{\text{salt}} \approx 17\ 181\ \text{cm}^3$$

The volume of salt in the bin is 17 181 cm^3.

4. a) $V = \pi r^2 h$

$$V = \pi\left(\frac{62.2}{2}\right)^2 (149.9)$$

$$V \approx 455\ 483\ \text{cm}^3$$

Convert centimetres to litres.

1 litre = 1000 cm^3

$$V = \frac{455\ 483}{1000}$$

$$V \approx 455\ \text{L}$$

The total volume is 455 litres.

b) $\frac{\text{Hot water tank capacity}}{\text{Total volume}} \times 100$

$\frac{270}{455} \times 100 \approx 59.3\%$

The rated water capacity of the tank is 59.3%, or just over one half its total volume.

A hot water tank has a rated capacity, but also has a discharge rate that is measured in either litres per second or gallons per minute. Have students research what the current discharge rate is for today's energy efficient hot water tanks and determine how quickly it would take the hot water tank in this question to be drained.

5. a) Area of bubble wrap = surface area of shaft

surface area of shaft = πdh

$A = \pi(40)(475)$

$A \approx 59\ 690\ \text{mm}^2$

Convert millimetres to centimetres.

1 cm^2 = (10 × 10) mm^2

$\frac{59\ 690}{100} \approx 597\ \text{cm}^2$

The area of bubble wrap needed is 597 cm^2.

b) Volume of box = volume of (shaft + bubble wrap)

Volume of box = 6 × 6 × 50

6 × 6 × 50 = 1800 cm^3

Volume of (shaft + bubble wrap) = $\pi r^2 h$

For the radius, use the radius of the shaft plus one layer of bubble wrap thickness.

$$r = \left(\frac{4}{2} + 1\right)$$

$$r = 3\ \text{cm}$$

Volume of (shaft + bubble wrap) = $\pi(3)^2(47.5)$

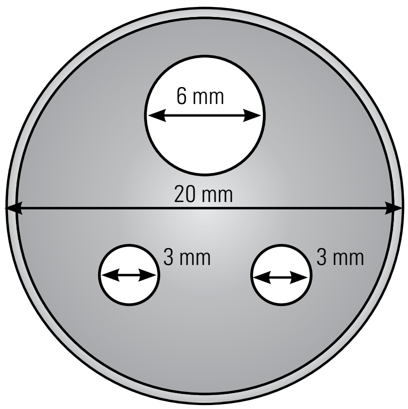
$V \approx 1343\ \text{cm}^3$

Volume of additional packing = 1800 − 1343

1800 − 1343 = 457

The volume of additional packaging is 457 cm^3.

6.



Total volume = (Cross-section of outer pipe − cross-section of inner holes) × length

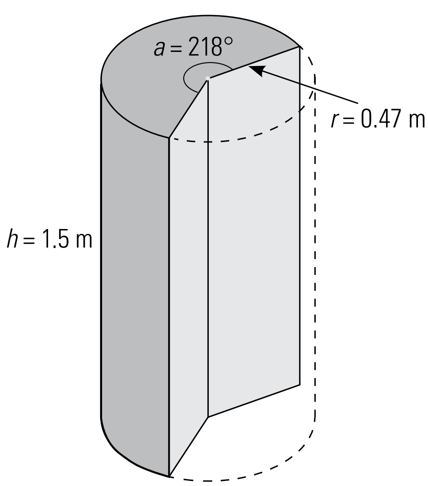
$$V = \left[\pi\left(\frac{20}{2}\right)^2 - \pi\left(\frac{6}{2}\right)^2 - 2\pi\left(\frac{3}{2}\right)^2\right] \times 1000$$

$$V \approx 271\ 748\ \text{mm}^3$$

For each metre of extrusion, 271 748 mm^3 of plastic are needed.

Extend Your Thinking

7.



Volume of a cylinder = area of base × height

The area of the base will be proportional to the total circular area by the ratio of the angle to the total angles in a circle, or $\frac{218°}{360°}$.

$$V = \pi r^2 h \times \left(\frac{218}{360}\right)$$

$$V = \pi(0.47)^2(1.5)\left(\frac{218}{360}\right)$$

$$V \approx 0.63$$

The volume of the section is 0.63 m^3.

PUZZLE IT OUT

CUBE

STUDENT BOOK, p. 147

SOLUTION

6 units.

The surface area of a cube can be represented by the formula $6s^2$. The volume of a cube can be represented by the formula s^3. The question is asking when $6s^2 = s^3$.

Students can solve this question using two methods. In the first method, students can try trial and error, using a side length of 1 in the formula and increasing by 1 until they find the solution.

$6(1)^2 \neq (1)^3$
$6(2)^2 \neq (2)^3$
$6(3)^2 \neq (3)^3$
$6(4)^2 \neq (4)^3$
$6(5)^2 \neq (5)^3$
$6(6)^2 = (6)^3$

In the second method, students can solve the equation algebraically.

$6s^2 = s^3$
$s^2(6) = s^2(s)$
$6 = s$

3.4 Volume and Capacity of Spheres, Cones, & Pyramids

TIME REQUIRED FOR THIS SECTION: 3 CLASSES

MATH *ON THE JOB*

STUDENT BOOK, p. 148

Choose a student to read through the scenario describing Kristi Hansen, a Red Seal plumber in Vancouver, BC. Have students create a diagram of the water tank so that they can visualize how much water it will contain.

Remind students that although the water tank has a diameter of 120 cm, they will need to find the diameter of the inside of the tank, without the insulating material.

SOLUTION

$V = \pi r^2 h$

$V = \pi(1600)(250)$

$V \approx 1\ 256\ 637\ \text{cm}^3$

The tank has a capacity of 1 256 637 cm^3.

To find the capacity in litres, divide by 1000.

1 256 637 ÷ 1000 = 1 256.637

The tank's capacity is 1 256.637 litres.

EXPLORE THE MATH

STUDENT BOOK, p. 148

To visualize how a material poured will form a cone, have a student pour a small bag of sugar or salt onto a plate and see how the pile slowly forms a cone.

The volume formulas for a cone and a pyramid seem to match those of a cylinder and a prism, respectively, except they have been multiplied by $\frac{1}{3}$. Ask students to think about why this relationship exists. How many cones could fit into a cylinder with the same diameter and height? How many pyramids could fit into a rectangular prism with the same base area and height?

Have students discuss similarities and differences between the spheres and cones. Point out to students that the formulas for a cone and pyramid have the same form. You can ask students the following questions to assess their understanding of these formulas.

1. Do you think a pyramid with an octagonal base would have the same formula as a cone or a pyramid?
2. How about an n-sided pyramid, where n can be any number? What shape does the base become as n becomes very large? What shape does the pyramid become?

SOLUTIONS

1. A pyramid with an octagonal base would have the same formula. However, the method for calculating the base would be different.
2. An n-sided polygon would have the same formula. Again, the method for calculating the base would be different. As n becomes large, the base become more like a circle, so that the pyramid approaches the shape of a cone.

DISCUSS THE IDEAS

MATHEMATICAL FORMULAS FOR CYLINDERS AND SPHERES

STUDENT BOOK, p. 149

Discuss the scenario involving lead fish sinkers. Have students examine the two shapes to determine that the height of the cylinder is the same as twice the sphere's radius.

SOLUTION

Volume of a sphere = $\frac{2}{3}\ (\pi r^2 h)$

$V = \frac{2}{3} \times \pi r^2 \times 2r$

$V = \frac{4}{3}\ \pi r^3$

Example 2

Have students do an internet search on the angle of repose of gravel or sand piles. Do construction companies have a specific angle of repose for their piles? Why would a company want to set an angle of repose for their piles?

ACTIVITY 3.6
USING MICROMETERS AND CALIPERS

STUDENT BOOK, p. 152

This activity relies on workplace tools. Encourage students to make the connection between volume/surface area calculations and their use in the workplace. This activity allows students to experience the mathematical concepts of diameter, radius, surface area, and volume in a hands-on, concrete way by measuring objects. Students can see and experience the direct relationship between an object's measurements and its surface area or volume. This is a good activity for students who have kinesthetic or visual learning styles. For the first part of this activity, you will need several materials. You will need enough micrometers for every two students in your class. Ensure that the micrometers are metric and not imperial. Before students begin the activity, give students several objects to measure that have a small diameter, such as a marble or a coin. Have students think about the surface area and volume of each object. You may want to practise reading measurements with students before beginning.

For the second part of this activity, you will need enough calipers for every two students. Again, ensure that the calipers are metric, and not imperial. Before students begin the activity, give students several objects that have an inner and outer diameter, such as a paper towel roll or a bracelet. Have students think about the volume of the object as subtracting the volume of the hollow cylinder from the volume of the larger cylinder.

You may want to practise reading measurements with students before beginning. Before this class, it is a good idea to use calipers and micrometers to measure the objects you will provide to students (marbles, coins, etc.). You can then compare these measurements to student measurements in order to ensure students are using the tools correctly and measuring accurately.

SOLUTIONS

1. a) Measurements will vary.

   b) Students should halve the diameter of their object in order to obtain its radius. Once they have obtained the object's radius, they can put this value into the surface area formula for a cylinder or sphere and calculate the answer.

   To determine the volume of the object, students can insert the same radius value into the formula for volume. They will need to take a second measurement to determine the object's height. Once students have the values for radius and height, they can calculate the volume of the cylinder or sphere.

2. a) Answers will vary.

   b) Students could use their inside and outside diameter measurements to calculate the volume of two cylinders (one cylinder's measurements would be based on the outside diameter measurements, the other would be based on the inside diameter measurement). They could subtract the volume of the smaller cylinder (the cylinder calculated using the inside diameter measurement) from the volume of the larger cylinder (the cylinder calculated using the outside diameter measurement). The resulting value would be the amount of material used to create the item.

ACTIVITY 3.7
USING DISPLACEMENT TO MEASURE VOLUME

STUDENT BOOK, p. 153

For this activity, you will need to provide students with a cup, a graduated cylinder, a pan, and several items to measure that will displace enough water to get a good reading, such as a tennis ball or a golf ball. Ensure that your materials are not absorbent and will not soak up any of the water. To do this activity, students will:

- place the cup on a pan and fill it with water to the very top;
- slowly drop their object into the cup and let the excess water pour into the pan; and
- pour the displaced water from the pan into the graduated cylinder to see how much water was displaced.

You can modify this activity to have students place the object into a filled graduated cylinder. Students can then take the object out to see how much water was displaced.

SOLUTIONS

1. Answers will vary among students.
   Some sources of error may be inaccurate measurement of liquid or dropping the object into the liquid too forcefully so that too much liquid spills out.
2. An advantage of using this method to calculate volume is that it can be used to measure very complex shapes.

Example 4

You may want to have students use this example as a project and build their own gnomes. Students can construct each shape using paper or items found around the classroom. They can present their gnomes to the class, showing how they found the volume of their gnomes. Encourage students to be creative with the designs of their gnomes.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 156

1. Volume of rectangular prism $= 53 \times 25 \times 30$
   $= 39\ 750 \text{ cm}^3$
   Volume of rectangular pyramid $= \frac{1}{3}(53 \times 25 \times 15)$
   $= 6625 \text{ cm}^3$

   Total volume = 39 760 + 6625
   Total volume = 46 375 cm^3

   The volume of Jayne's model is 46 375 cm^3 .

2. a) $V = \frac{4}{3}\pi r^3$
   $V = \frac{4}{3}\pi\left(\frac{6}{2}\right)^3$
   $V = 113 \text{ mm}^3$
   The volume of the ball bearing is 113 mm^3.

   b) Doubling the radius results in an increase in volume of 2^3 or 8.

   c) The volume will increase by a factor of 4^3 or 64.
3. a) 14.70 mm

   b) 7.64 mm

   c) 12.48 mm
4. a) 2.325 cm

   b) 4.22 cm

   c) 1.14 cm
5. a) Volume of cube:
   $V = \ell^3$
   $V = 54^3$
   $V = 157\ 464 \text{ mm}^3$

   Volume of sphere $= \frac{4}{3}\pi r^3$
   $V = \frac{4}{3}\pi\left(\frac{54}{2}\right)^3$
   $V \approx 82\ 448 \text{ mm}^3$

   Volume of marble ground away = volume of cube − volume of sphere
   $V = 157\ 464 - 82\ 448$
   $V = 75\ 016 \text{ mm}^3$

   b) Percentage wasted (w)
   $\frac{\text{volume ground away}}{\text{volume of cube}} \times 100$
   $w = \frac{75\ 016}{157\ 464} \times 100\%$
   $w \approx 47.6\%$

   Almost one half of the marble is wasted.

Extend Your Thinking

6. volume of pyramid $= \frac{1}{3}\ell^2 h_p$
   volume of cone $= \frac{1}{3}\pi r^2 h_c$

volume of cone = $\frac{1}{3}\pi\left(\frac{\ell}{2}\right)^2 h_c$

volume of pyramid = volume of cone

$\frac{1}{3}\ell^2 h_p = \frac{1}{3}\pi\left(\frac{\ell}{2}\right)^2 h_c$

Divide both sides by $\frac{1}{3}\ell^2$.

$h_p = \left(\frac{\pi}{4}\right) h_c$

$h_p = 0.785\ h_c$

The height of the pyramid is slightly more than $\frac{3}{4}$ of the height of the cone.

7. a) Volume of two half-cone ends = volume of one cone

$$V_{\text{ends}} = \frac{1}{3}\pi r^2 h$$

$$V_{\text{ends}} = \frac{1}{3}\pi\left(\frac{51.4}{2}\right)^2 \times 18$$

$$V_{\text{ends}} \approx 12\ 450\ \text{m}^3$$

$$V_{\text{triangular prism}} = \frac{1}{2}(\text{width} \times \text{height}) \times \text{length}$$

$$V = \frac{1}{2}(51.4 \times 18) \times 50$$

$$V = 23\ 130\ \text{m}^3$$

Calculate the total volume.

$12\ 450 + 23\ 130 = 35\ 580\ \text{m}^3$

The stockpile contains 35 580 m^3 of ore.

b) Let the angle of repose equal a.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan a = \frac{\text{height}}{\text{half width}}$$

$$\tan a = \frac{18}{25.7}$$

$$a = \tan^{-1}\left(\frac{18}{25.7}\right)$$

$$a \approx 35°$$

The angle of repose is 35°.

c) Volume of cone = $\frac{1}{3}\pi r^2 h$

Use the tangent ratio to find a relation between the radius and the height:

$\frac{h}{r}$ = tangent (angle of repose)

$h = r \times \tan 35°$

Substitute the expression for h into the volume formula.

Volume = $\frac{1}{3}\pi r^2 \times r \times \tan 35°$

Volume = $\frac{1}{3}\pi r^3 \tan 35°$

Use algebraic manipulation to solve for r^3.

$$r^3 = \frac{3\ (\text{volume of cone})}{\pi \times \tan 35°}$$

$$r^3 = \frac{(3 \times 35\ 580)}{\pi \times \tan 35°}$$

$$r \approx 36.5\ \text{m}$$

$$h = 36.5 \tan 35°$$

$$h \approx 25.6\ \text{m}$$

The height would be 25.6 m and the diameter would be 73 m.

Students should realize that the angle of repose remains the same.

REFLECT ON YOUR LEARNING

SURFACE AREA, VOLUME, AND CAPACITY

STUDENT BOOK, p. 159

Ask students to review and reflect on the list of new skills and knowledge they have encountered in this chapter.

PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 160

1. a) Volume to surface area of a cube of side length ℓ :

Surface area = $6\ell^2$

Volume = ℓ^3

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{\ell^3}{6\ell^2}$$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{\ell}{6}$$

b) Volume to surface area of a sphere of radius r:

Volume = $\frac{4}{3}\pi r^3$

Surface area = $4\pi r^2$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{\left(\frac{4}{3}\pi r^3\right)}{4\pi r^2}$$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{r}{3}$$

c) Relationship between the length of the cube and the radius of the sphere:

surface area of cube = surface area of sphere

$6\ell^2 = 4\pi r^2$

$\ell^2 = \frac{2}{3}\pi r^2$

$\ell = r \times \sqrt{\frac{2}{3}\pi}$

$\ell \approx 1.447r$

d) Volume of cube = volume of sphere

$\ell^3 = \frac{4}{3}\pi r^3$

$\ell = r\left(\frac{4}{3}\pi\right)^{\frac{1}{3}}$

$\ell \approx 1.612r$

2. a) The pipeline is a cylinder 5280 ft long with a diameter of 4 ft.

Volume = $\pi r^2 \ell$

$V = \pi(2)^2(5280)$

$V \approx 66\ 350$ ft^3

One mile of pipeline holds 66 350 ft^3 of oil.

b) $66\ 350 \times 7.4805 \approx 496\ 331$

There are 496 331 US gallons of oil in one mile of pipeline.

c) $\frac{496\ 331}{42} \approx 11\ 817$

There are 11 817 barrels (bbl) of oil per mile of pipeline.

d) $11\ 817 \times 800 = 9\ 453\ 600$

There are 9 453 600 barrels of crude oil in the entire pipeline.

3. a) Volume of large pyramid = $\frac{1}{3} \times 25^2 \times 25$

$V \approx 5208$ cm^3

Volume of small pyramid = $\frac{1}{3} \times 7.5^2 \times 7.5$

$V = 140.625$ cm^3

Volume of rectangular prism = $25 \times 25 \times 40$
$V = 25\ 000$ cm^3

Total volume = $25\ 000 + 5208 - 140.625$
$V = 30\ 067.375$ cm^3

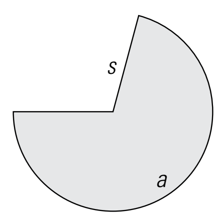
The volume of the bin is 30 067 cm^3.

b) $\frac{30\ 067}{2250} \approx 13.4$ kg

The bin holds 13.4 kg of coffee.

Students should realize they can subtract the small pyramid from the large pyramid to find the volume of the bottom of the bin.

4. a)



b) Edge a is the length of the circumference of the base of the cone, or $2\pi r$.

5. a) Surface area of top = πr^2

$SA = \pi(900)^2$

$SA \approx 2\ 544\ 690$ mm^2

Surface area of cylinder = $2\pi rh$

$SA = 2\pi(900)(1570)$

$SA \approx 8\ 878\ 141$ mm^2

The surface area of the lateral face of a cone equals πrs where s equals the slant height.

Use the Pythagorean theorem to find s.

$s^2 = r^2 + h^2$
$s^2 = 900^2 + 1930^2$
$s \approx 2130$ mm

Surface area of cone = $\pi(900)(2130)$

$SA \approx 6\,022\,433$ mm^2

Total surface area:

2 544 690 + 8 878 141 + 6 022 433 = 17 445 264 mm^2

The amount of sheet metal needed is 17 445 264 mm^2.

b) 1 m^2 = (1000 × 1000) mm^2

1 m^2 = 1 000 000 mm^2

$\frac{17\,445\,264}{1\,000\,000} \approx 17.445$

The amount of sheet metal needed, in metres, is 17.445 m^2 .

6. a) Volume of one basket = $\frac{1}{2} \times \frac{4}{3}\pi r^3$

$V = \frac{1}{4} \times \frac{4}{3}\pi\left(\frac{50}{2}\right)^3$

$V \approx 32\,725$ cm^3

Convert to litres.

1 litre = (10 × 10 × 10) cm^3

Capacity of one basket:

$\frac{32\,725}{1000} = 32.725$ L

Soil needed for 48 baskets:

48 × 32.725 ≈ 1571 L

Ara will need 1571 L of soil.

b) Bags:
1571 ÷ 60 ≈ 26.2, rounded up to 27 bags
27 × 13.50 = 364.50
Bags of soil would cost $364.50.

Bulk soil:
Use the given conversion factor.
one cubic yard = 27 × 28.23
one cubic yard = 762.21 L

1571 ÷ 762.21 ≈ 2.06 cubic yards
2.06 × 41.50 = 85.49
Bulk soil costs $85.49.

Ara should buy the bulk potting soil.

7. The awning has a top, sloped face, and two ends. The back and bottom are open.

Total surface area = sum of areas of faces

Area of top = 20 × 1.5
$A = 30$ ft^2

Area of sides (both):
$2\left[(2 \times 1.5) + \frac{1}{2}(2)(2.5)\right]$

$A = 11$ ft^2

Area of sloped face:

First calculate the width using the Pythagorean theorem.

$w^2 = 2^2 + 2.5^2$

$w = \sqrt{2^2 + 2.5^2}$

$w \approx 3.2$ ft

$A = 3.2 \times 20$

$A = 64$ ft^2

Total area = 64 + 11 + 30

$A = 105$ ft^2

The total area of fabric Seth needs is 105 ft^2.

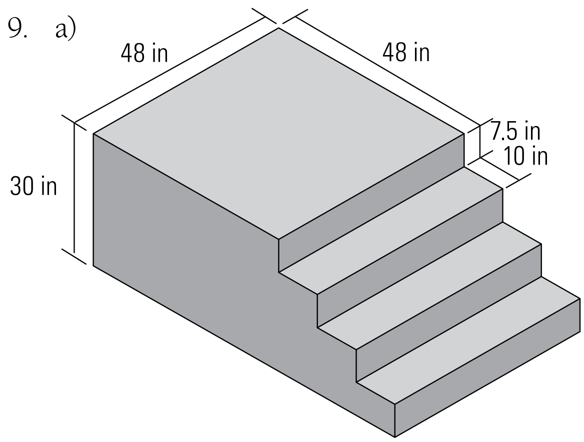
8. Surface area = $\frac{1}{2}(4\pi r^2)$

$SA = \frac{1}{2} \times 4\pi\left(\frac{7}{2}\right)^2$

$SA \approx 77.0$ ft^2

The surface area is 77.0 ft^2.

9. a)



Calculate the cross-sectional area and multiply by the width of the stairs.

Surface area:

(10 × 7.5) + (10 × 15) + (10 × 22.5) + (48 × 30)

$SA = 1890 \text{ in}^2$

Volume:

Volume = 1890 × 48

Volume = $90\ 720 \text{ in}^3$

$$\text{Volume} = \frac{\left(\frac{90\ 720}{1728}\right)}{27}$$

volume ≈ 1.94 cubic yards

The volume of concrete needed is 1.94 cubic yards.

b) Surface area:

SA of ends + SA of treads + SA of risers + SA of landing

Total surface area = 2(1890) + 3(10 × 48) + 4(7.5 × 48) + 48 × 48

$SA = 8964 \text{ in}^2$

$SA = 62.25 \text{ ft}^2$

The surface area that needs to be covered is 62.25 ft^2.

10. a) Volume = $\frac{1}{3}$(area of base) × (height)

$$V = \frac{1}{3}(230.56)^2 \times 138.75$$

$$V \approx 2\ 458\ 554 \text{ m}^3$$

The volume of the stone used to build the pyramid is $2\ 458\ 554 \text{ m}^3$.

b) 2.56 × 2 458 554 ≈ 6 293 898

The weight of the pyramid is about 6 293 898 tonnes or 6.3 million metric tonnes.

Extension

You may wish to give students further practise in calculating the volume of complex objects using Blackline Master 3.10 (p. 221).

SOLUTIONS

$V_{rectangle}$ = length × width × height
$V_{rectangle} = 20 \times 13 \times 13$
$V_{rectangle} = 3380 \text{ m}^3$

$V_{cylinder} = \pi r^2 h$
$V_{cylinder} = \pi \times (6)^2 \times (7)$
$V_{cylinder} \approx 792 \text{ m}^3$

$V_{rectangle}$ = length × width × height
$V_{rectangle} = 8 \times 8 \times 16$
$V_{rectangle} = 1024 \text{ m}^3$

$V_{pyramid} = \frac{1}{3}$(length of base × width of base × height)

$V_{pyramid} = \frac{1}{3}(8 \times 8 \times 12)$

$V_{pyramid} = 256 \text{ m}^3$

Total volume = 3380 + 792 + 1024 + 256

Total volume = 5452 m^3

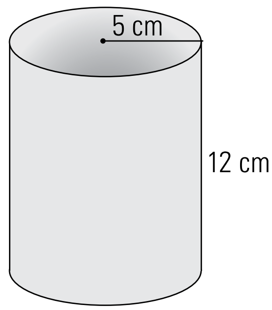
SAMPLE CHAPTER TEST

Name: ______________________ Date: ______________________

Part A: Multiple Choice

Choose the best response to each of the following questions.

1. Ray runs a stationery store. He sells a canister to hold pencils and pens, as shown below.



He protects each canister with plastic wrap. What is the minimum amount of plastic wrap Ray will need to cover the canister?

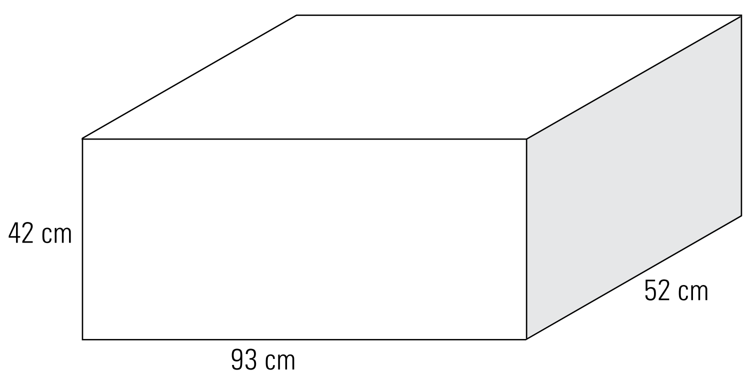
a) 188.50 cm^2

b) 455.53 cm^2

c) 534.07 cm^2

d) 942.48 cm^2

2. Mick is a carpenter and is building a brace for a wall. One of the pieces he needs to create is in the shape of a rectangular prism, as shown below.



How much wood will Mick need to create the piece, in cubic metres?

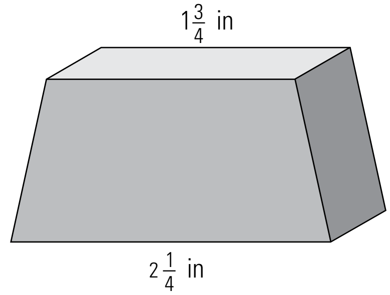
a) 0.02 m^3

b) 0.2 m^3

c) 21 852 m^3

d) 203 112 m^3

3. Lira runs a jewellery store. She has created a small jewellery box in the shape of an oblique trapezoidal prism, as shown below.



Lira decides to create another box that can hold eight times the amount of the original box. Which of the following strategies can Lira use to create the new box?

a) Multiply each of the original side lengths by 2.

b) Multiply each of the original side lengths by 8.

c) Square the value of the original side lengths.

d) Cube the value of the original side lengths.

4. Sayeli is a chef at her family's restaurant. Many of her dishes use rice, so she buys a large box of rice in the shape of a cube with a volume of 15 625 cm^3. What is the side length of the box?

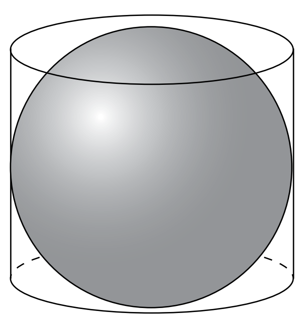
a) 25 cm

b) 125 cm

c) 1953.1 cm

d) 5208.3 cm

5. Nick's Sporting Goods sells basketballs in cylindrical boxes. Each ball is tightly packed into each box.



How much space inside the cylindrical box is taken up by the sphere?

a) $\frac{1}{3}$

b) $\frac{1}{2}$

c) $\frac{2}{3}$

d) $\frac{3}{4}$

Part B: Short Answer

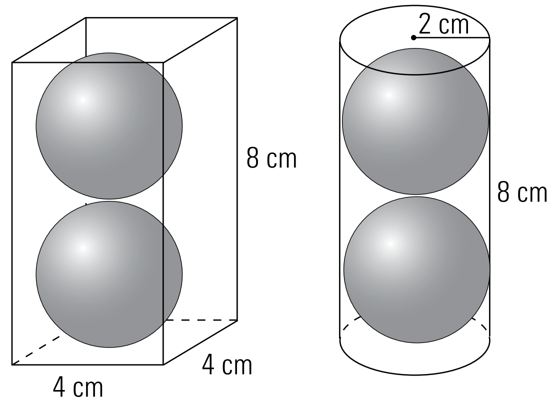
1. Sylvio has installed a cylindrical water tank in an apartment building with a radius of 15 feet and a height of 30 feet. How much water will the tank hold?‘

2. Atsidi is a painter. She has been hired to paint a large conference room. The room is in the shape of a rectangular prism with a length of 25 feet, a width of 32 feet, and a height of 10 feet. How much paint will she need to paint the four walls of the room, in square feet?

3. Henry is shipping small cubic boxes with a side length of 5 inches. He needs to ship 64 boxes to one of his customers. What shape of box would require the least material to make? What would be the dimensions of the box?

Part C: Extended Answer

1. Luke is designing a package to hold tennis balls. He creates two packages, one in the shape of a right rectangular prism, and the other in the shape of a cylinder. Each package can hold two balls tightly.



a) Which of the packages will require less material to create? Explain your answer.

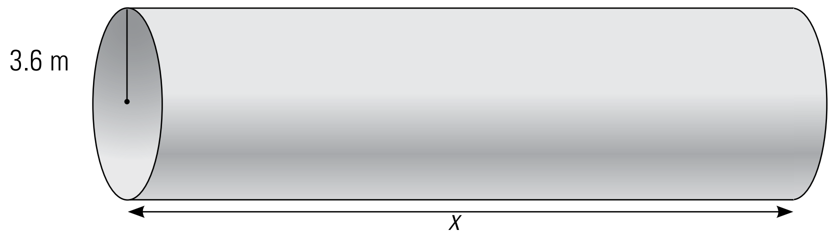
b) If Luke creates 10 packages, how much less material will be required to create the package you identified in a)?

2. Jorge is an artist. He has been contracted to create a large work of art for the park grounds in his town. He has proposed two different pieces to the town:
   - A rectangular prism with a height of 12 metres, a width of 3 metres, and a length of 4 metres.
   - A sphere with a diameter of 6 metres.

   a) If he creates a hollow prism or a sphere, which shape will require less material to create? Explain your answer.

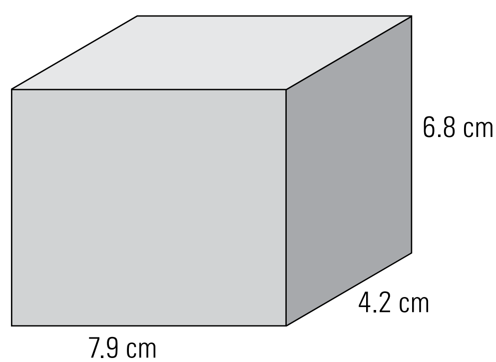
   b) If he creates a solid prism or a sphere, which shape will require less material to fill? Explain your answer.

3. David is a millwright. He is installing a large pipe in the ground with a volume of 879.44 m^3.



The pipe has a radius of 3.6 metres. What is the length of the pipe, *x*?

4. Darrin is a candle maker. For his next candle, he buys a rectangular brick of wax, as shown below, and melts it. He shapes the melted wax into a spherical candle.



What is the approximate radius of the largest candle Darrin can make?

5. Yolanda is a city planner. She has planned to create a rectangular landfill with a depth of 85 ft and a base with dimensions of 700 ft by 200 ft.

   a) How many cubic feet of garbage will the landfill be able to hold?

   b) When the landfill is created, approximately how many cubic yards of dirt will the city need to haul away? (1 foot = $\frac{1}{3}$ yard)

SAMPLE CHAPTER TEST SOLUTIONS

Part A: Multiple Choice

1. $SA = 2\pi r^2 + 2\pi rh$

   $SA = 2\pi(5) + 2\pi(5)(12)$

   $SA \approx 534.07 \text{ cm}^2$

   The answer is c).

2. $V = \ell \times w \times h$

   $V = 42 \times 93 \times 52$

   $V = 203\ 112 \text{ cm}^3$

   $203\ 112 \div (100)^3 \approx 0.2 \text{ m}^3$

   The answer is b).

3. A box that has 8 times the volume will have side lengths that have been doubled. This is because 2 times 2 times 2 equals 8.

   The answer is a).

4. Since the box is in the shape of a cube, each of the side lengths are equal. This means that students can cube root the volume to determine the side length.

   $\sqrt[3]{15\ 625} = 25$

   The answer is a).

5. A sphere has $\frac{2}{3}$ of the volume of its circumscribing cylinder.

   The answer is c).

Part B: Short Answer

1. $V = \pi r^2 h$

   $V = \pi(15)^2(30)$

   $V \approx 21\ 205.75 \text{ ft}^3$

   The tank will hold $21\ 205.75 \text{ ft}^3$.

2. Two of the walls will have an area of 25 ft times 10 ft, or 250 ft^2 each.

   Two of the walls will have an area of 32 ft times 10 ft, or 320 ft^2 each.

   The total area of the four walls would be 1140 ft^2.

3. A cubic box would require the least material. To fit 64 boxes, the boxes could be placed 4 times 4 times 4 in the box, which would make the dimensions of the larger box 20 in. by 20 in. by 20 in.

Part C: Extended Answer

1. a) Surface area of prism = $8 \times 4 \times 4 + 2 \times 4 \times 4$
   Surface area of prism = 160 cm^2

   Surface area of cylinder = $2\pi(2)^2 + 2\pi(2)(8)$
   Surface area of cylinder $\approx 125.7 \text{ cm}^2$

   The cylinder will require less material to create.

   b) 10 prisms = 1600 cm^2

   10 cylinders = 1256.6 cm^2

   The cylinders would use 343.4 cm^2 less material than using the prisms.

2. a) Surface area of prism =
   $2(12 \times 3) + 2(3 \times 4) + 2(12 \times 4)$
   Surface area of prism = 192 m^2

   Surface area of sphere = $4\pi(3)^2$
   Surface area of sphere $\approx 113.10 \text{ m}^2$

   The sphere will require less material to create.

   b) Volume of prism:

   $12 \times 3 \times 4 = 144 \text{ m}^3$

   Volume of sphere:

   $\frac{4}{3}\pi(3)^3 \approx 113.10 \text{ m}^3$

   The sphere will require less material to fill.

3. $$V = \pi r^2 x$$

   $$879.44 = \pi(3.6)^2 x$$

   $$879.44 \div [\pi(3.6)^2] = x$$

   $$21.6 \approx x$$

   The length of the pipe is 21.6 m.

4. Volume of rectangular brick:

$V = \ell \times w \times h$

$V = 7.9 \times 4.2 \times 6.8$

$V = 225.624 \text{ cm}^3$

Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

$$225.624 = \frac{4}{3}\pi r^3$$

$$225.624 \div \left(\frac{4}{3}\pi\right) = r^3$$

$$53.86 \approx r^3$$

$$\sqrt[3]{53.86} \approx r$$

$$3.78 \approx r$$

The radius of the largest candle Darrin can make is approximately 3.78 cm.

5. a) $V = 85 \times 700 \times 200$

$V = 11\ 900\ 000 \text{ ft}^3$

b) $1 \text{ foot} = \frac{1}{3} \text{ yard}$

$$(1 \text{ foot})^3 = \left(\frac{1}{3} \text{ yard}\right)^3$$

$$1 \text{ ft}^3 = \frac{1}{27} \text{ yd}^3$$

$11\ 900\ 000 \text{ ft}^3 = 440\ 740.7407 \text{ yd}^3$

The city will need to haul away approximately 440 741 yd^3 of dirt.

BLACKLINE MASTER 3.1

CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research;
- the accuracy of your calculations and drawings;
- the effectiveness of your uses of technology for research, organizing, and presenting;
- the creativity you brought to planning and presenting; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel? ______________________

Are there areas in which you could improve? ______________________

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 3.2

CHAPTER PROJECT: MEASUREMENTS AND CALCULATIONS

Name: ______________________________ Date: ______________________________

Item: ____________________________________

item

package 1

package 2

package 3

PACKAGING MEASUREMENTS				
	Item	*Package 1*	*Package 2*	*Package 3*
Length				
Height				
Width				
Surface area				
Volume				
Amount of material used				
Cost				

BLACKLINE MASTER 3.3

CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❐ Did you research and record package dimensions on different continents?	
❐ Did you research and record the materials commonly used to make your packaging?	
❐ Are each of your packages different shapes? Does one incorporate two shapes?	
❐ Did you research and record how to make your packaging appeal to a customer?	
❐ Did you calculate the volume of each package and the cost to make it?	
❐ Did you label each of your packages with its dimensions?	
❐ Did you prepare a poster, handout, or electronic presentation?	
❐ Other notes	

BLACKLINE MASTER 3.4

GRAPH PAPER (0.5 CM × 0.5 CM)

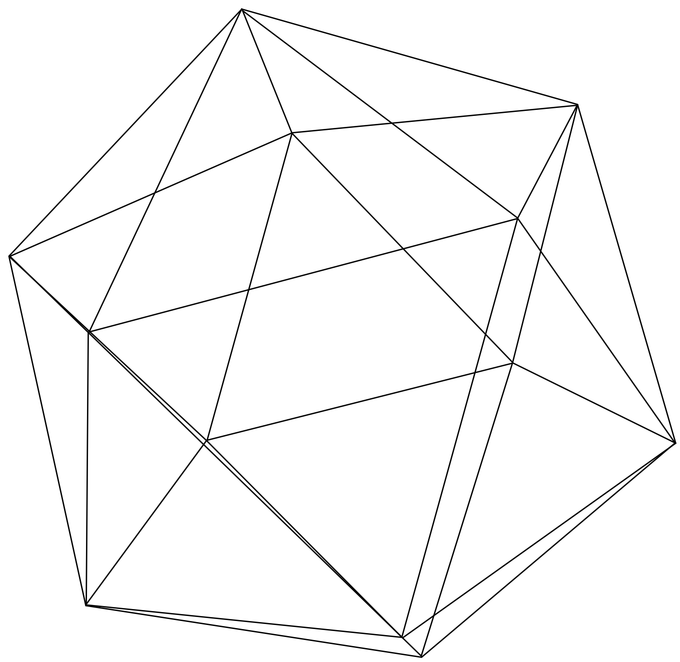
Name: ______________________ Date: ______________________

BLACKLINE MASTER 3.5

CALCULATING THE SURFACE AREA OF A GEODESIC SPHERE

Name: ______________________________ Date: ______________________________

This geodesic sphere is called an icosahedron.



It is made up of identical equilateral triangles.

1. How many triangles make up the shape?

2. If the length of one side of each triangle is 2 m, calculate the surface area of the icosahedron.

BLACKLINE MASTER 3.6

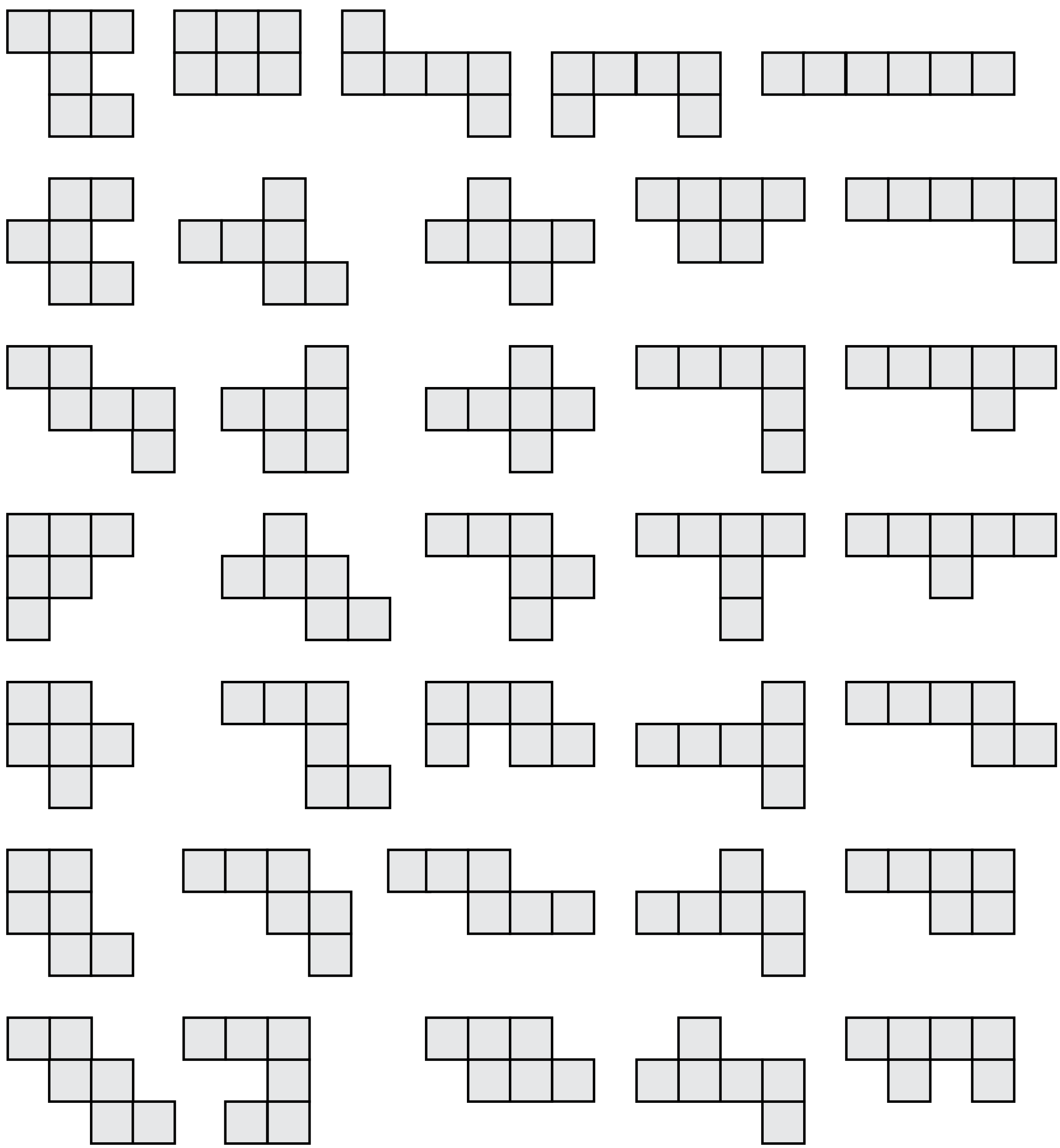
ISOMETRIC GRAPH PAPER (0.5 CM × 0.5 CM)

Name: ______________________________ Date: ______________________________

BLACKLINE MASTER 3.7

HEXOMINOS

Name: ______________________________ Date: ______________________________



BLACKLINE MASTER 3.8

MEASURING AND CALCULATING THE SURFACE AREA OF A COMPLEX OBJECT

Name: ______________________ Date: ______________________

Calculate the surface area of a complex object by breaking it down into simpler shapes. You will be assigned a complex object commonly found in your classroom. Work with a partner and use the table below to record the important dimensions of the parts of the object, appropriate equations, your calculations, and the total surface area of your object.

MEASURING AND CALCULATING THE SURFACE AREA OF A COMPLEX OBJECT				
Object				
Units of measurement used				
Segment description	*Equation for surface area*	*Important dimensions*	*Measured values of dimensions*	*Calculated area*
Total surface area (add up all the segment areas)				

If others in your class calculated the surface area of the same complex object, compare your results.

ALTERNATIVE CHAPTER PROJECT—RENOVATE AND REDECORATE YOUR BEDROOM

TEACHER MATERIALS

GOALS: To understand how three-dimensional measurement and surface area/volume calculations are used to complete practical home improvements, as well as by those working in the home improvement and decorating business. Students will incorporate their understanding of dimensional analysis in their calculations.

OUTCOME: In this project, students will use their understanding of surface area, three-dimensional figures, and volume to determine the materials necessary for home renovation and improvement. Students will also calculate the cost of materials needed.

PREREQUISITES: Students will need to recall their prior knowledge of proportions and similar figures. Students reviewed proportional reasoning in grade 10 and were exposed to similar figures in grade 9. Students will use their knowledge of three-dimensional measurement when they calculate the surface area of the walls in their room and the amount of materials needed to redo the room and build two new items of furniture.

ABOUT THIS PROJECT: There is plenty of room in this project for student creativity, especially if they choose to make additional changes to their room. Students can work individually to complete this project. Using measurements from their own bedroom, they will use their math skills to calculate the amount and price of materials needed to make over the room, as well as build two new pieces of furniture. Students will apply what they have learned about surface area and volume of three-dimensional objects to create a 3-D sketch of their room.

Students should be given a few class periods to work on this project. This will allow for questions/feedback from the teacher as well as allowing the teacher to observe the quality of work as it is done, rather than at the end of the chapter. Interim guidance can help the students complete the culminating activity more successfully.

1. Start to plan

When you introduce this project, you may want to bring in some home renovation magazines or catalogues to pass around. This will give students a better idea of the variety of materials they can choose to use for this project, as well as the cost of these items. After you have gone over the introductory part of the project, tell students that as homework, they are responsible for measuring and recording the dimensions of their bedroom – this includes the floor, ceiling, walls, windows, and doorway. To encourage students to make changes to their room beyond what is listed in the project, the class can brainstorm and list extra features that they would like to include in their renovation.

2. Research your renovation T

Provide students with class time to sketch and label their room with its dimensions, as well as research the different costs, types, and amounts of materials they will use. Students will use their understanding of three-dimensional measurement to determine the amounts of each item needed for the project. Students will need to record their work in a table or a spreadsheet. When students are calculating the amount of materials needed for their renovation, remind them that it is best to purchase a little extra.

Encourage students to incorporate different shapes into their furniture design. You can bring in books on furniture design to give students a better idea of how furniture builders can use practical skills to create objects that are both artistic and functional.

At the end of this segment of the project, discuss progress with your students to ensure that all requirements have been met. Hand out the chapter project checklist, Blackline Master 3.2A (p. 216), and ensure that students are aware of the work they must complete and hand in.

You can also hand out Blackline Master 3.6 (p. 207). It contains isometric graph paper which students can use for their sketches.

3. Complete your renovating and redecorating plans

To complete the project, students need to calculate and record the volume of their storage unit and end table. Each student should submit the list that includes the different materials used in their renovation, as well as the amounts and costs of these materials. Students should also submit the three-dimensional drawings of their room and the two pieces of furniture they chose to build. These should be labelled with their dimensions.

ASSESSING THE PROJECT

1. Start to plan

- Ensure that students are aware of all of the materials they will need to include in their renovation, and the project components they will need to hand in. Record your observations on student progress. Provide students with numeric information on how they will be assessed using a scheme that meets your reporting needs.

2. Research your renovation

- Have students complete the checklist of all items that should be in their project to allow them to reflect on their progress. As students calculate the surface area of their room, ensure that they are subtracting the surface area of any openings that won't require paint or other materials, such as windows and doors.

3. Make a presentation

- Use the assessment rubric (p. 212) as a gauge to accompany a numerical grading rubric you have created. Remind students that all submitted drawings must be labelled with dimensions.

<table>
<tr><th colspan="5">PROJECT ASSESSMENT RUBRIC: RENOVATE AND REDECORATE YOUR BEDROOM</th></tr>
<tr><th></th><th>Not yet adequate</th><th>Adequate</th><th>Proficient</th><th>Excellent</th></tr>
<tr><td colspan="5">Conceptual Understanding</td></tr>
<tr><td>• Explanations show an understanding of three-dimensional measurement</td><td>shows very limited understanding; explanations are omitted or inappropriate</td><td>shows partial understanding; explanations are often incomplete or somewhat confusing</td><td>shows understanding; explanations are appropriate</td><td>shows thorough understanding; explanations are effective and thorough</td></tr>
<tr><td colspan="5">Procedural Understanding</td></tr>
<tr><td>Accurately:
▪ records the necessary materials for the project
▪ determines the type, amount, and cost of materials needed
▪ draws and labels room and furniture dimensions
▪ researches appropriate materials and their costs
▪ determines the surface area and/or volume for room components and furniture</td><td>limited accuracy; major errors or omissions
For example:
• most materials are not recorded
• material cost, type, and amount missing or incorrectly calculated
• drawing is missing, or labels are missing or incorrect
• surface area/ volume calculations missing or incorrect
• research is incorrect or not included in the project
• project is incomplete</td><td>partially accurate; some errors or omissions
For example:
• records most necessary materials
• one or two material costs, types, and amounts are incorrectly calculated
• drawings are labelled incorrectly in one or two places
• surface area/ volume calculations are correct
• research is mostly complete
• project could use more work to ensure calculations are done correctly</td><td>generally accurate; few errors or omissions
For example:
• records necessary materials
• calculations regarding material cost, type, and amount are correct
• drawings are labelled correctly
• surface area/ volume calculations are correct
• research is complete
• project is complete and correct</td><td>accurate and precise; very few or no errors
For example:
• records necessary materials
• calculations regarding material cost, type, and amount are correct
• drawings are labelled correctly
• surface area/ volume calculations are correct
• research is complete
• project is complete and correct, and student has added extra renovations/ decorations to the project</td></tr>
<tr><td colspan="5">PROBLEM-SOLVING SKILLS</td></tr>
<tr><td>• Uses appropriate strategies to solve problems successfully and explain the solutions</td><td>uses few effective strategies; does not solve problems</td><td>uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions</td><td>uses appropriate strategies to successfully solve most problems and explain solutions</td><td>uses effective and often innovative strategies to successfully solve problems and explain solutions</td></tr>
<tr><td colspan="5">COMMUNICATION</td></tr>
<tr><td>• Presents work and explanations clearly, using appropriate mathematical terminology</td><td>does not present work and explanations clearly; uses few appropriate mathematical terms</td><td>presents work and explanations with some clarity, using some appropriate mathematical terms</td><td>presents work and explanations clearly, using appropriate mathematical terms</td><td>presents work and explanations precisely, using a range of appropriate mathematical terms</td></tr>
</table>

ALTERNATIVE CHAPTER PROJECT — RENOVATE AND REDECORATE YOUR BEDROOM

STUDENT MATERIALS

PROJECT OVERVIEW

What would your ultimate bedroom look like? With the mathematical knowledge that you will acquire from this chapter, you will have the skills needed to remake your bedroom. This project will also allow you to perform tasks that home improvement and decorating professionals carry out at work, such as calculating the volume of paint needed to cover a room, the surface area of a floor that needs to be tiled, or the amount of fabric needed to make a set of curtains.

START TO PLAN

Think of the changes you would like to make to your existing bedroom. Would you paint the walls? Redo the floors? Buy new furniture? Would you redecorate using a theme? You are about to use your personal preferences to make the following improvements to your room. Start a list with three columns, labelled "material," "amount," and "price." Record the colours and textures you would choose and list the materials you would need to do the following renovations.

- paint or refinish the ceiling with stipple or other material;
- redo the walls with paint or wallpaper;
- refinish the floors with carpet, laminate flooring, or tile;
- install and finish new baseboards; and
- design a storage unit and an end table. The end table must incorporate two different shapes, one of which is a cone, sphere, or cylinder. The storage unit can be a trunk or a shelf. You must calculate its volume to determine the amount of material it can hold.

You may also want to consider changing the light switch coverings, electrical outlets, window coverings, and door, but this is optional. To get started with this renovation, you will have to measure the actual dimensions of your bedroom's walls, ceiling, floor, and windows.

PROJECT CHECKLIST

Your final project will include the following:

- a three-dimensional drawing of your renovated room, with its measurements labelled;
- a three-dimensional drawing of the two pieces of furniture you will build, labelled with measurements. You will need to label the storage unit or shelf with its volume; and
- a list that includes the type, amount, and cost of the materials needed to renovate your room and build your two items of furniture.

RESEARCH YOUR RENOVATION

T

Drawing is the next step in your bedroom renovation. Sketch a three-dimensional picture of your bedroom and label its floor, ceiling, and walls with its measurements. Next, calculate the surface area of each of these components. Record these values on your drawing.

Then, make a three-dimensional sketch of the two pieces of furniture you will build. Label each one with its dimensions. On the list of types of materials you made in the first section of the project, record the amount of each material you will need. Next, visit a hardware store or look online to research how much it will cost to buy these materials. Record this information. You may find that you have to purchase more materials than you will use. For example, you may have to buy two cans of paint when you only need one-and-a-half cans of paint to cover your walls.

After you have decided on the types of materials you will use to redo your room, you can update your drawing by adding colour and texture that reflects these materials to your drawing. You can also add in furnishings.

COMPLETE YOUR RENOVATING AND REDECORATING PLANS

Are you satisfied with how your room looks? Did it cost more or less than you thought to change its appearance? To complete your project, submit the list that includes the different materials used in your renovation, the amounts, and the costs to your teacher. Submit the three-dimensional drawings of your room and the two pieces of furniture you chose to build, labelled with their dimensions. Remember to label the storage unit or shelf with its volume.

BLACKLINE MASTER 3.1A

ALTERNATIVE CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research;
- the accuracy of your calculations and drawings;
- the effectiveness of your uses of technology for research and organizing;
- the creativity you brought to planning and presenting; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel? ______________________

Are there areas in which you could improve? ______________________

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 3.2A

ALTERNATIVE CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ Did you accurately measure your bedroom walls, ceiling, and floor?	
❒ Did you research and record the cost, amount, and type of materials you would need to renovate and redecorate your bedroom?	
❒ Have you produced a three-dimensional drawing of your room, labelled with its measurements?	
❒ Do you have a three-dimensional drawing of two items of furniture, labelled with their measurements?	
❒ Did you calculate the volume of your storage unit or shelf?	
❒ Did you calculate the surface area of your room?	
❒ Did you add any extra features to your room, such as a rug, light switch covers, curtains, or artwork?	
❒ Other notes	

BLACKLINE MASTER 3.9

REVIEWING PRIOR CONCEPTS

Name: ______________________________ Date: ______________________________

Order of Operations

Determine the value of each expression below using the order of operations.

1. $5^2 \times 3 - (84 - 37)$

2. $(22 - 25)^3 \div [(13 - 7) + 3]$

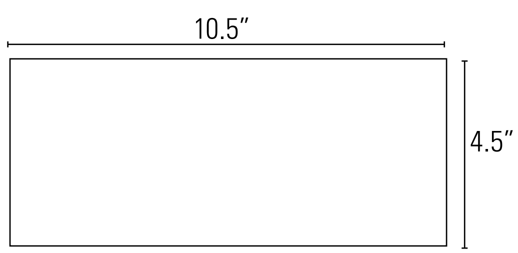
3. $\left(\frac{36}{9}\right)^2 \times 2 - 15 \div (-3)$

4. $(-4)^3 + (5 - 11)^2 \div 12 + 20$

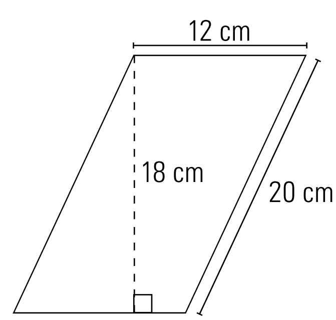
Finding the Area of Composite Figures

Find the area of the figures below.

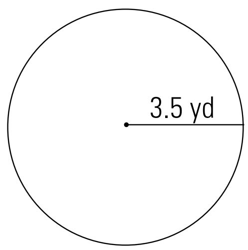
5.



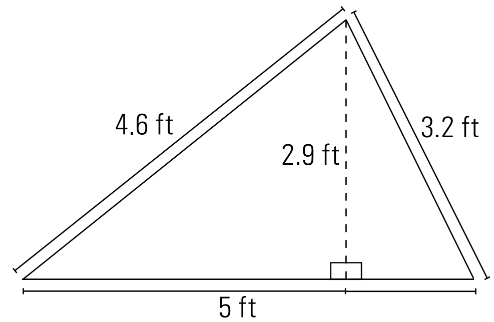
6.



7.



8.



Working with Formulas

Determine the value of the expression for the given value of each variable.

9. $4\pi r^2$ ($r = 3.4$)

10. $\frac{1}{3}\pi r^2 h$ ($r = 5.2$, $h = 8$)

11. $\pi rs + \pi r^2$ ($r = 3$, $s = 4.3$)

12. $2\pi r^2 + 2\pi rh$ ($r = 6.7$, $h = 12.3$)

Converting Measurements Within and Between the SI and Imperial Systems

Convert the given measurement into the indicated unit.

13. 4.56 km; metres

14. 56.64 yd; inches (1 yard = 36 inches)

15. 27.2 feet; cm (1 foot ≈ 30.48 cm)

16. 89.2 miles; km (1 mile = 1.609344 km)

BLACKLINE MASTER 3.9: SOLUTIONS

Order of Operations

1. $5^2 \times 3 - (84 - 37)$

   $= 25 \times 3 - 47$
   $= 75 - 47$
   $= 28$

2. $(22 - 25)^3 \div [(13 - 7) + 3]$

   $= (-3)^3 \div (6 + 3)$
   $= -27 \div 9$
   $= -3$

3. $\left(\frac{36}{9}\right)^2 \times 2 - 15 \div (-3)$

   $= 4^2 \times 2 - 15 \div (-3)$
   $= 16 \times 2 - (-5)$
   $= 32 + 5$
   $= 37$

4. $(-4)^3 + (5 - 11)^2 \div 12 + 20$

   $= -64 + (-6)^2 \div 12 + 20$
   $= -64 + 36 \div 12 + 20$
   $= -64 + 3 + 20$
   $= -41$

Finding the Area of Composite Figures

5. $A = (10.5)(4.5)$

   $A = 47.25 \text{ in}^2$

6. $A = (12)(18)$

   $A = 216 \text{ cm}^2$

7. $A = \pi(3.5)^2$

   $A \approx 38.48 \text{ yd}^2$

8. $A_1 = \frac{1}{2}(5)(2.9)$

   $A = 7.25 \text{ ft}^2$

Working with Formulas

9. $4\pi r^2$ $(r = 3.4)$

   $= 4\pi(3.4)^2$
   ≈ 145.27

10. $\frac{1}{3}\pi r^2 h$ $(r = 5.2, h = 8)$

    $= \frac{1}{3}\pi(5.2)^2(8)$
    ≈ 226.53

11. $\pi rs + \pi r^2$ $(r = 3, s = 4.3)$

    $= \pi(3)(4.3) + \pi(3)^2$
    $\approx 40.53 + 28.27$
    ≈ 68.8

12. $2\pi r^2 + 2\pi rh$ $(r = 6.7, h = 12.3)$

    $= 2\pi(6.7)^2 + 2\pi(6.7)(12.3)$
    $\approx 282.05 + 517.80$
    ≈ 799.85

Converting Measurements Within and Between the SI and Imperial Systems

13. 4.56 km; metres

    1 km = 1000 m
    4.56 km = 4560 m

14. 56.64 yd; inches (1 yard = 36 inches)

    1 yard = 36 inches
    56.64 yards = 2039.04 inches

15. 27.2 feet; cm (1 foot ≈ 30.48 cm)

    1 foot ≈ 30.48 cm
    27.2 feet ≈ 829.056 cm

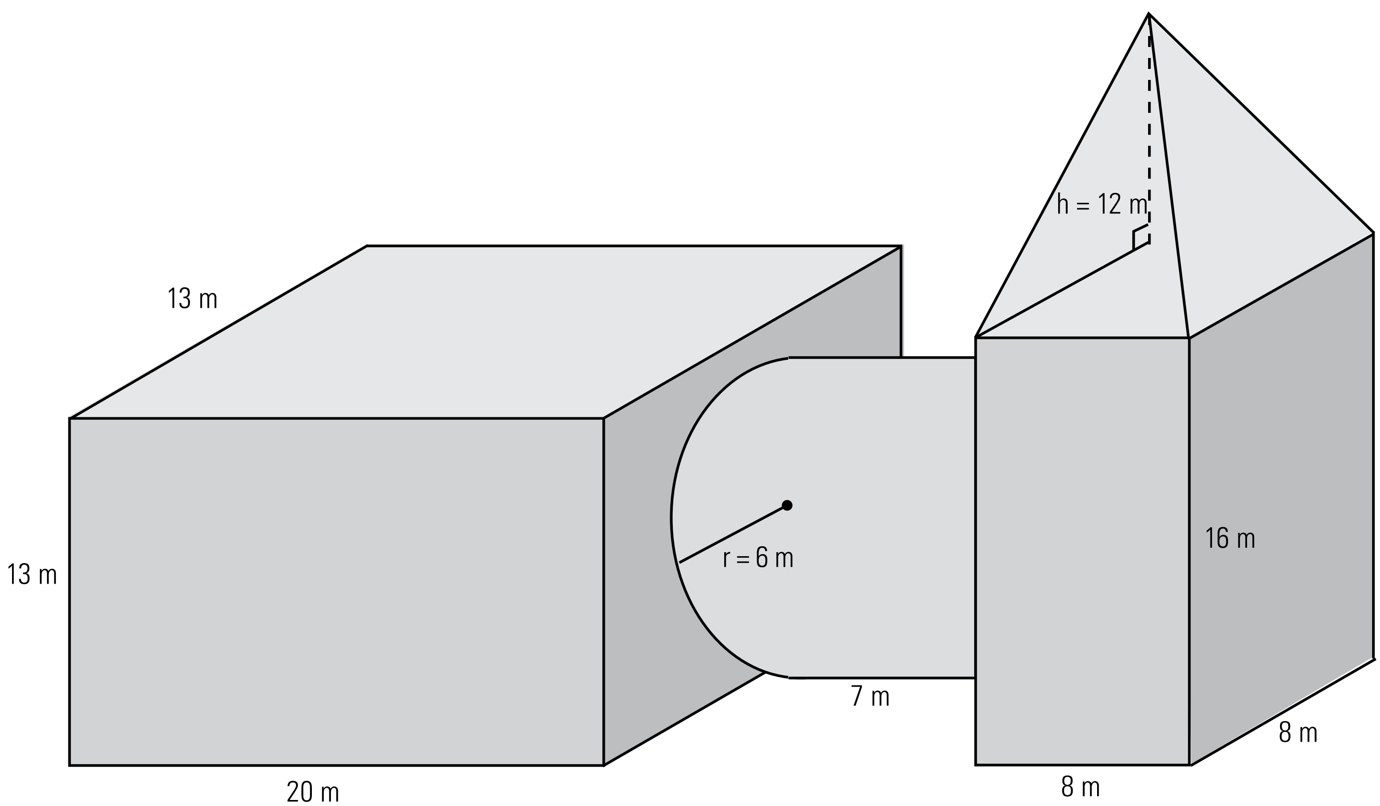
16. 89.2 miles; km (1 mile = 1.609344 km)

    1 mile = 1.609344 km
    89.2 miles ≈ 143.55 km

BLACKLINE MASTER 3.10

PRACTISE CALCULATING VOLUME OF COMPLEX OBJECTS

Name: ______________________________ Date: ______________________________



Chapter 4

Trigonometry of Right Triangles

INTRODUCTION

STUDENT BOOK, pp. 164–207

In this chapter, students learn about situations that involve more than one triangle, in both two and three dimensions. They build on their knowledge of trigonometry from grade 10, and use sine, cosine, and tangent in a variety of ways. Two- and three-triangle problem-solving strategies are developed. Students begin with distances, lengths, and angles, and progress to using right triangles to solve problems set in three-dimensional situations.

GEOMETRY, GRADES 10–12

This chart illustrates the development of the Geometry strand in the Apprenticeship and Workplace pathway through senior secondary school. The highlighted sections include the outcomes chapter 4 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop spatial sense.	**General Outcome** Develop spatial sense.	**General Outcome** Develop spatial sense.
Specific Outcome *It is expected that students will:*	**Specific Outcome** *It is expected that students will:*	**Specific Outcome** *It is expected that students will:*
Develop an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.	Solve problems that involve two and three right triangles.	Solve problems by using the sine law and cosine law, excluding the ambiguous case.
Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.		Solve problems that involve triangles, quadrilaterals, and regular polygons.
Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.		Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including translations, rotations, reflections, and dilations.
General Outcome Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.
Solve problems that require the manipulation and application of formulas related to the Pythagorean theorem and primary trigonometric ratios.		

CURRICULUM AND CHAPTER OVERVIEW

General Outcome: Geometry
Develop spatial sense.

Specific Outcome: Solve problems that involve two and three right triangles.

Section 4.1:
Solving for Angles, Lengths, and Distances

- Math on the Job
- Explore the Math
- Activity 4.1 Ancient Trigonometry: Sine Ratios
- Discuss the Ideas: Solving Problems with Right Triangles
- Activity 4.2: Conduct a Survey
- Mental Math and Estimation
- Build Your Skills
- Puzzle It Out: Stained Glass Sailboat

Section 4.2:
Solving Complex Problems in the Real World

- Math on the Job
- Explore the Math
- Mental Math and Estimation
- Discuss the Ideas: Using Trigonometry to Calculate Height
- Activity 4.3: Fabricate a Range Hood
- Build Your Skills
- The Roots of Math: Navigation in History

Chapter Project: Plan a Community-Friendly Park
Reflect on Your Learning
Practise Your New Skills

THE MATHEMATICAL IDEAS

TRIGONOMETRY OF RIGHT TRIANGLES

This chapter provides practice with the three trigonometry functions (sine, cosine, and tangent) in a variety of ways that require students to use advanced problem-solving strategies. Real-life scenarios are presented that require calculations that use two or three right-angled triangles.

Two-triangle problem-solving strategies are developed. In this chapter, students learn a variety of ways that problems can be broken down into right-angled triangles to solve. The Pythagorean theorem and good spatial sense are essential prerequisites for the problem-solving in this chapter.

Section 4.1 begins with simple distance problems in a variety of concrete formats. It expands to think about a variety of applications of triangles and ways to solve problems by breaking them into two or three triangles. In Section 4.2, students consider a three-dimensional situation and further explore two- and three-triangle situations.

T Students will need their calculators throughout the chapter to solve the trigonometry ratios for angles and distances, but must always write the correct ratio and substitute the numbers into the formulas. It is advisable for students to always identify the ratio of interest for each problem type first.

WHY ARE THESE CONCEPTS IMPORTANT?

Good spatial sense, the ability to break down complex problems, and the ability to visualize situations with multiple components are essential skills for many trades. Though students may suggest that they do not know many tradespeople who actually use multi-triangle trigonometry, the skills of breaking down a problem into a set of simpler problems that can be solved easily with known skills is an essential one for many trades, from the spatial planning of cabinetmakers to the calculations performed by metal fabricators.

PRIOR SKILLS AND KNOWLEDGE

Students will apply their knowledge of triangle geometry and sine, cosine, and tangent from previous years to a variety of multi-triangle problem-solving situations. They will also apply their knowledge of the Pythagorean theorem as they solve problems.

REVIEWING PRIOR CONCEPTS

Some students may benefit from reviewing concepts that have been covered in previous years. You may want to give some students specific review exercises in the following concepts and processes:

- the sine ratio;
- the cosine ratio;
- the tangent ratio; and
- Pythagorean theorem.

Blackline Master 4.9 contains review questions and solutions. It can be found at the end of this chapter of the teacher resource (p. 284).

PLANNING CHAPTER 4

This chapter will take one to two weeks of class time to complete. Class period estimates are based on a class length ranging from 60 to 75 minutes. Actual time may vary depending on each class's needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	165	Introduce the Project: Plan a Community-Friendly Park	20 minutes	Blackline Master 4.1 (p. 264)
4.1	166	Math on the Job: Surveyor	10 minutes	
4.1	167 168 170	Explore the Math Example 1 Activity 4.1: Ancient Trigonometry: Sine Ratios	1 class	 Protractor, calculator Blackline Master 4.4, 4.5 (pp. 267–268)
4.1	171	Discuss the Ideas: Solving Problems with Right Triangles	15 minutes	
4.1	172 177	Examples 2, 3 Activity 4.2: Conduct a Survey	1 class	 Tape measure, compass, notebook, pencil, calculator
4.1	177	Mental Math and Estimation	15 minutes	
4.1	177	Build Your Skills	1 class	Calculator
4.1	183	Project: Map Your Park	45 minutes	Blackline Master 4.3 (p. 266)
4.1	184	Puzzle It Out: Stained Glass Sailboat	30 minutes	Blackline Master 4.6, 4.7 (pp. 269–270)
4.2	185 185	Math on the Job: Airline Pilot Explore the Math	1 class	
4.2	186 189	Examples 1, 2 Mental Math and Estimation	30 minutes	
4.2	190 191	Discuss the Ideas: Using Trigonometry to Calculate Height Examples 3, 4	45 minutes	Notebook, pencil, calculator
4.2	194	Activity 4.3: Fabricate a Range Hood		Cardboard, scissors, pencils, erasers, protractor, ruler
4.2	195	Build Your Skills	1 class	
4.2	200	The Roots of Math: Navigation in History	30 minutes	Blackline Master 4.8 (p. 272)
4.2	201	Project: Research and Build Your Outdoor Theatre	1 class	Blackline Master 4.2 (p. 265)
4.2	201 202	Reflect on Your Learning Practise Your New Skills	1 hour	
		Chapter Test (p. 258 of this resource)	1 hour	

PLANNING FOR ASSESSMENT

Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Discuss progress of chapter project to determine if students need clarification or extra help. • Ask students to explain the process they used to arrive at a solution, to determine if they are performing the correct steps and calculations. • At the end of a lesson, ask students to write down one item they would like to review. Collect their feedback and review accordingly. • Display, in poster format, samples of strong and weak responses to guide student work.	• Monitor project work (both in groups and individually). • Observe student participation in discussions. • Allow students to self-monitor their work by checking solutions with a partner. Encourage partners to explain to each other how they arrived at a solution. • Observe interaction of individuals in groups.
Assessment as Learning	• After completing Build Your Skills, have students record the questions they had difficulty with. • After completing a lesson, discuss as a class the lesson's objective and ask for feedback on areas students would like to review. • Ask students to create a two-or three-triangle problem and its solution.	• Check homework. • Listen to ideas, build on ideas, and encourage each student to participate in class discussions. • Ask students to explain to the class how they solved a question. • Talk about what was learned and how it can be used. • Provide feedback on assignments and quizzes.
Assessment of Learning	• Give a quiz and set aside time to provide feedback to students on their quiz results. • After providing feedback, allow students to revise an assignment. • Record correct and incorrect responses to homework questions.	• Use results from quizzes or homework assignments to determine material that needs to be reviewed. • Presentations of projects may include peer, self, and teacher evaluation.
Learning Skills/ Mathematical Disposition	• Throughout the unit, observe for proper mathematical discussion. Choose a mathematical term to explore during a lesson. • Observe for student understanding of mathematical relationships and procedures—particularly, whether students can break a problem down into parts.	• Keep a checklist of student interaction. • Inform students that part of their grade will be based on class participation. • Use peer evaluations and ratings to determine classmates' participation and input on projects.

PROJECT — PLAN A COMMUNITY-FRIENDLY PARK

GOALS: In this chapter project, students apply their knowledge of trigonometry to design elements of a new, accessible park. The project can be adapted to suit a rural area with a small population by having students design an educational nature walk. Students will use calculations involving two and three right-angle triangles to determine the area of the park and measurements related to their theatre, such as angle of depression and inclination, as well as seat distance from the stage.

OUTCOME: Students compile a scale map of the park as well as a scale model of an outdoor theatre that will be located within the park. If you would like to devote more time to this segment of the chapter, you can have students create a diorama of their project.

PREREQUISITES: To complete this project, students need an understanding of proportional reasoning. Students will also need to understand and calculate trigonometric ratios. They will need to be able to measure angles, calculate the slope of an incline, as well as calculate angle of depression and angle of inclination.

To complete the project, students will also need to be able to visualize creative designs and understand how to use right angle triangles to measure aspects of a design.

ABOUT THIS PROJECT: Students can work alone, in pairs, or in a group to complete the project. They can begin with an artistic sketch of their park and refine it through calculations regarding path angles, path slopes, the area of the park, and their outdoor theatre's dimensions. The map can be drawn on a large sheet of paper and the theatre can be built with foam, paper, or clay. The following website gives instructions on how to build model buildings.

www.ehow.com/how_5102046_build-scale-model-building.html

If you choose to have students build a diorama, they could use papier-maché or modelling clay. The website below provides instructions on building papier-maché models.

www.familycrafts.about.com/cs/papermache/a/blpmpastes.htm

An alternative project, "Survey a Youth Wilderness Base Camp," is included on pp. 273–283.

1. Start to plan

STUDENT BOOK, p. 165

To begin, you may want to have students discuss park layouts that they know about. Are these parks accessible by people of different ages and physical abilities? Why or why not? You can brainstorm or record these features. Remind students to include enough space for people to gather and travel between the features.

As homework, ask students to pace off and record reasonable distances between their park pathways and seating for an open-air theatre. Also ask them to research and record the slopes of wheelchair accessible paths. They will need this information for the next section of the project.

Some urban and rural parks, recognized for their features, are listed below. If students are having difficulty thinking of features to include in the park, you can refer them to these websites.

Banff National Park, Banff, Canada

www.discoverlakelouise.com/

Jardin du Luxembourg, Paris, France

www.aviewoncities.com/paris/jardinduluxembourg.htm

Beihai Park, Beijing, China

www.travelchinaguide.com/attraction/beijing/beihai.htm

2. Map your park

STUDENT BOOK, p. 183

At the beginning of this section, you may want to review how to make a scale drawing. Using the Blackline Master 4.3 (p. 266), ask students to calculate the area of their park. Remind students that as they design the park, they are to incorporate the land's natural features into their design. Next, hand out large pieces of paper for students to draw their map on, and have them choose a scale for their map. (If you prefer, you can assign a scale of one centimeter equals 10 metres of parkland to make the map fit on a piece of paper that is 40 cm by 100 cm.)

Have students place their park features and paths and measure and label their paths with angles and slopes. In order to be accessible by people using mobility scooters, the paths must not turn at angles less than 60 degrees.

The maximum slope of a wheelchair ramp is generally 1:12, or one inch of rise to 12 inches of slope. Even this slope may be difficult to navigate, so no slopes should be greater than this. Let students know that a comfortable slope for wheelchairs and scooters to navigate is 1:20.

To calculate the area of the park, encourage students to try breaking the shape of park down into different shapes, including right triangles. One right triangle is formed with the 300 m measurement as the hypotenuse. A second right triangle has the 650 m measurement as the hypotenuse. A rectangle with two sides measuring 410 m is the third shape that composes the park

For the first triangle, use the sine ratio to find the length of the leg opposite to the 37° angle, x.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 37° = \frac{x}{300}$$

$$300 \sin 37° = x$$

$$180.5 \approx x$$

Use the Pythagorean theorem to find the length of the leg adjacent to the 37° angle, y.

$$a^2 + b^2 = c^2$$

$$y^2 + 180.5^2 = 300^2$$

$$y^2 = 300^2 - 180.5^2$$

$$y = \sqrt{57419.5}$$

$$y \approx 239.6$$

Find the first triangle's area.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(180.5)(239.6)$$

$$A = 21\,623.9$$

Next, solve the second triangle. Add 180.5 to 410 to find the length of the long leg of the second triangle.

$$180.5 + 410 = 590.5$$

Use the Pythagorean theorem to find the length of the second leg, b.

$$a^2 + b^2 = c^2$$

$$590.5^2 + b^2 = 650^2$$

$$b^2 = 650^2 - 590.5^2$$

$$b = \sqrt{73809.75}$$

$$b \approx 271.7$$

The second leg measures 271.7 m.

Find the area of the second triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(271.7)(590.5)$$

$$A = 80\,219.4$$

Find the area of the rectangle.

$$A = \ell w$$

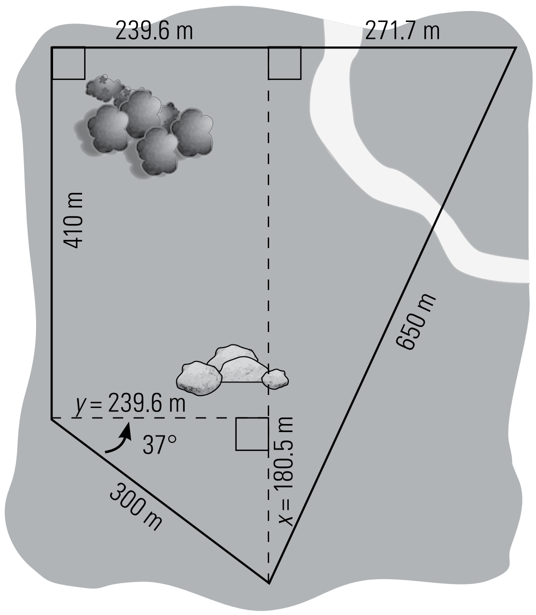
$$A = 410 \times 239.6$$

$$A = 98\,236$$

Add the three areas to find the total area of the park.

$$21\,623.9 + 80\,219.4 + 98\,236 = 200\,079.3$$

The area is 200 079.3 m^2.



3. Research and build your outdoor theatre

STUDENT BOOK, p. 201

For the semi-circular theatre design, you may have to review some circle geometry with students. Mention that the seating is arranged in a semi-circle and that the radius from the centre of the circle to the edge will be the same all the way around, or that the diameter times pi will give the circumference of the circle.

If you have students research and build their outdoor theatres in class, you may want to divide this section of the project over two or more periods, with one used for researching theatre dimensions and one or more for building the theatre. Seats in Roman theatres were built at around 34 to 39-degree angles. Modern theatres also have seats built at this angle. One way students can make their theatres accessible to people with limited physical mobility is by providing floor seating in front of the stage.

After students have performed this research and chosen the angle and slope for the theatre's seating, you can demonstrate how students can use a large triangle with the same slope as a guide when they are building the theatre. If one end of the triangle's hypotenuse is placed at the base of the stage and the seats line up along the triangle's hypotenuse, the slope of the seating will be correct and consistent.

After students have completed their theatre, have them sketch it and calculate and record the angle of inclination or depression from a seat that is at the back, middle, and front of the theatre to the centre of the front edge of the stage. Also have students calculate and record how far each of the three seats is from the centre of the edge of the stage.

ASSESSING THE PROJECT

1. Start to plan

- Decide as a class, or by yourself as a teacher, what the assessment tools will be and explain this to the students before they begin. What percentage of the mark will be for creativity of design? How much will be assigned to accuracy of calculations? How much for planning, compared to execution?
- Students often become more engaged with a project when they have a hand in creating the assessment tool.

2. Map your park

- Have students calculate the area of the park using the Blackline Master 4.3 (p. 266). After they have calculated the area of the park, students can begin to map their park. As practise, you may want to have students draw a rough sketch of their park before drawing their map. If students choose their own scale for their map, ensure that the scale is appropriate before they have completed it.
- Emphasize to students that the angles of the pathways have to make sense with the distances given and should be drawn accurately and to scale. As students draw

their maps, you can circulate and look at the different features they have chosen to include within their park.

3. **Research and build your outdoor theatre**
   - Ensure that students have used sound trigonometric calculations in their presentation of their theatre plans. Their sketch of the theatre should include angle of depression, inclination, and distance calculations. All of the measurements in the sketch of the theatre should match the scale model. Make sure students have drawn the map of their park to scale.

PROJECT ASSESSMENT RUBRIC				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Explanations show an understanding of three-dimensional measurement	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are often incomplete or somewhat confusing	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and thorough
Procedural Understanding				
Accurately: ▪ chooses an appropriate scale for map ▪ draws a scale map ▪ determines appropriate dimensions for scale theatre and seating ▪ designs a scale theatre and seating ▪ calculates path angles ▪ calculates angle of depression and inclination for theatre seating, as well as seat distance from stage ▪ researches accessible path slopes and historical theatre design	limited accuracy; major errors or omissions For example: • map scale is not appropriate • map is not drawn to scale • theatre dimensions are not appropriate or accurate • theatre is not created • path angles are missing or calculated incorrectly • calculations are incorrect or missing • research is missing or incorrect • project is incomplete	partially accurate; some errors or omissions For example: • map scale is appropriate • two or three map features are not drawn to scale • theatre dimensions are mostly to scale • path angles are calculated correctly • calculations are present, with one or two errors • project could use more work to ensure calculations are correct, and theatre/map conform to a scale	generally accurate; few errors or omissions For example: • map scale is appropriate • map features are drawn to scale • theatre dimensions are appropriate and drawn to scale • path angles are calculated correctly • calculations are complete and correct • project is complete and correct	accurate and precise; very few or no errors For example: • map scale is appropriate • map features are designed to be community-friendly and are drawn to scale • theatre dimensions are appropriate and to scale; model is creative • path angles are calculated correctly • calculations are complete and correct • project is complete, correct, and creative
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

4.1 Solving for Angles, Lengths, and Distances

TIME REQUIRED FOR THIS SECTION: 2–3 CLASSES

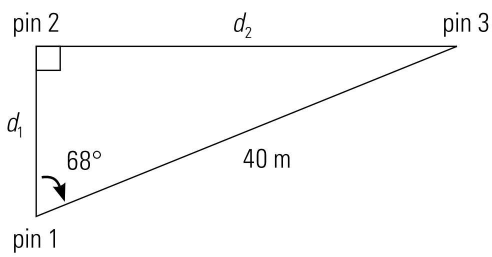
STUDENT BOOK, pp. 166–184

MATH *ON THE JOB*

STUDENT BOOK, p. 166

Start this chapter by reviewing what students have learned about trigonometry in past years. They should know how to label the sides of a triangle as opposite, adjacent, and hypotenuse, and should know the sine, cosine, and tangent ratios. Remind students that the angles of a triangle have a sum of 180 degrees.

Discuss which trigonometric functions will allow them to find the distances.



SOLUTION

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 68° = \frac{d_1}{40}$$

$$40(\cos 68°) = \left(\frac{d_1}{40}\right)40$$

$$40(\cos 68°) = d_1$$

$$14.98 \approx d_1$$

Pin 2 is 14.98 m away from pin 1.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 68° = \frac{d_2}{40}$$

$$40(\sin 68°) = \left(\frac{d_2}{40}\right)40$$

$$40(\sin 68°) = d_2$$

$$37.09 \approx d_2$$

Pin 3 is 37.09 m away from pin 2.

EXPLORE THE MATH

STUDENT BOOK, p. 167

Continue the discussion about triangles in the real world. If your students are tactile, consider taking the class time to cut out cardboard triangles and brainstorm different real-life situations that arise involving triangles. Have students do this in small groups and draw up lists of scenarios, then present these to the class. Have students discuss how they might solve multi-triangle problems. This will get their problem-solving ideas flowing in a low-stress context in which they can begin to think laterally, building on prior knowledge.

If they become stuck, have them flip through the chapter, looking at diagrams and examples, and have them try to arrange their tactile triangles in those patterns and discuss the solutions.

After students have examined the right triangles they have drawn, ask them to share their conclusions. The minimum information needed to solve a right triangle is two side lengths, or one side length and one angle (other than the right angle).

If students need help recalling the trigonometric ratios, review these with them.

$$\text{sine ratio} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine ratio} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent ratio} = \frac{\text{opposite}}{\text{adjacent}}$$

ACTIVITY 4.1

ANCIENT TRIGONOMETRY: SINE RATIOS

STUDENT BOOK, p. 170

Students should find that the ratios of the sides are proportional to the angles. The radius OA is the same as radius OB or OC, and so the sine ratio of half of the central angle is given by the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.

Students should be able to visualize the right-angled triangle within the circle at this level.

Blackline Master 4.4 (p. 267) provides circle outlines for this activity. An extension activity is provided on Blackline Master 4.5 (p. 268).

SOLUTION

Answers to the charts will vary.

KINESTHETIC LEARNERS: Kinesthetic learners may grasp this activity better if they are given a coffee lid, pins, string, a protractor, and a ruler. The coffee lid often has a divot marking the centre of the circle. Tie a knot in the string and pin it to the centre.

Have them create a variety of radii/chord pairs with the string. Use the pins to attach the string at the edge of the lid, then across to the other side. A second piece of string can be pinned on the third side of the triangle.

Students could mark the string to compare the different triangles formed by different angles, and thus help them understand the ratios involved.

DISCUSS THE IDEAS

SOLVING PROBLEMS WITH RIGHT TRIANGLES

STUDENT BOOK, p. 171

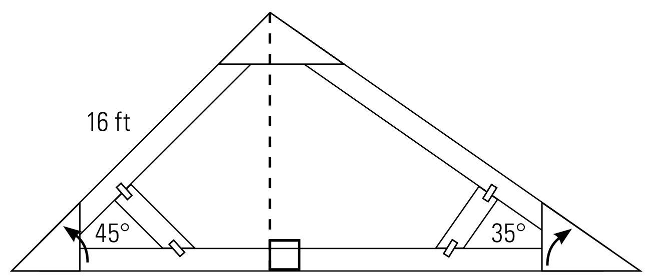
Have students work in pairs. They may have an easier time visualizing the practical questions posed if they use the cardboard triangles they made from the earlier activity in Explore the Math.

SOLUTIONS

1. Because the interior angles of a triangle must always add up to 180 degrees, you can find the size of the third angle.

$$45 + 35 + x = 180$$
$$x = 180 - 45 - 35$$
$$x = 100$$

2. The peak angle of the truss is 100 degrees, not 90 degrees, so this is not a right-angle triangle.
3. By dropping a vertical line from the peak to the base, the truss can be represented by two right-angle triangles.



4. No, because there is only one piece of information for the right-hand triangle.
5. You know one angle and one side length, therefore you can solve the triangle.
6. $$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 45° = \frac{\text{height}}{16}$$

$$16 \times \sin 45° = \text{height}$$

$$11.3 \text{ ft} \approx \text{height}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 45° = \frac{\text{baseL}}{16}$$

$$16 \times \cos 45° = \text{baseL}$$

$$11.3 \text{ ft} \approx \text{baseL}$$

7. You could also use the Pythagorean theorem to solve the triangle. Students could also use their knowledge of isosceles triangles to recognize that the base and the height of the left-hand triangle would be the same.

$$(\text{baseL})^2 + (\text{height})^2 = (\text{hypotenuse})^2$$

$$(\text{baseL})^2 = 16^2 - 11.3^2$$

$$\text{baseL} \approx 11.3 \text{ ft}$$

8. Yes. Now that the height has been calculated from the left-hand triangle, and given that the two triangles share this side in common, you have an angle and a length for the right-hand triangle. This is enough information to solve the triangle.

9.

$$\tan 35° = \frac{\text{height}}{\text{baseR}}$$

$$\tan 35° = \frac{11.3}{\text{baseR}}$$

$$\frac{11.3}{\tan 35°} = \text{baseR}$$

$$\text{baseR} \approx 16.1 \text{ ft}$$

$$\sin 35° = \frac{11.3}{\text{hypotenuse}}$$

$$\text{hypotenuse} = \frac{11.3}{\sin 35°}$$

$$\text{hypotenuse} \approx 19.7 \text{ ft}$$

10.

$$\text{hypotenuse}^2 = 11.3^2 + 16.1^2$$

$$\text{hypotenuse}^2 = 127.69 + 259.21$$

$$\text{hypotenuse}^2 = 386.90$$

$$\text{hypotenuse} \approx 19.7 \text{ ft}$$

The length calculated using the Pythagorean theorem agrees with the length calculated using the sine ratio.

11. The total base length of the irregular triangle is the sum of the two base lengths of the right triangles.

Base length = baseL + baseR

Base length = 11.3 + 16.1

Base length = 27.4 ft

12. You can solve a non-right triangle by dividing it into two right triangles using the method above if you know two angles and one length.

You can provide students with additional practise in solving problems with multiple triangles using Blackline Master 4.10 (p. 287).

ACTIVITY 4.2
CONDUCT A SURVEY

STUDENT BOOK, p. 177

This problem will be particularly appealing to kinesthetic and tactile learners. Allow groups to choose which members physically measure the spaces, draw a scale map, and identify the shapes that make up the plot. Encourage all members to perform the calculations, comparing results as a self-check.

SOLUTION

To solve this problem, students will need to identify shapes within their chosen plot: squares, rectangles, and especially triangles. To make use of the skills in this chapter, they should divide triangles to form right-angle triangles.

Because of the irregular shape of the surveyed plot, students may have difficulty determining how to calculate the area. You may want to discuss that there are two ways of finding the area of an irregular shape:

- You can divide the irregular shape into regular shapes and add up the areas of the parts; or
- You can draw a rectangle around the irregular shape and calculate its area, then find the regular shapes between the rectangle and the irregular shape. Calculate the areas of the regular shapes and subtract these from the area of the rectangle.

Extension

T Technology can be used in an extension to this activity. Students can use aerial maps on the internet to calculate the perimeter and area of a building or property in the community.

Aerial maps are available on the following website: http://maps.google.ca

Students can enter the address of a building or property that they wish to investigate. They may choose to examine, for example, the dimensions of the school property, the block they live on, or a community park.

Using the scale provided on the aerial image, students can find the dimensions of their chosen location, and then use this information to calculate the perimeter and area.

Mental Math and Estimation

STUDENT BOOK, p. 177

SOLUTION

In a 45-45-90 triangle, the two sides adjacent to the 90-degree angle are equal (it is an isosceles right triangle). Choosing either of the 45-degree angles:

$$\tan 45° = \frac{\text{opposite}}{\text{adjacent}}$$

Because the side lengths are equal,

$$\tan 45° = \frac{\text{opposite}}{\text{opposite}}$$

$$\tan 45° = \frac{\text{adjacent}}{\text{adjacent}}$$

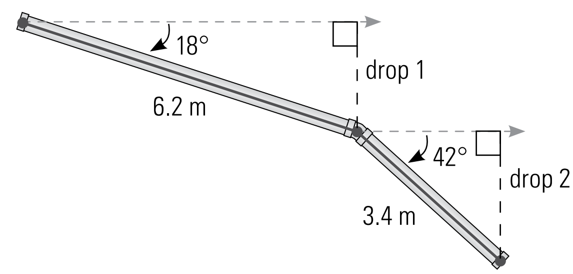
$$\tan 45° = 1.000$$

So tan 45° is exactly equal to 1.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, pp. 177–182

SOLUTIONS

1. 

$$\frac{drop\ 1}{6.2} = \sin 18°$$

$$drop\ 1 = 6.2 \times \sin 18°$$

$$drop\ 1 \approx 1.9 \text{ m}$$

$$\frac{drop\ 2}{3.4} = \sin 42°$$

$$drop\ 2 = 3.4 \times \sin 42°$$

$$drop\ 2 \approx 2.3 \text{ m}$$

total drop = 1.9 + 2.3

total drop = 4.2

The total drop is 4.2 m.

2. The angles are equal if the sine ratios are equal.

$$\sin a = \frac{13}{25}$$

$$\sin a = 0.52$$

$$a \approx 31.3°$$

$$\sin b = \frac{23}{43}$$

$$\sin b = 0.53$$

$$b \approx 32.3°$$

These angles are close, but not exactly the same, so the reflection is not quite perfect. Jack probably needs to fine-tune the adjustments.

3. $\tan 27° = \dfrac{\text{opp}}{\text{adj}}$

$\tan 27° = \dfrac{4.8}{a}$

$4.8 = \tan 27° \times a$

$\dfrac{4.8}{\tan 27°} = a$

$9.4 \approx a$

Similarly,

$\tan \theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 37° = \dfrac{4.8}{b}$

$4.8 = b \tan 37°$

$\dfrac{4.8}{\tan 37°} = b$

$6.4 \approx b$

$\tan \theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 47° = \dfrac{4.8}{c}$

$4.8 = c \tan 47°$

$\dfrac{4.8}{\tan 47°} = c$

$4.5 \approx c$

The stakes are about 9.4 m, 6.4 m, and 4.5 m from the tree.

4. $\dfrac{h_1}{440} = \sin 67°$

$h_1 = \sin 67° \times 440$

$h_1 \approx 405$

$\dfrac{h_2}{300} = \sin 35°$

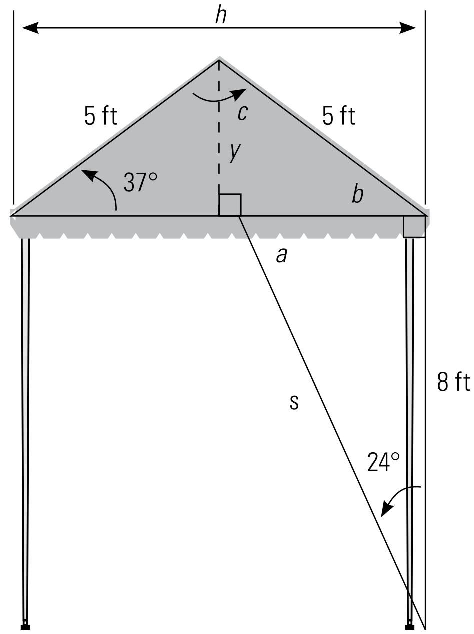
$h_2 = \sin 35° \times 300$

$h_2 \approx 172$

Total elevation: 1400 + 405 + 172 = 1977

The elevation at the top of the hill is 1977 m.

5.



$a = 90° - 24°$

$a = 66°$

$b = 37°$

$c = 180° - 37° - 37°$

$c = 106°$

$\sin 37° = \dfrac{y}{5}$

$y = 3 \text{ ft}$

$\cos 24° = \dfrac{8}{s}$

$s = \dfrac{8}{\cos 24°}$

$s \approx 8.8 \text{ ft}$

$5^2 = \left(\dfrac{h}{2}\right)^2 + y^2$

$h = 2\sqrt{5^2 - 3^2}$

$h = 8 \text{ ft}$

6. $\dfrac{h}{50} = \sin\ 40°$

$h = \sin 40° \times 50$

$h \approx 32$

$$\tan x = \frac{32}{30}$$

$$x = \tan^{-1}\left(\frac{32}{30}\right)$$

$$x \approx 47°$$

The angle of the downward slope, x, is 47°.

7. a) First, solve the triangle.

Side a: 1700 − 1000 = 700 mm

Next, use the tangent ratio to solve side b.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\frac{b}{700} = \tan 68°$$

$$b = 700 \tan 68°$$

$$b \approx 1732.56$$

Side b is 1732.56 mm long.

Next, find the triangle's area.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 700 \times 1732.56$$

$$A = 606\,396$$

The triangle's area is 606 396 mm^2.

Subtract the area of the triangle from the rectangle.

$$A = (1700 \times 2400) - 606\,396$$
$$A = 3\,473\,604 \text{ mm}^2$$

Divide by 1 000 000 to convert to square metres.

$$3\,473\,604 \div 1\,000\,000 = 3.473\,604$$

The area is approximately 3.5 m^2.

b) To find the area of the sloped face, first use the cosine ratio to calculate the length of c.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\frac{700}{c} = \cos 68°$$

$$c = \frac{700}{\cos 68°}$$

$$c \approx 1868.63$$

The length of c is 1868.63 mm.

Multiply the base by the height.

$$A = 1200 \times 1868.63$$
$$A \approx 2\,242\,356$$

Divide by 1 000 000 to convert to square metres.

$$2\,242\,356 \div 1\,000\,000 = 2.242\,356$$

The area of the sloped face is 2.2 m^2.

8. a) Use the tangent ratio to find the lengths. Begin with EK.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 30° = \frac{\text{EK}}{4}$$

$$4 \tan 30° = \text{EK}$$

$$2.31 \approx \text{EK}$$

Length JD:

$$\frac{\text{JD}}{8} = \tan 30°$$

$$\text{JD} = \tan 30° \times 8$$

$$\text{JD} \approx 4.62$$

Length IC:

$$\frac{\text{IC}}{12} = \tan 30°$$

$$\text{IC} = \tan 30° \times 12$$

$$\text{IC} \approx 6.93$$

Length HB:

$$\frac{\text{HB}}{16} = \tan 30°$$

$$\text{HB} = \tan 30° \times 16$$

$$\text{HB} \approx 9.24$$

Length GA:

$$\frac{\text{GA}}{20} = \tan 30°$$

$$\text{GA} = 20 \tan 30°$$

$$\text{GA} \approx 11.55$$

ROOF TRUSS DIMENSIONS			
Segment	*Horizontal length (ft)*	*Segment*	*Vertical length (ft)*
FK	4	KE	2.31
FJ	8	JD	4.62
FI	12	IC	6.93
FH	16	HB	9.24
FG	20	GA	11.55

b) To calculate the length of AF, use the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 30^\circ = \frac{20}{\text{AF}}$$

$$\frac{20}{\cos 30^\circ} = \text{AF}$$

$$23.09 \approx \text{AF}$$

AF must be 23.09 feet long.

9. a) To find David's bearing, use the tangent ratio to calculate angle A.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \text{A} = \frac{43.46}{50}$$

$$\text{A} = \tan^{-1}\left(\frac{43.46}{50}\right)$$

$$\text{A} \approx 41^\circ$$

David is traveling on a bearing of 41°.

b) Calculate angle B by subtracting the new heading from 180°.

$180^\circ - 130^\circ = 50^\circ$

Next, let x be the west distance between B1 and B2. Use the tangent ratio to find the distance between the two points.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

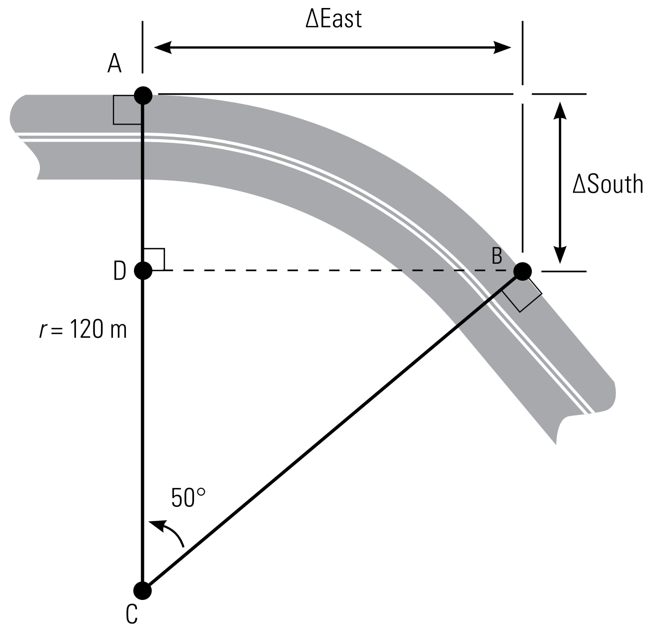
$$\tan 50^\circ = \frac{x}{50}$$

$$50 \tan 50^\circ = x$$

$$59.59 \approx x$$

B2 is about 59.59 m west of B1.

10. First, draw point D, which creates the right angle triangles ABD and BCD. AD is equivalent to the distance south and BD is equivalent to the distance east.



The hypotenuse BC and the angle C are known so triangle BCD can be solved.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 50^\circ = \frac{\text{BD}}{120}$$

$$120 \sin 50^\circ = \text{BD}$$

$$91.93 \approx \text{DB}$$

DB is 91.93 m long.

Use the cosine ratio to solve for CD.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 50^\circ = \frac{\text{CD}}{120}$$

$$120 \cos 50^\circ = \text{CD}$$

$$77.13 \approx \text{CD}$$

CD is 77.13 m long.

Use the length CD and the length of the radius to solve for AD.

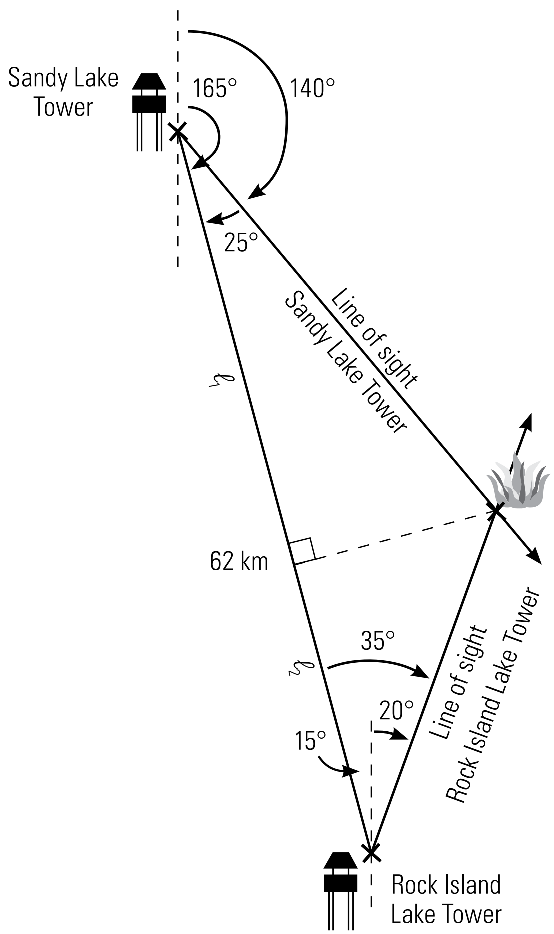
$$AD = 120 - 77.13$$

$$AD = 42.87 \text{ m}$$

Therefore, point B is 91.93 m east and 42.87 m south of point A.

Extend Your Thinking

11. The triangle formed by the two towers is not a right triangle, but you can determine all of the interior angles from the bearings.



Interior angle at Sandy Lake Tower:

$$165° - 140° = 25°$$

Interior angle at Rock Lake Tower:

$$180° - 165° + 20° = 35°$$

Divide the triangle into two right triangles where the lines of sight are the hypotenuses and they share a common side length of d.

$$\ell_1 + \ell_2 = 62$$

Use the tangent ratio for each triangle, and find expressions for ℓ_1 and ℓ_2.

For ℓ_1:

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 25° = \frac{d}{\ell_1}$$

$$\ell_1 \tan 25° = d$$

$$\ell_1 = \frac{d}{\tan 25°}$$

For ℓ_2:

$$\tan 35° = \frac{d}{\ell_2}$$

$$\ell_2 \tan 35° = d$$

$$\ell_2 = \frac{d}{\tan 35°}$$

Substitute the expressions for ℓ_1 and ℓ_2 into the equality and solve for d.

$$\ell_1 + \ell_2 = 62$$

$$\frac{d}{\tan 25°} + \frac{d}{\tan 35°} = 62$$

$$d\left(\frac{1}{\tan 25°} + \frac{1}{\tan 35°}\right) = 62$$

$$d = \frac{62}{\left(\frac{1}{\tan 25°} + \frac{1}{\tan 35°}\right)}$$

$$d \approx 17.354$$

Now the distances from the towers to the smoke can be calculated using the sine ratio.

Let Sandy Lake line of sight equal x.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 25° = \frac{17.354}{x}$$

$$\frac{17.354}{\sin 25°} = x$$

$$41.063 \approx x$$

The Sandy Lake tower is 41.063 km from the smoke.

Note: This problem can be solved by using scale drawings. If students have difficulty with the problem, pass this information on to them as a hint.

Let the Rock Island Lake line of sight equal y. Use the sine ratio to solve.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 35° = \frac{d}{y}$$

$$\sin 35° = \frac{17.354}{y}$$

$$\frac{17.354}{\sin 35°} = y$$

$$30.256 \approx y$$

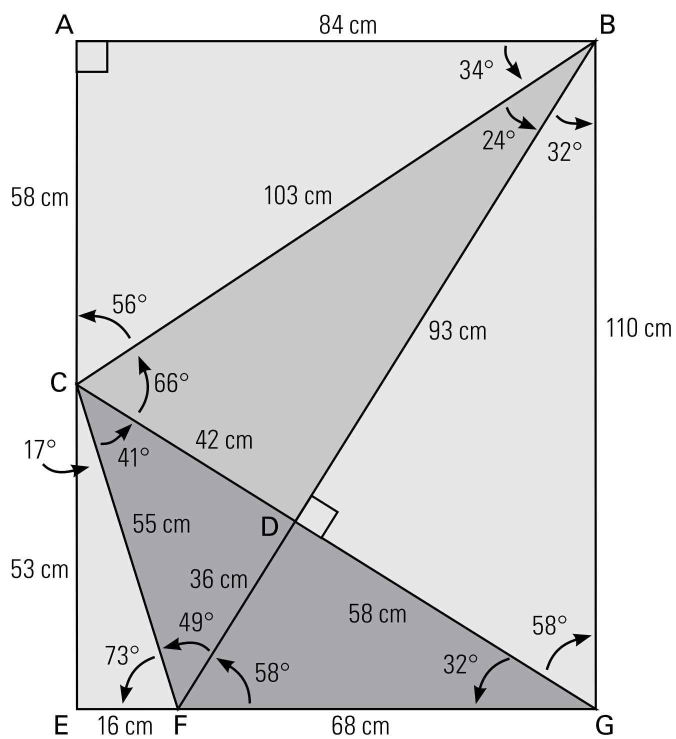
The Rock Island Lake Tower is 30.256 km from the smoke.

PUZZLE IT OUT

STAINED GLASS SAILBOAT

STUDENT BOOK, p. 184

A copy of the diagram is provided in Blackline Master 4.6 (p. 269). An additional pattern is provided in Blackline Master 4.7 (p. 270).



SOLUTIONS

$$\cos 58° = \frac{DF}{FG}$$

$$\cos 58° = \frac{DF}{68}$$

$$DF = \cos 58° \times 68$$

$$DF \approx 36 \text{ cm}$$

$$\angle FGD = 90° - 58°$$

$$\angle FGD = 32°$$

$$CF = \sqrt{CD^2 + DF^2}$$

$$CF = \sqrt{42^2 + 36^2}$$

$$CF \approx 55 \text{ cm}$$

$$\cos\angle CFD = \frac{36}{55}$$

$$\angle CFD = \cos^{-1}\left(\frac{36}{55}\right)$$

$$\angle CFD = \cos^{-1}(0.6545)$$

$$\angle CFD \approx 49°$$

$$\angle CFE = 180° - 58° - 49$$

$$\angle CFE = 73°$$

$$\angle ECF = 180° - 73° - 90°$$

$$\angle ECF = 17°$$

$$\sin\angle FCD = \frac{36}{55}$$

$$\angle FCD = \sin^{-1}\left(\frac{36}{55}\right)$$

$$\angle FCD \approx 41°$$

This is correct because 49° plus 41° plus 90° equals 180°.

$$\cos 73° = \frac{EF}{55}$$

$$EF = \cos 73° \times 55$$

$$EF \approx 16 \text{ cm}$$

This could also be solved by sin 17°.

$$\sin 73° = \frac{CE}{55}$$

$$CE = \sin 73° \times 55$$

$$CE \approx 53 \text{ cm}$$

This could also be solved by cos 17°.

$\angle BGD = 90° - 32°$

$\angle BGD = 58°$

$\angle GBD = 180° - 90° - 58°$

$\angle GBD = 32°$

$$DG = \sqrt{FG^2 - DF^2}$$

$$DG = \sqrt{68^2 - 36^2}$$

$$DG \approx 58 \text{ cm}$$

$$\tan 58° = \frac{BD}{58}$$

$$BD = \tan 58° \times 58$$

$$BD \approx 93 \text{ cm}$$

$$\cos 32° = \frac{93}{BG}$$

$$BG = \frac{93}{\cos 32°}$$

$$BG \approx 110 \text{ cm}$$

$$\tan \angle DBC = \frac{42}{93}$$

$$\angle DBC = \tan^{-1}\left(\frac{42}{93}\right)$$

$$\angle DBC \approx 24°$$

$$\tan \angle DCB = \frac{93}{42}$$

$$\angle DCB = \tan^{-1}\left(\frac{93}{42}\right)$$

$$\angle DCB \approx 66°$$

$$\cos 66° = \frac{42}{BC}$$

$$BC = \frac{42}{\cos 66°}$$

$$BC \approx 103 \text{ cm}$$

Check:

$$42^2 + 93^2 = 103^2$$

$$10\,413 \approx 10\,609$$

Yes, close enough (with rounding error).

$\angle ACB = 180° - 17° - 41° - 66°$
$\angle ACB = 56°$

$\angle ABC = 90° - 24° - 32°$
$\angle ABC = 34°$

Check:

$56° + 34° = 90°$

$$\cos 56° = \frac{AC}{103}$$

$$AC = \cos 56° \times 103$$

$$AC \approx 58 \text{ cm}$$

$$\cos 34° = \frac{AB}{103}$$

$$AB = \cos 34° \times 103$$

$$AB \approx 85 \text{ cm}$$

Checks:

$AB = EF + FG$
$85 = 16 + 68$
$85 \approx 84$

Yes, within rounding error.

$BG = AC + CE$
$110 = 58 + 53$
$110 \approx 111$

Yes, within rounding error.

4.2 Solving Complex Problems in the Real World

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 185–207

MATH *ON THE JOB*

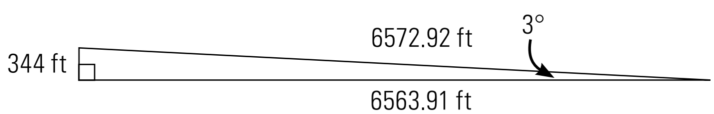
STUDENT BOOK, p. 185

After reading through the Math on the Job, mention to students that some other professionals who use trigonometry to perform their jobs include metal fabricators who design ventilation systems, plumbers who install pipes, and building inspectors.

Students will likely find it easier to solve the problem after they have visualized it by sketching it. You can have them work in groups of three or four to produce a sketch and answer the question. You can have the groups present their sketches and solutions to the class. After the groups present, you can go over the solution and request that the groups revise their answers or modify their sketches if necessary.

Help students get a feel for what their answers will be before they make any calculations.

SOLUTION



To find Judith's distance from her touchdown point, use the tangent ratio.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 3° = \frac{344}{x}$$

$$\frac{344}{\tan 3°} = x$$

$$6563.91 \approx x$$

Along the ground, Judith is about 6563.91 feet from her touchdown point.

Next, use the Pythagorean theorem to determine how far, in the air, Judith is from her touchdown point.

$$a^2 + b^2 = c^2$$

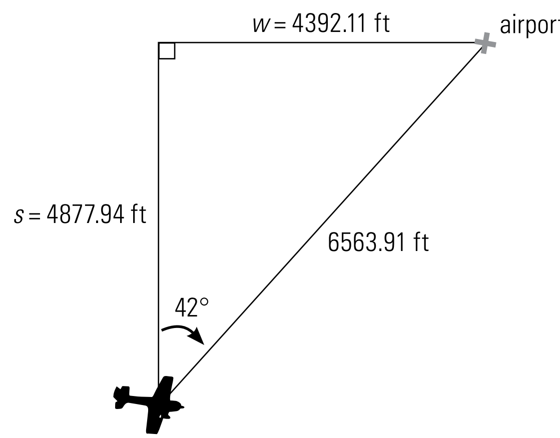
$$344^2 + 6563.91^2 = c^2$$

$$\sqrt{43\,203\,250.49} = c^2$$

$$6572.92 \approx c$$

In flight, Judith is about 6572.92 feet from her landing point.

To find how far south and west Judith is, sketch the scenario.



Next, use the sine ratio to find the west distance.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 42° = \frac{w}{6563.91}$$

$$\sin 42° \times 6563.91 = w$$

$$4392.11 \approx w$$

The distance is 4392.11 feet.

Next, use the cosine ratio to find the south distance.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 42° = \frac{s}{6563.91}$$

$$6563.91 \times \cos 42° = s$$

$$4877.94 \approx s$$

The south distance is 4877.94 feet.

EXPLORE THE MATH

STUDENT BOOK, p. 185

The distance south and west have a hypotenuse that is the same as the ground distance.

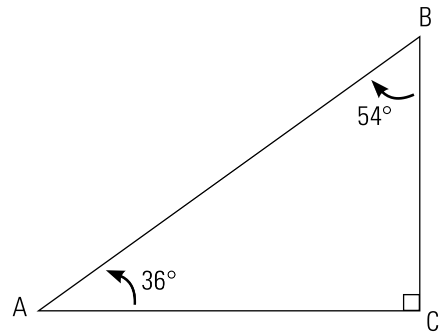
Mental Math and Estimation

STUDENT BOOK, p. 189

This question provides an opportunity to review the fundamentals of trigonometry. By this point, students should be familiar with trigonometric ratios. They should recognize that the sine of an angle equals the side of the triangle opposite the angle over the triangle's hypotenuse, and the cosine of an angle equals the side of the triangle adjacent to the angle over the triangle's hypotenuse.

Students should also realize that the two acute angles of a right triangle should add up to 90 degrees, and that 36 plus 54 equals 90. They should make the connection that the sine of 36° is equivalent to the cosine of 54°, so the cosine of 54 degrees will also be 0.5878. Both the sine and cosine values will be expressed using the same ratio.

If students have difficulty with the problem, encourage them to sketch a triangle and label its angles and sides.



SOLUTION

$$\frac{\text{CB}}{\text{hyp}} = \sin 36°$$

$$\text{CB} = \sin 36° \times \text{hyp}$$

$$\frac{\text{CB}}{\text{hyp}} = \cos 54°$$

$$\text{CB} = \cos 54° \times \text{hyp}$$

Therefore the sine of 36 degrees is equal to the cosine of 54 degrees. Since the values are equal, the cosine of 54 degrees equals 0.5878.

DISCUSS THE IDEAS

USING TRIGONOMETRY TO CALCULATE HEIGHT

STUDENT BOOK, p. 190

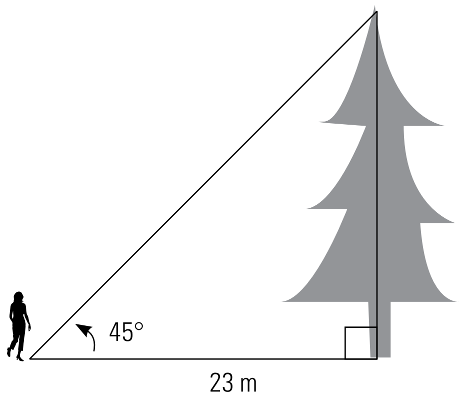
Ask students to work with a partner in this Discuss the Ideas.

After completing this Discuss the Ideas, you may want to focus on mathematical literacy by holding a short review quiz on the mathematical terminology used in the chapter. Ask students to write a definition for five to seven of the terms you think they need to review. You can also provide them with a mathematical question and ask them to solve the question, as well as write down the steps they took to arrive at a conclusion.

Quiz terms could include tangent ratio, Pythagorean theorem, angle of depression, transit, and three-triangle problem. As a class, you can read the student definitions aloud, and use them to construct a final definition for the term.

SOLUTIONS

1. To solve a right triangle, the arborist needs two pieces of information, such as the distance from the base of the tree and the angle of elevation.
2. The easiest ones for her to measure are:
   - her distance from the tree
   - the angle from a horizontal line of sight to the line of sight to the top of the tree.
3.



4. The height of the tree is the side opposite the 45-degree angle, and the distance to the tree is the side adjacent to the 45-degree angle. She can therefore use the tangent ratio to find the height of the tree:

$$\tan 45° = \frac{\text{height of tree}}{\text{distance to tree}}$$

$$\text{height of tree} = \text{distance to tree} \times \tan 45°$$

5. She knows that the tangent of a 45-degree angle is equal to 1, so she doesn't need to use a calculator. Her distance from the tree will be equal to the height above her line of sight.
6. Use the previous reasoning.

$$\text{height of tree} = \text{distance to tree} \times (1.0)$$

$$\text{height of tree} = 23 \times 1.0$$

$$\text{height of tree} = 23 \text{ m}$$

7. She must add the distance from the ground to her eye level.

ACTIVITY 4.3

FABRICATE A RANGE HOOD

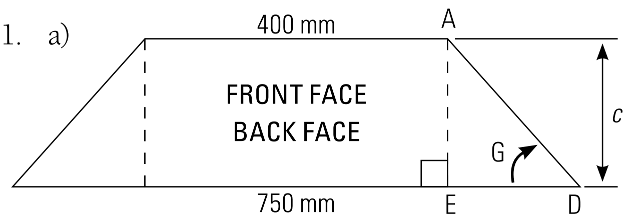
STUDENT BOOK, p. 194

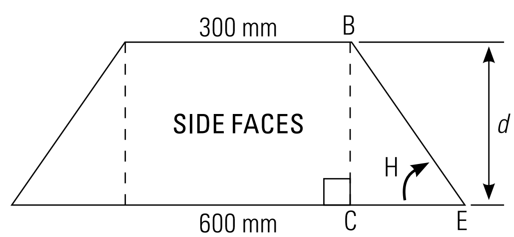
This activity provides students with an opportunity to use trigonometry, as well as make a connection to their prior knowledge about surface area, covered in chapter 3. This activity should be completed in pairs, so that all students have the opportunity to take an active role in drawing the range hood and determine its dimensions. Students who learn more productively when visuals are present may excel at this activity.

To begin, ask students to draw the table in their notebooks that they will use to record the range hood dimensions. Next, have them calculate the dimensions of the different pieces. As this activity may initially look complex, encourage students to start by calculating the dimensions of the larger pieces. Remind students that it is important to make accurate calculations and measurements so that the pieces will fit together soundly.

SOLUTION

1. a)





To solve the slope of the side faces, first solve triangle BEC.

$$b = \frac{1}{2}(600 - 300)$$

$$b = 150$$

Side b measures 150 mm.

Next, use the cosine ratio to determine c.

$$\frac{\text{adj}}{\text{hyp}} = \cos\theta$$
$$\frac{b}{c} = \cos 40°$$
$$c\cos 40° = b$$
$$c = \frac{b}{\cos 40°}$$
$$c = \frac{150}{\cos 40°}$$
$$c \approx 196$$

Side c measures about 196 mm.

Solve triangle AED by first determining the length of a.

$$a = \frac{1}{2}(750 - 400)$$
$$a = 175$$

Side a measures 175 mm.

Solve side e by using the tangent ratio.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 40° = \frac{e}{b}$$
$$b\tan 40° = e$$
$$150\tan 40° = e$$
$$126 \approx e$$

Side e measures approximately 126 mm.

Use the Pythagorean theorem to calculate d.

$$a^2 + e^2 = d^2$$
$$175^2 + 126^2 = d^2$$
$$46\,501 = d^2$$
$$\sqrt{46\,501} = d$$
$$216 \approx d$$

Next, calculate the slope of the side face using the tangent ratio.

$$\tan F = \frac{e}{a}$$
$$\tan F = \frac{126}{175}$$
$$F = \tan^{-1}\left(\frac{126}{175}\right)$$
$$F \approx 35.8°$$

The slope is approximately 35.8°.

b) Calculate the corner angles using the tangent ratio.

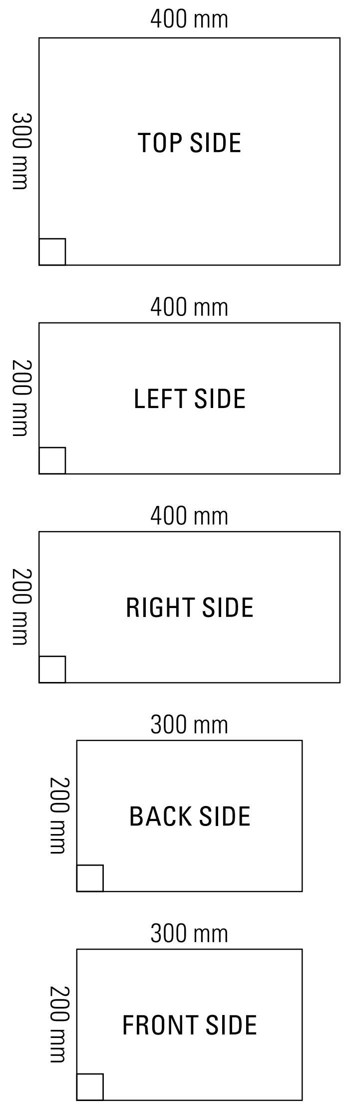
$$\tan G = \frac{c}{a}$$
$$\tan G = \frac{196}{175}$$
$$G = \tan^{-1}\left(\frac{196}{175}\right)$$
$$G \approx 48.2°$$

Next calculate the angle at H.

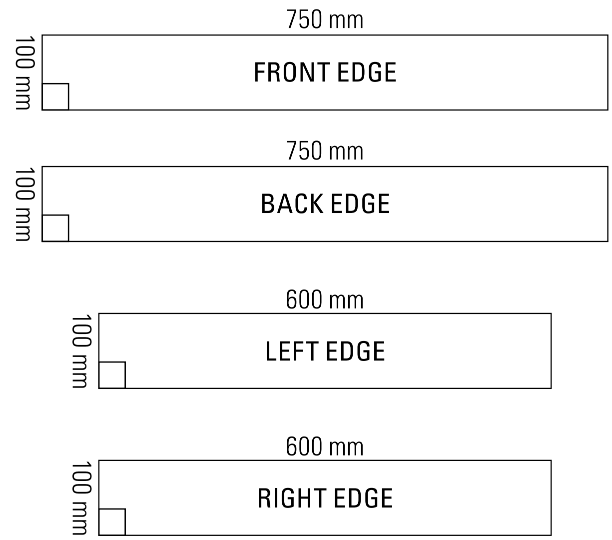
$$\tan H = \frac{d}{b}$$
$$\tan H = \frac{216}{150}$$
$$H = \tan^{-1}\left(\frac{216}{150}\right)$$
$$H \approx 55.2°$$

The angle at G is 48.2° and the angle at H is 55.2°.

2. Top of hood:

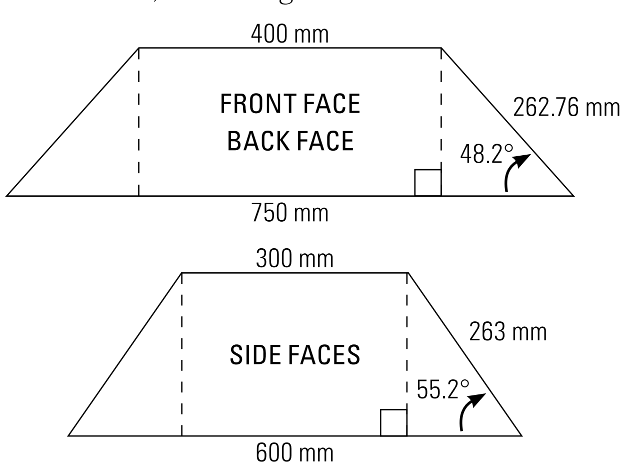


Bottom of hood:



Sloped section of hood:

Each of the faces is a trapezoid. The front face has a base length of 750 mm, a top length of 400 mm, and a height *c*. The side face has a base length of 600 mm, a top length of 300 mm, and a height *d*.



3.

RANGE HOOD DIMENSIONS		
Dimension	*Formula Used*	*Calculated Value*
a	$\frac{1}{2}(750 - 400)$	175 mm
b	$\frac{1}{2}(600 - 300)$	150 mm
c	$\cos 40° = \frac{b}{c}$	196 mm
d	$a^2 + e^2 = d^2$	216 mm
e	$\tan 40° = \frac{e}{b}$	126 mm
F	$\tan F = \frac{e}{a}$	35.8°
G	$\tan G = \frac{c}{a}$	48.2°
H	$\tan H = \frac{d}{b}$	55.2°

Extension

Activity 4.3 provides the opportunity for students to witness how, using a systematic approach, three-dimensional objects can be broken down into pieces, or fabricated from them. Ducts such as the one constructed in this activity are incorporated into larger ventilation systems. If you wish to extend the activity, you could provide students with the measurements of a room that needs a ventilation system installed, and ask them to add the necessary ventilation pipes to their hood.

Students could research the shapes of these pipes, and the way they are constructed, as homework. The pipes could be fabricated of square pieces, or be cylindrical in shape. Either way, students now possess the mathematical knowledge necessary to determine the dimensions and surface area of the ventilation pipes. Examples of the shapes ventilation pipes could take are given below.

http://www.tsmhouston.com/images/industrial-ventilation-systems.jpg

http://www.fieldair.co.uk/uploadedimages/15449a.jpg

This website shows a diagram of how ventilation hoods can be attached to ventilation pipes.

http://www.wbdg.org/design/ensure_health.php

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 195

1. $$\frac{h_1}{2210} = \sin 15°$$

$$h_1 = \sin 15° \times 2210$$

$$h_1 \approx 572$$

$$\frac{h_2}{2460} = \sin 23°$$

$$h_2 = \sin 23° \times 2460$$

$$h_2 = 961$$

$$\frac{h_3}{890} = \sin 34°$$

$$h_3 = \sin 34° \times 890$$

$$h_3 \approx 498$$

$$8060 - 572 - 961 - 498 = 6029$$

The elevation at Anna's final position is 6029 ft.

2. $$\frac{h}{140} = \tan 14°$$

$$h \approx 35$$

The distance to the top of the rock formation from the top of the canyon edge is 35 ft.

$$\frac{d}{140} = \tan 62°$$

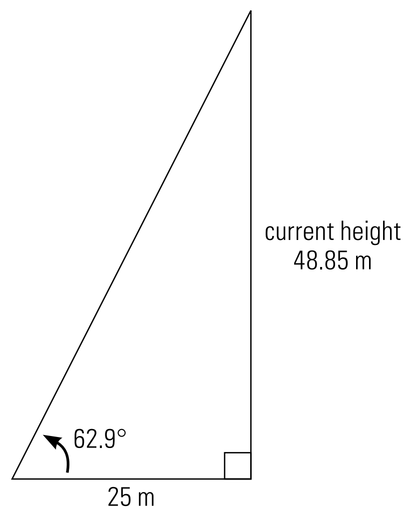
$$d \approx 263$$

$$35 + 263 = 298$$

The height of the rock formation is 298 ft.

If he adds ten feet of rope, he needs at least 308 feet of rope.

3.



To find the current height of the cliff, use the values of the right triangle that has the cliff and ground as its legs. Its hypotenuse is a line drawn from the Jordan's position on the ground to the top of the cliff. Insert these values into the tangent ratio.

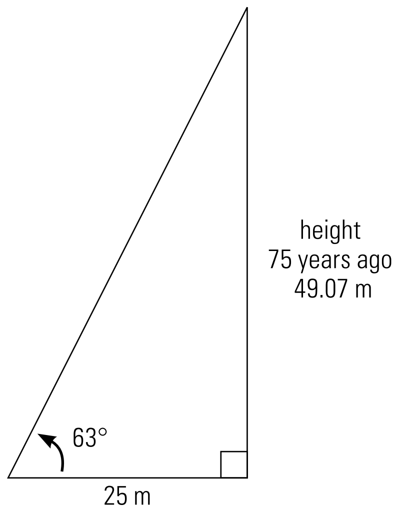
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 62.9° = \frac{h}{25}$$

$$25 \tan 62.9° = h$$

$$48.85 \approx h$$

The cliff is 48.85 m high.



To find the amount of erosion, subtract its current height from the original height.

$49.07 - 48.85 = 0.22$

The cliff has lost 0.22 m, or 22 cm, to erosion.

To find the angle of elevation to the top of the cliff 75 years ago, use the tangent ratio. Use the values of the right triangle that has the cliff (as it was 75 years ago) and ground as its legs. Its hypotenuse is a line drawn from the Jordan's position on the ground, to the top of the cliff.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \frac{49.07}{25}$$

$$\tan\theta = 1.9628$$

$$\theta = \tan^{-1}(1.9628)$$

$$\theta \approx 63°$$

The angle of elevation 75 years ago was 63°.

4. AC = 12 inches

CD = 12 − 3 − 3

CD = 6 inches

$$\tan(\text{horizontal angle}) = \frac{CD}{AC}$$

$$\text{horizontal angle} = \tan^{-1}\left(\frac{CD}{AC}\right)$$

$$\text{horizontal angle} = \tan^{-1}\left(\frac{6}{12}\right)$$

$$\text{horizontal angle} \approx 26.6°$$

BD = 12 − 3 − 3

BD = 6 inches

$$AD^2 = AC^2 + CD^2$$

$$AD^2 = 12^2 + 6^2$$

$$AD = \sqrt{144 + 36}$$

$$AD \approx 13.4 \text{ inches}$$

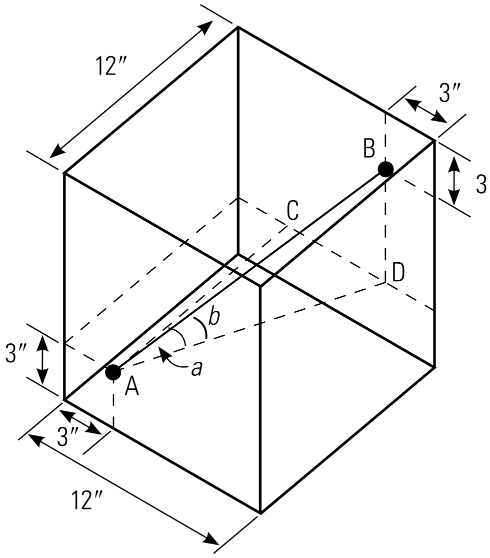
$$\tan(\text{vertical angle}) = \frac{BD}{AD}$$

$$\text{vertical angle} = \tan^{-1}\left(\frac{BD}{AD}\right)$$

$$\text{vertical angle} = \tan^{-1}\left(\frac{6}{13.4}\right)$$

$$\text{vertical angle} \approx 24.1°$$

Alan must drill a horizontal angle of 26.6° and a vertical angle of 24° to make a hole from point A to point B.



5. Use the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 4° = \frac{d}{36.2}$$

$$36.1 \approx d$$

Next, use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$
$$24^2 + b^2 = 36.1^2$$
$$b^2 = 36.1^2 - 24^2$$
$$b = \sqrt{727.21}$$
$$b \approx 27$$

The plane is 27 km west of the tower.

6. a) Find the distance from point B to point C at the bottom of the cliff using the tangent ratio.

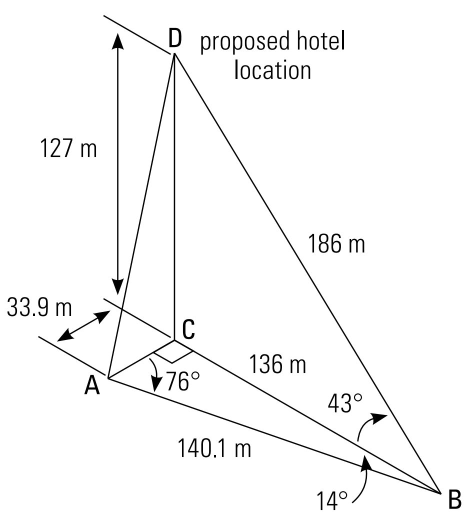
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 76° = \frac{\text{BC}}{33.9}$$
$$33.9\tan 76° = \text{BC}$$
$$136 \text{ m} \approx \text{BC}$$

The distance from point B to point C is 136 m.

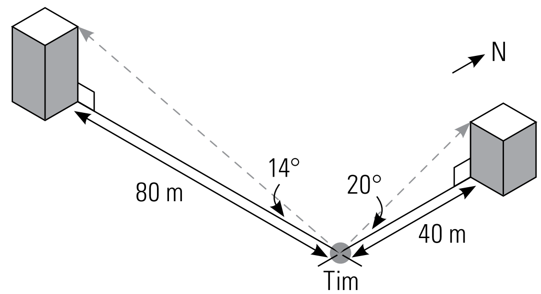
b) Find the height of the cliff.

$$\tan 43° = \frac{h}{136}$$
$$127 \approx h$$

The height of the cliff is 127 m.



7. a) Tim looks straight north and sees the top of a building 40 m distant at an angle of elevation.



b) Use the tangent ratio to find the taller building.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan 20° = \frac{b_1}{40}$$
$$14.6 \approx b_1$$
$$\tan 14° = \frac{b_2}{80}$$
$$19.9 \approx b_2$$

Building two is taller. Subtract to find how much taller the second building is.

$19.9 - 14.6 = 5.3$

The second building is taller by 5.3 m.

c) Use the Pythagorean theorem to find the horizontal diagonal distance.

$$\sqrt{a^2 + b^2} = c^2$$
$$40^2 + 80^2 = c^2$$
$$\sqrt{8000} = c$$
$$89 \approx c$$

The straight-line distance is 89 m.

d) Use the tangent ratio to find the angle of elevation.

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan\theta = \frac{5.3}{89}$$

$$\theta = \tan^{-1}\left(\frac{5.3}{89}\right)$$

$$\theta \approx 3.4°$$

The angle of elevation from the lower to the higher building is 3.4°.

Extend Your Thinking

8. a) To calculate angle A, we need to calculate ℓ_1 and ℓ_3. Calculate ℓ_1 using the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 15° = \frac{\ell_1}{2000}$$

$$2000\cos 15° = \ell_1$$

$$1932 \approx \ell_1$$

ℓ_1 measures 1932 mm.

Calculate ℓ_3 using the sine ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 70° = \frac{\ell_3}{\ell_1}$$

$$\ell_1 \sin 70° = \ell_3$$

$$1932\sin 70° = \ell_3$$

$$1815 \approx \ell_3$$

Next, use the sine ratio to calculate the angle the back leg will make with the ground.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{\ell_3}{2000}$$

$$\sin A = \frac{1815}{2000}$$

$$\sin A = 0.9075$$

$$A = \sin^{-1}(0.9075)$$

$$A \approx 65°$$

The back leg makes a 65° angle with the ground.

b) To calculate the length of string, ℓ_6, we need to calculate ℓ_2, ℓ_4, and ℓ_5.

Calculate ℓ_2 using the sine ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 15° = \frac{\ell_2}{2000}$$

$$2000\sin 15° = \ell_2$$

$$518 \approx \ell_2$$

ℓ_2 measures 518 mm.

Calculate ℓ_5 using the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 70° = \frac{\ell_5}{\ell_1}$$

$$\ell_1 \cos 70° = \ell_5$$

$$1932\cos 70° = \ell_5$$

$$661 \approx \ell_5$$

ℓ_5 measures 661 mm.

Finally, calculate ℓ_4 using the cosine ratio.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 65° = \frac{\ell_4}{2000}$$
$$2000\cos 65° = \ell_4$$
$$845 \approx \ell_4$$

ℓ_4 measures 845 mm.

We can now use the Pythagorean theorem to calculate ℓ_6.

$$a^2 + b^2 = c^2$$
$$(\ell_2)^2 + (\ell_4 + \ell_5)^2 = (\ell_6)^2$$
$$518^2 + (845 + 661)^2 = (\ell_6)^2$$
$$2\,536\,360 = (\ell_6)^2$$
$$\sqrt{2\,536\,360} = \ell_6$$
$$1593 \approx \ell_6$$

To maintain a 20° tilt on the easel, the strings connecting the back leg to the front legs must be 1593 mm long.

9.

GUY ROPE EFFECTIVENESS

Guy Rope Angle	*% Effectiveness*	*Sine of Angle*	*Cosine of Angle*	*Tangent of Angle*
60	50	0.866	0.500	1.732
45	71	0.707	0.707	1.00
30	87	0.500	0.866	0.577
0	100	0.000	1.00	0.00

The percent effectiveness is the cosine of the angle expressed as a percentage.

After you finish this problem, you can explain to students that many trades rely on using codes, specifications, and handbooks of tables rather than doing complex calculations on site. Knowing some of the calculations that go into making up these publications will help them use them; they will have a better sense of which parts of the codes and specifications are the right ones to use under various circumstances, and why.

THE ROOTS OF MATH

NAVIGATION IN HISTORY

STUDENT BOOK, p. 200

Studying GPS technologies is usually a high-interest topic with a lot of good, easy to read information on the internet. You can use Blackline Master 4.8 (p. 272) to help students take notes in addition to the questions in the book.

SOLUTIONS

1. Answers include but are not limited to: cars, GPS on smart phones, Google Earth, navigation.
2. Answers include but are not limited to: government surveillance, rescue missions (e.g., avalanche GPS devices), national defense, wildlife tracking, vehicle fleet tracking.
3. Students may begin with comparing stars and satellites. For millennia, people in civilizations around the world navigated by correlating their positions to the positions of various stars, and this required a great deal of knowledge about star movements in the 24-hour period and throughout the year. GPS systems, however, interact electronically with satellites so that the user does not need to make any calculations or know how fast the satellites move or where they are in the sky.

PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 202

1. a) Find the direction, in degrees, of each girl from the north-south line.

Anne-Christine

$$\tan \theta_c = \frac{200}{420}$$

$$\theta_c = \tan^{-1}\left(\frac{200}{420}\right)$$

$$\theta_c = 25° \text{ towards west}$$

Anna-Mieke

$$\tan \theta_m = \frac{130}{300}$$

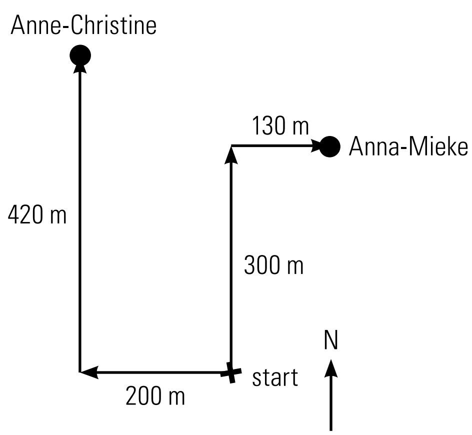
$$\theta_m = \tan^{-1}\left(\frac{130}{300}\right)$$

$$\theta_m = 23° \text{ towards east}$$

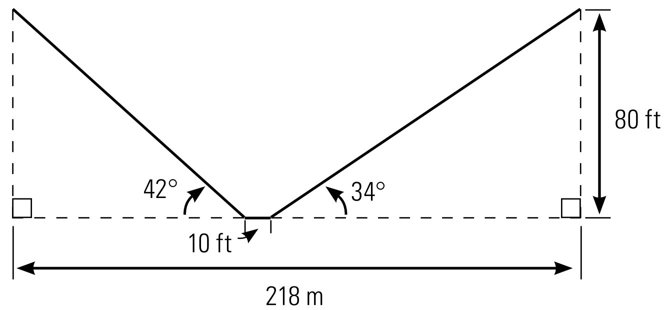
b) Find the distance between the two girls.

$$d = \sqrt{120^2 + 330^2}$$

$$d \approx 351 \text{ m}$$



2. Using the tangent ratio, find the total width of the ride on both sides.



$$\tan 42° = \frac{80}{w_1}$$

$$w_1 \approx 89 \text{ ft}$$

$$\tan 34° = \frac{80}{w_2}$$

$$w_2 \approx 119 \text{ ft}$$

Add to find the total width.

$$89 + 119 + 10 = 218$$

The total width of the ride is 218 ft.

3. Determine the angle for the reflected beam.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{24}{45}$$

$$\theta = \sin^{-1}\left(\frac{24}{45}\right)$$

$$\theta \approx 32$$

$$32° \neq 38°$$

The angle of reflection does not equal the angle of incidence; therefore the mirror is warped.

4. Find the height of the mountain using the tangent ratio.

$$\tan 70° = \frac{h}{x}$$
$$h \approx 2.747x$$
$$\tan 75° = \frac{h}{x - 300}$$
$$h = (x - 300)\tan 75°$$
$$h \approx 3.732x - 1119.6$$
$$2747x - 3732x = -1119.6$$
$$1119.6 = 0.985x$$
$$x \approx 1137 \text{ m}$$
$$h \approx 2.747x$$
$$h \approx 2.747 \times 1137$$
$$h \approx 3123 \text{ m}$$

The mountain is about 3123 m high.

5. a) For the distances *a* and *b*, complete the following calculations.

Length, m	*Angle*	*a = length × cos (angle)*	*b = length × sin (angle)*
6.40	38.7	4.99	4.00
4.47	26.6	4.00	2.00
6.26	28.6	5.50	3.00

b) To calculate the area of the patio, the shape can be broken into triangles and rectangles in many different ways. The simplest way, however, is to enclose the entire area in one large rectangle and then subtract the corner triangles.

Rectangle area = $(a_1 + a_2) \times (b_2 + a_3)$

Rectangle area = (4.99 + 4.00) × (2.00 + 5.50)

Rectangle area = 8.99 × 7.50

Rectangle area = 67.425 m^2

$$\text{Area of Triangle } T_1 = \frac{1}{2}(4.99 \times 4.00)$$
$$\text{Area of } T_1 = 9.98 \text{ m}^2$$
$$\text{Area of } T_2 = \frac{1}{2}(4.00 \times 2.00)$$
$$\text{Area of } T_2 = 4.00 \text{ m}^2$$
$$\text{Area of } T_3 = \frac{1}{2}(5.50 \times 3.00)$$
$$\text{Area of } T_3 = 8.25 \text{ m}^2$$

Total area = Rectangle area − T_1 − T_2 − T_3
Total area = 67.425 − 9.98 − 4.00 − 8.25
Total area = 45.195 m^2

The total surface area of the patio is 45.195 m^2.

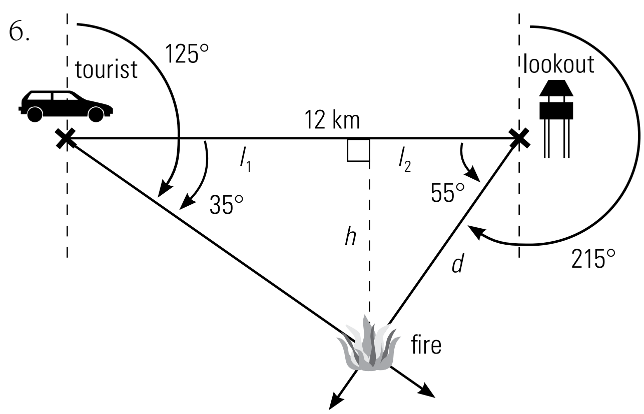
c) Area of one paver = 0.30 × 0.20
Area of one paver = 0.06 m^2

$$\text{Number of pavers} = \frac{\text{Total area}}{\text{Area of one paver}}$$
$$\text{Number of pavers} = \frac{45.2}{0.06}$$
$$\text{Number of pavers} \approx 754$$

Add 10% for wasteage:
Total number of pavers to order = 754 × 1.10
Total number of pavers to order = 829.4

Leslie should order 830 pavers.

6.



$$\frac{h}{\ell_1} = \tan 35°$$
$$h = \ell_1 \tan 35°$$
$$\frac{h}{\tan 35°} = \ell_1$$

$$\frac{h}{\ell_2} = \tan 55°$$

$$h = \ell_2 \tan 55°$$

$$\frac{h}{\tan 55°} = \ell_2$$

$$\ell_1 + \ell_2 = 12.0 \text{ km}$$

Substitute the expressions for ℓ_1 and ℓ_2 into the above equality and solve or h:

$$\frac{h}{\tan 35°} + \frac{h}{\tan 55°} = 12.0$$

$$h\left(\frac{1}{\tan 35°} + \frac{1}{\tan 55°}\right) = 12.0$$

$$h = \frac{12.0}{\left(\frac{1}{\tan 35°} + \frac{1}{\tan 55°}\right)}$$

$$h \approx 5.64 \text{ km}$$

Now calculate the distance d from the smoke to the lookout tower.

$$\frac{h}{d} = \sin(55°)$$

$$h = d\sin(55°)$$

$$\frac{h}{\sin 55°} = d$$

$$\frac{5.64}{\sin 55°} = d$$

$$d \approx 6.89 \text{ km}$$

The smoke is 6.89 km from the lookout tower.

7. a) Find the angle of inclination

$$\sqrt{3^2 + 5^2} \approx 5.8 \text{ mi}$$

Convert to feet.

5.8 mi × 5280 ft/mi = 30 624 ft

$$\tan x = \frac{2000}{30\,624}$$

$$x = \tan^{-1}\left(\frac{2000}{30\,624}\right)$$

$$x \approx 3.7°$$

b) Find the airplane's straight-line distance to the takeoff point.

$$\cos 3.7° = \frac{30\,624}{y}$$

$$y\cos 3.7° = 30\,624$$

$$y = \frac{30\,624}{\cos 3.7°}$$

$$y \approx 30\,688 \text{ ft}$$

8. a) Use the Pythagorean theorem to calculate the horizontal distance:

$b^2 = 80^2 + 250^2$
$b^2 = 68\,900$
$b \approx 262.49$ m

b)

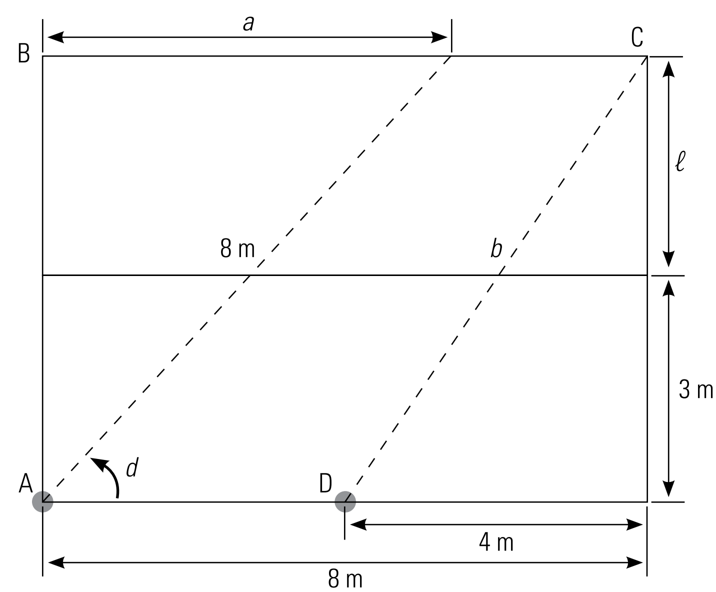
$$\tan(1.754) = \frac{\text{elevation}}{b}$$

$$b\tan(1.754) = \text{elevation}$$

$$262.49\tan(1.754) = \text{elevation}$$

$$\text{elevation} \approx 8.04 \text{ m}$$

9. This problem can best be solved by "unfolding" the garage roof (in a similar manner to unfolding a three-dimensional shape into a net to calculate the surface area, as seen in chapter 3).



The wall and roof of the garage, when unfolded, form a rectangle of width 8 m and height (ℓ + 3), where ℓ is the slope length of the roof:

$$\frac{2.50}{\ell} = \cos(30)$$

$$2.50 = \ell \cos(30)$$

$$\frac{2.50}{\cos(30)} = \ell$$

$$\ell \approx 2.887 \text{ m}$$

The height of the (wall and roof) rectangle is 5.887 m.

a) The air hose forms the hypotenuse of a right triangle of sides 5.887 and a m. We can use the Pythagorean theorem to calculate a.

$$a^2 + 5.887^2 = 8.0^2$$

$$a^2 \approx 64.0 - 34.657$$

$$a^2 \approx 29.343$$

$$a \approx 5.417 \text{ m}$$

She has completed 5.4 metres of the top row of shingles.

b) $$\tan(d) = \frac{3 + \ell}{a}$$

$$\tan(d) = \frac{5.887}{5.417}$$

$$\tan(d) = 47.38°$$

The hose makes an angle of 47.38° with the ground.

c) When the compressor is repositioned, we can calculate the length of hose she needs.

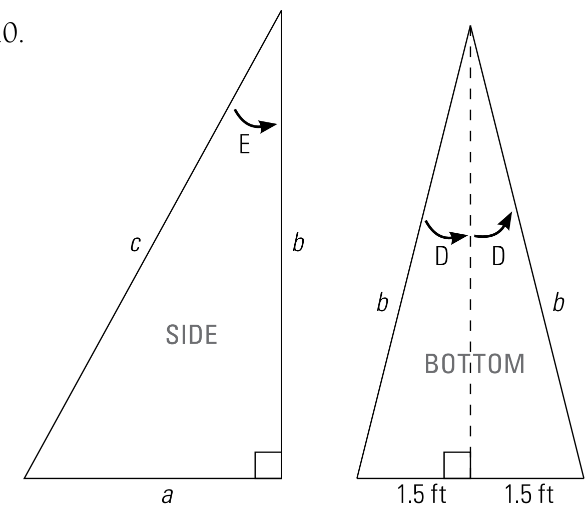
$$4^2 + 5.887^2 = b^2$$

$$b^2 \approx 50.657$$

$$b \approx 7.117 \text{ m}$$

The hose is long enough when she places the compressor at the middle of the side.

10.



$$a^2 = 3^2 + 1.5^2$$
$$a^2 = 11.25$$
$$a \approx 3.35 \text{ ft}$$

$$b^2 = 1.5^2 + 6^2$$
$$b^2 = 38.25$$
$$b \approx 6.18 \text{ ft}$$

$$c^2 = 3^2 + 6^2$$
$$c^2 = 45$$
$$c \approx 6.71 \text{ ft}$$

$$\tan \mathrm{D} = \frac{1.5}{6}$$

$$\mathrm{D} = \tan^{-1}\left(\frac{1.5}{6}\right)$$

$$\mathrm{D} \approx 14.0°$$

$$\tan \mathrm{E} = \frac{3.35}{6.18}$$

$$\mathrm{E} = \tan^{-1}\left(\frac{3.35}{6.18}\right)$$

$$\mathrm{E} \approx 28.5°$$

11. a) Calculate the length of strapping she needs to cut to hang the bottom of the first section from the beam if Jessica is using $\frac{3}{4}$-inch pipe.

drop of first section

$$\sin 22° = \frac{drop_1}{6}$$

$$drop_1 \approx 2.2 \text{ ft}$$

The strapping will need to be at least twice as long as the drop, which gives 4.4 feet. There needs to be enough to wrap around the pipe (about 2 inches, using the circumference formula to estimate) and enough to attach to the wall stud above (at least another two inches). This gives at least 4 feet 9 inches.

b) Calculate the total drop over the complete installation.

$$\sin 22° = \frac{drop_1}{6}$$
$$drop_1 \approx 2.2 \text{ ft}$$

$$\sin 25° = \frac{drop_2}{4.5}$$
$$drop_2 \approx 1.9 \text{ ft}$$

$$\sin 42° = \frac{drop_3}{3.33}$$
$$drop_3 \approx 2.2 \text{ ft}$$

total drop = 2.2 + 1.9 + 2.2

total drop = 6.3 ft

total drop ≈ 6 ft 4 in.

c) Calculate the length of the first wall.

$$\cos 22° = \frac{\ell_1}{6}$$
$$\ell_1 = 6\cos 22°$$
$$\ell_1 \approx 5.6 \text{ ft}$$

$$\cos 25° = \frac{\ell_2}{4.5}$$
$$\ell_2 = 4.5\cos 25°$$
$$\ell_2 \approx 4.1 \text{ ft}$$

5.6 + 4.1 = 9.7 ft

Calculate the length of the second wall.

$$\cos 42° = \frac{\ell_3}{3.3}$$
$$\ell_3 = 3.3\cos 42°$$
$$\ell_3 \approx 2.5 \text{ ft}$$

Calculate the horizontal diagonal distance between the two pipe ends

$$d = \sqrt{9.7^2 + 2.5^2}$$
$$d \approx 10 \text{ m}$$

Calculate the diagonal vertical distance.

$$d = \sqrt{10^2 + 6.3^2}$$
$$d \approx 11.8 \text{ m}$$

The diagonal distance from the top to the bottom of the pipe is 11.8 ft.

12. a) For the horizontal offset:

$$\frac{x}{250.000} = \tan(0.57)$$
$$x = 250.000\tan(0.57)$$
$$x \approx 2.487 \text{ mm}$$

b) For the vertical offset, first calculate h, then y.

Calculate h.

$$\frac{250.000}{h} = \cos(0.57)$$
$$250.000 = h\cos(0.57)$$
$$\frac{250.000}{\cos(0.57)} = h$$
$$h \approx 250.012 \text{ mm}$$

Calculate y.

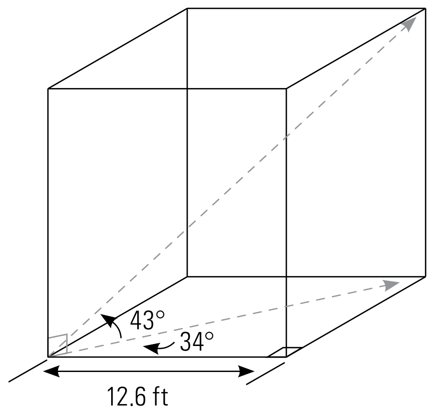
$$\frac{y}{h} = \tan(0.38)$$
$$y = h\tan(0.38)$$
$$y = 250.012\tan(0.38)$$
$$y \approx 1.658 \text{ mm}$$

SAMPLE CHAPTER TEST

Name: ______________________________ Date: ______________________________

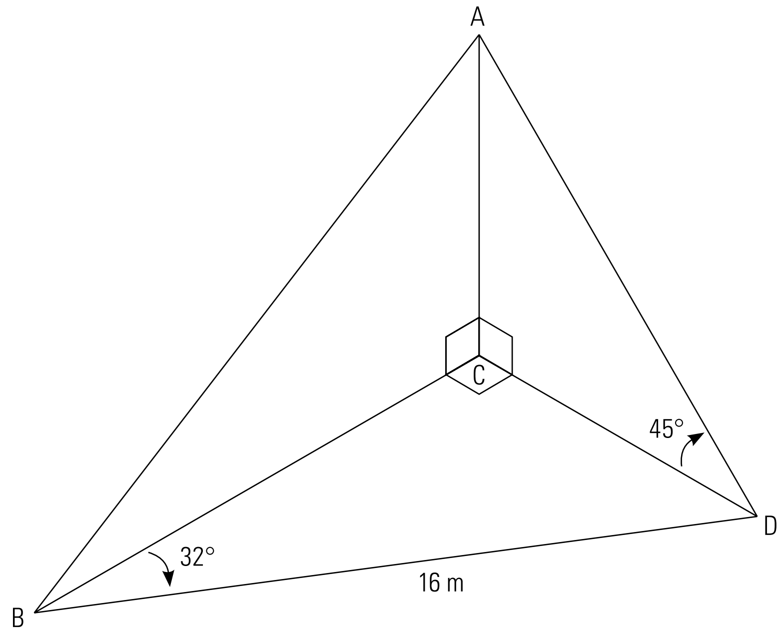
Part A: True or False

Use the diagram to answer the true and false questions.



1. The diagonal distance on the bottom of the box could be found by $\sin 34° = \frac{x}{12.6}$.
2. The length of the box could be found by $\tan 43° = \frac{x}{12.6}$.
3. It is necessary to make calculations with the 34° measurement before any calculations could be made with the 43° measurement.
4. There is enough information on the diagram to calculate all the dimensions and angles of the box.
5. The 43° angle is needed to find the height of the box.
6. The 43° angle is needed to find the length of the box.
7. The length of the bottom diagonal is 15.2 ft.
8. The height of the box could be found with the ratio $\tan 43° = \frac{x}{12.6}$.
9. The length of the three-dimensional diagonal is 20.8 ft.
10. The length of the three-dimensional diagonal is found by $\cos 43° = \frac{x}{16.9}$.

Part B Short Answer



1. Determine the length of CD, given that angle BCD is a right angle.

2. Determine the expression that you would use to calculate distance CB.

3. What is the height of AC?

4. What is the straight-line distance from A to D?

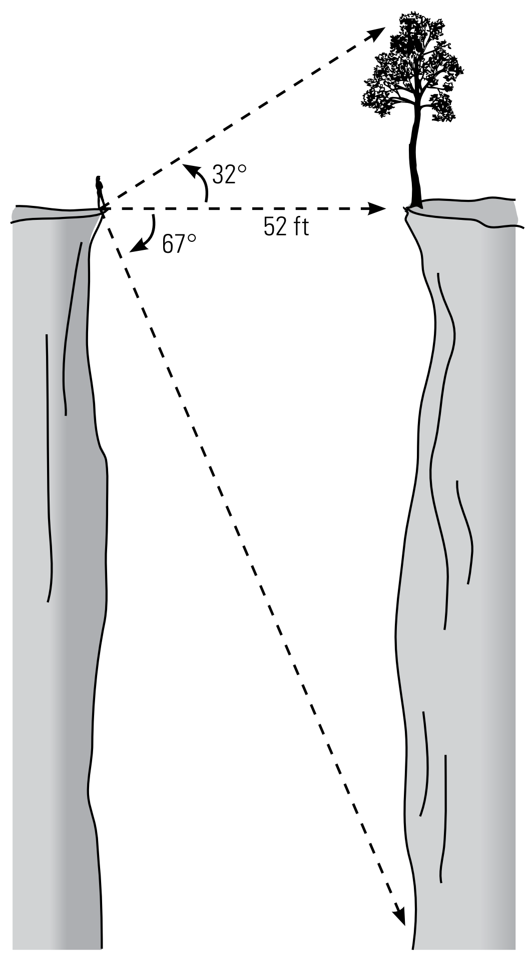
Part C Long Answer

1. An airplane is 37 km north and 81 km east of an airport. It is 6 km up in the air.
   a) What is the straight-line distance to the airport?

b) What is the angle along the ground that the airplane is at, as a direction east of north?

c) What is the angle of elevation of the airplane from the point of view of the airport?

2. A man stands on a cliff and looks across the canyon in front of him. On the other side of the canyon, he sees a tree. The tree is 52 ft away from him. From where the man stands, the angle of elevation to the top of the tree is 32°. From his position, the angle of depression to the bottom opposite side of the canyon is 67°.



   a) What is the height of the tree?

b) What is the total distance from the bottom of the canyon to the top of the tree?

3. A laser beam hits a mirror at an angle of 42° and when it is reflected, the beam creates a diagonal across a space this is 90 cm from the mirror in 100 cm of width. Is the mirror true or warped?

SAMPLE CHAPTER TEST: SOLUTIONS

Part A: True or False

1. False. $\cos 34° = \frac{12.6}{x}$
2. False. $\tan 34° = \frac{x}{12.6}$
3. True.
4. True.
5. True.
6. False.
7. True.
8. False. $\tan 43° = \frac{x}{15.2}$
9. True. $\cos 43° = \frac{15.2}{x}$
10. False.

Part B: Short Answer

1. $\sin 32° = \frac{CD}{16}$

$CD = 16 \times \sin 32°$

$CD \approx 8.5$ m

2. $\cos 32° = \frac{CB}{16}$

3. 8.5 m, since ACD is an isosceles triangle and the equal side is 8.5 m.

4. $\cos 45° = \frac{8.5}{AD}$

$AD = \frac{8.5}{\cos 45°}$

$AD \approx 12$ m

Part C: Extended Answer

1. a) $\sqrt{37^2 + 81^2} \approx 89$ km

$\sqrt{89^2 + 6^2} \approx 89.2$ km

b) $\tan\theta = \frac{81}{37}$

$\tan\theta \approx 2.189$

$\theta = \tan^{-1}(2.189)$

$\theta \approx 65°$

c) $\tan\theta = \frac{6}{89}$

$\tan\theta \approx 0.0674$

$\theta = \tan^{-1}(0.0674)$

$\theta \approx 3.9°$

2. a) $\tan 32° = \frac{h}{52}$

$h = 52 \times \tan 32°$

$h \approx 32.5$ ft

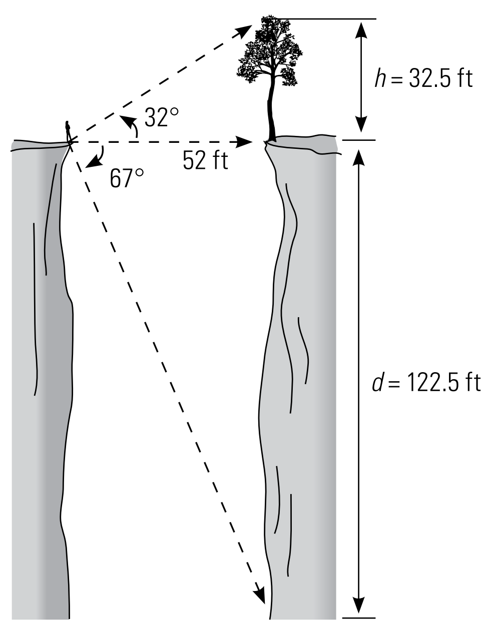
b) $\tan 67° = \frac{d}{52}$

$d = 52 \times \tan 67°$

$d \approx 122.5$ ft

$d + h = 32.5 + 122.5$

$d + h = 155$ ft



3. $\tan\theta = \frac{90}{100}$

$\theta = \tan^{-1}\left(\frac{90}{100}\right)$

$\theta \approx 42°$

The angle of inclination equals the angle of reflection; therefore the mirror is true.

BLACKLINE MASTER 4.1

CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ What features did your park include? How will these features attract people of different ages and with different hobbies?	
❒ How did you incorporate natural features, such as rocks and bodies of water, into your park?	
❒ Are your map and park features drawn to scale?	
❒ Is your theatre and seating built to scale? What slope did you choose for your theatre's seating?	
❒ Do you have a diagram of your theatre that includes angle of depression, inclination, and distance calculations for its seating?	
❒ Did you label your park paths with their angles and slopes?	

BLACKLINE MASTER 4.2

CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research, calculations, and measurements;
- the accuracy of your calculations;
- the effectiveness of your uses of technology for research;
- the creativity you brought to planning and presenting; and
- your completion of all the assigned tasks on time.

How do you feel you have done overall, given the criteria above?

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel?

Are there areas in which you could improve?

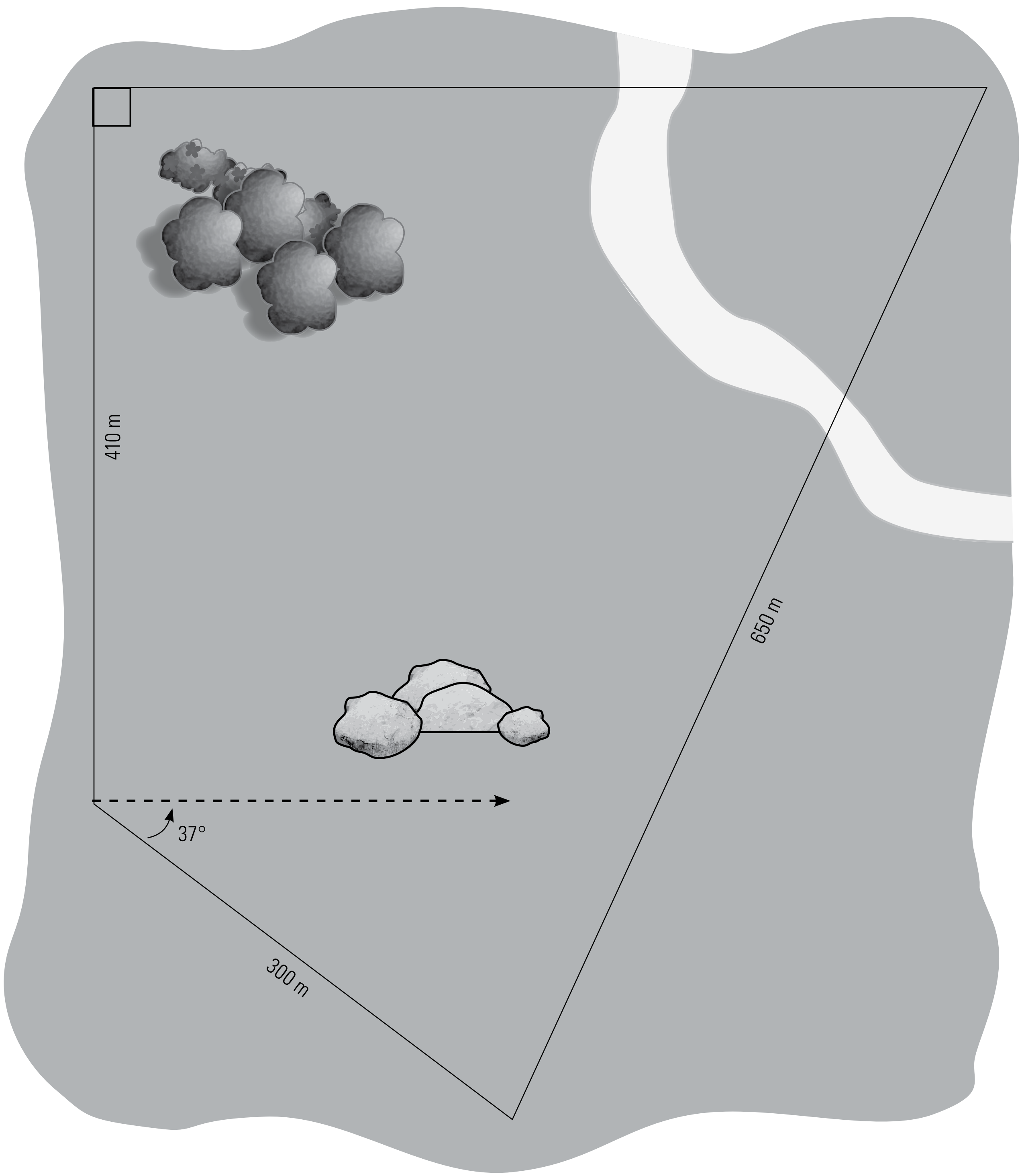
If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 4.3

CHAPTER PROJECT: PLAN A COMMUNITY-FRIENDLY PARK

Name: ______________________ Date: ______________________

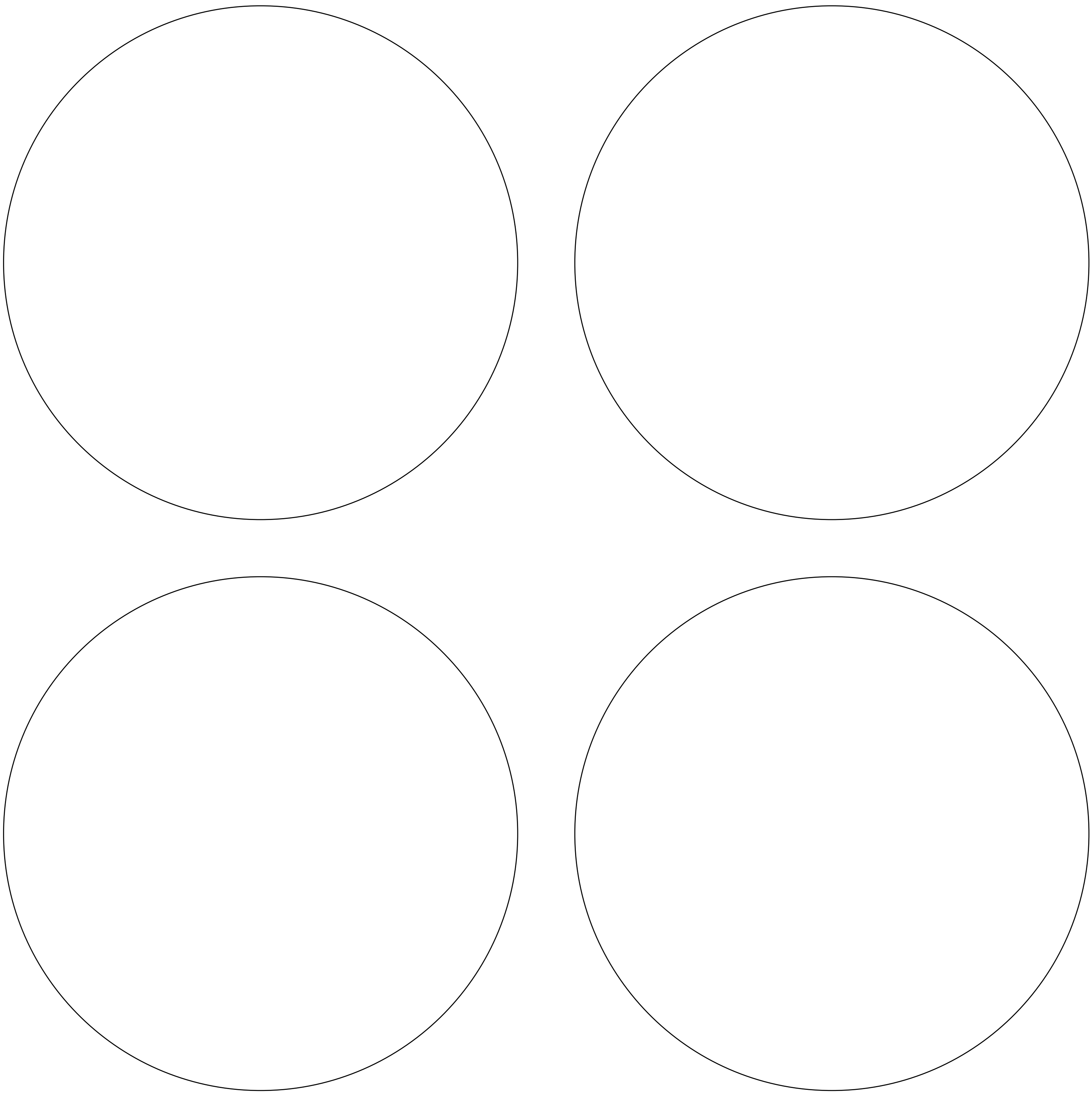


BLACKLINE MASTER 4.4

ANCIENT TRIGONOMETRY RATIOS WITHIN CIRCLES

Name: ____________________________ Date: ____________________________

Use these circles to sketch and measure your angles, chords, and radii for the ratio table in Activity 4.1.



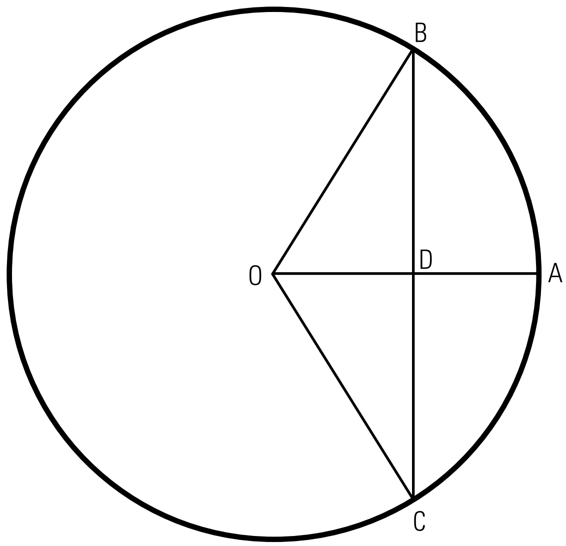
BLACKLINE MASTER 4.5

EXTENSION ACTIVITY FOR TRIGONOMETRY RATIOS WITHIN CIRCLES

Name: ______________________ Date: ______________________

Part A

Look at this diagram.



1. What two sides would you measure to find the sine ratio of triangle OBD?
2. What two sides would you measure to find the cosine ratio of triangle OBD?
3. What two sides would you measure to find the tangent ratio of triangle OBD?

Part B

4. Draw four angles with their chords and bisecting radii to form right triangles OBD as above. Measure the lengths of the sides of each triangle, then write the trigonometry ratios in the table as a fraction of one side over the other in each case.

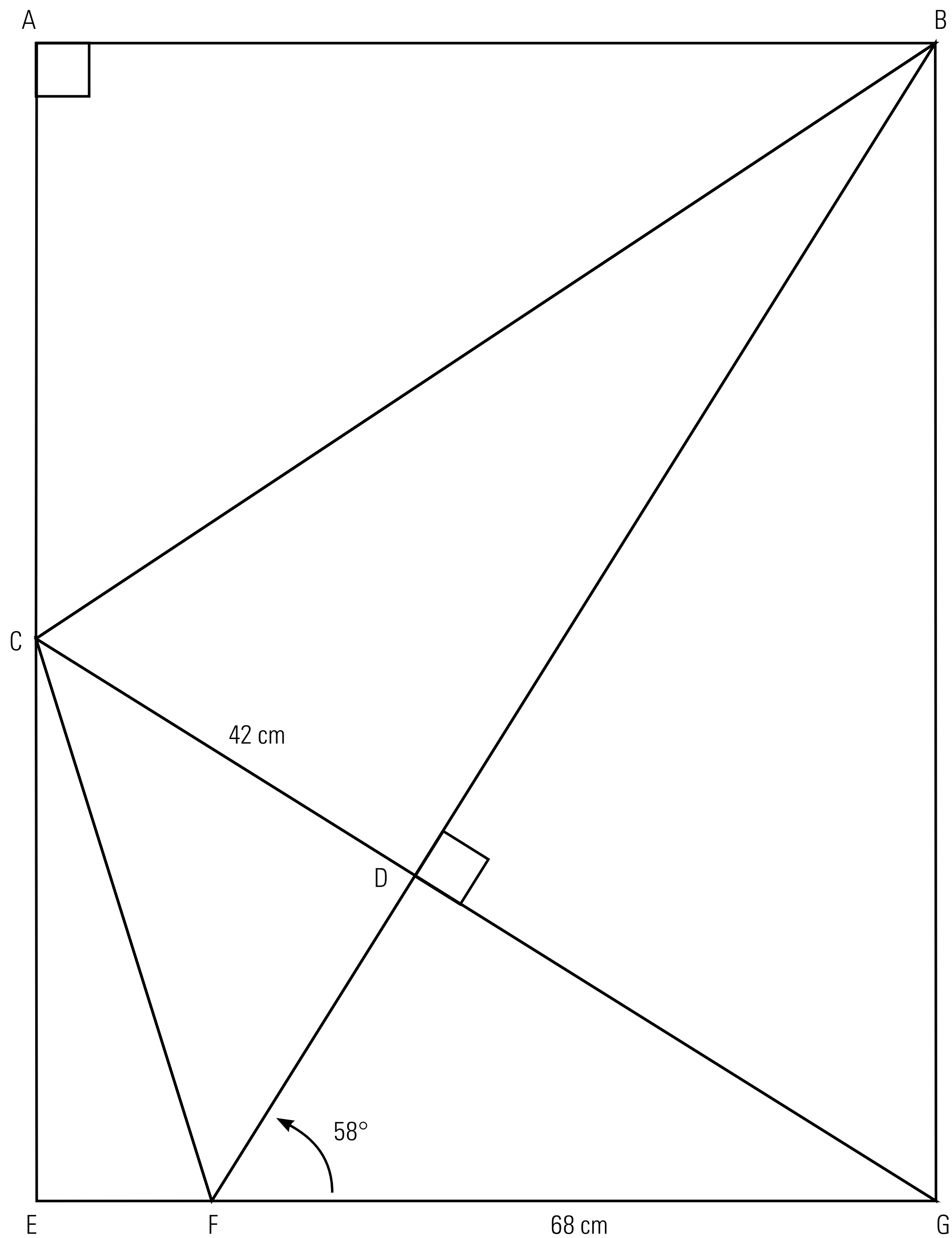
side OD	*side OB*	*side BD*	*sine ratio*	*calc'd*	*cosine ratio*	*calc'd*	*tangent ratio*	*calc'd*	*angle measure*

5. Measure the angles that you have drawn. Use your calculator to find the sine, cosine, and tangent of each angle. Record those values in the table under the three "calc'd" columns.
6. Compare your measurements to your calculations. What did you find?

BLACKLINE MASTER 4.6

STAINED GLASS SAILBOAT

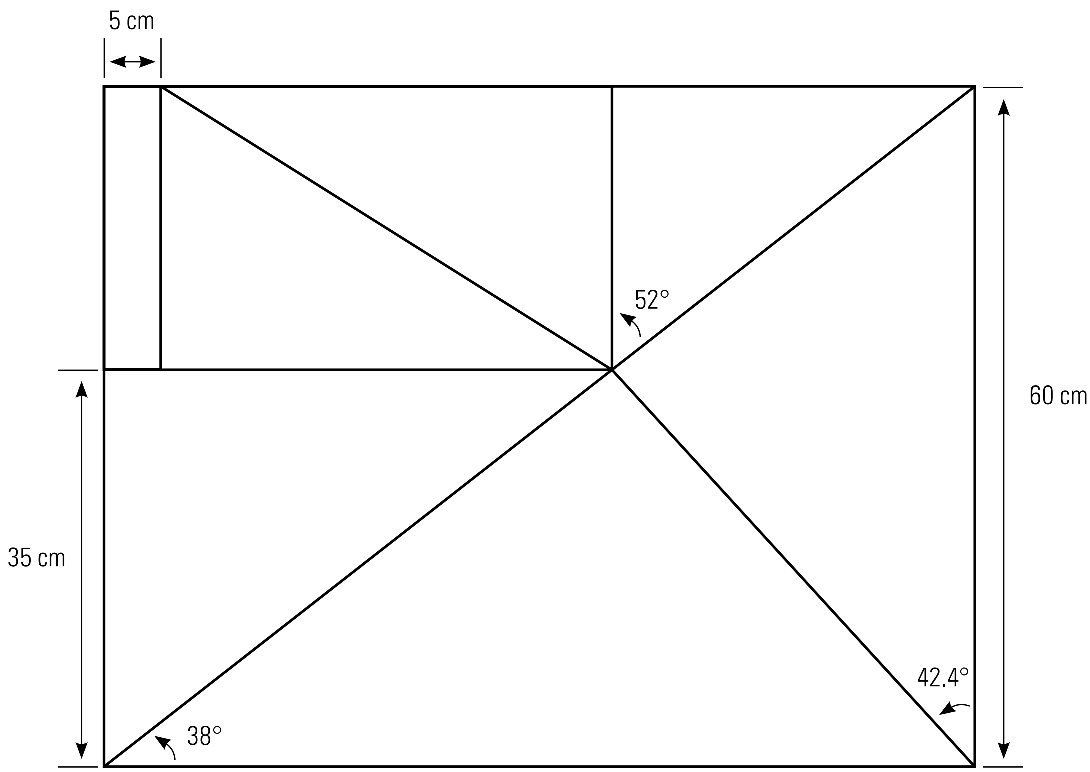
Name: ____________________ Date: ____________________



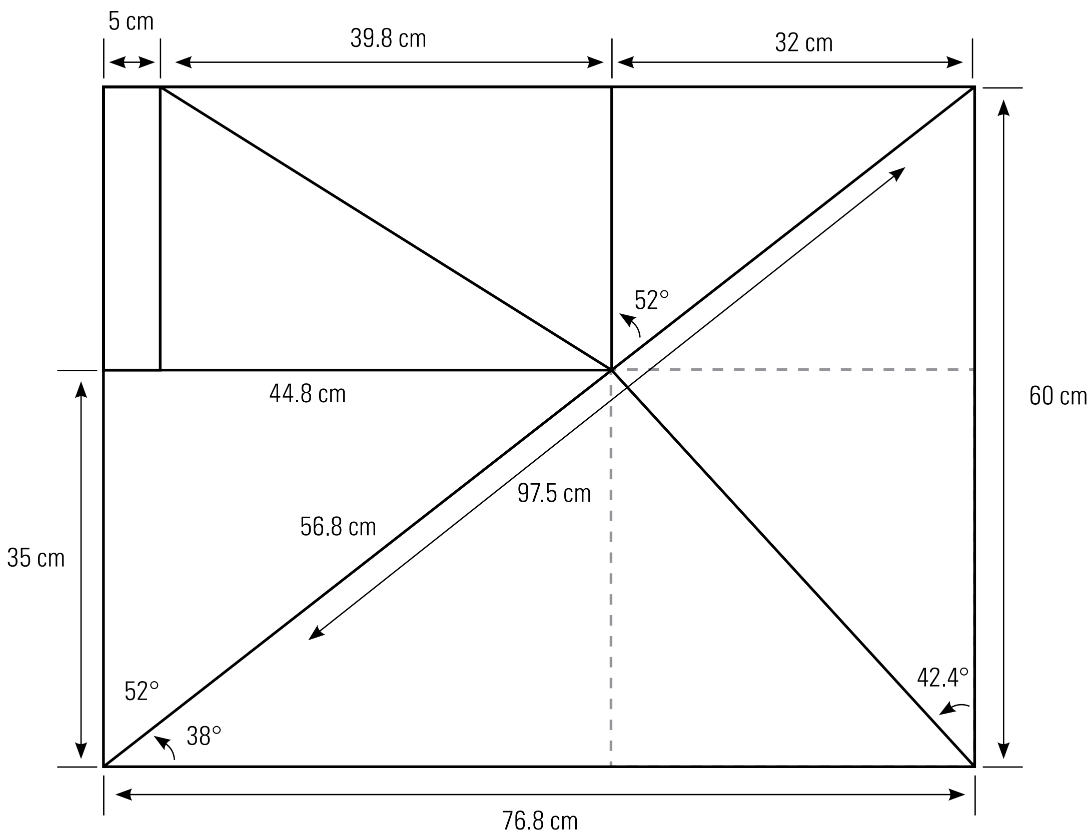
BLACKLINE MASTER 4.7

ALTERNATIVE PATTERN FOR THE PUZZLE IT OUT GAME

Name: ______________________________ Date: ______________________________



BLACKLINE MASTER 4.7: SOLUTION



BLACKLINE MASTER 4.8

THE ROOTS OF MATH AND GPS RESEARCH

Name: ______________________ Date: ______________________

Use this concept map to help you make notes about the Roots of Math article and about GPS technology as you do internet research. Remember to accurately copy the addresses of pages that you gather information from or want to return to.

Traditional navigation methods page addresses:	GPS technology theory page addresses:
Traditional methods compared to modern method of navigation page addresses:	GPS technology devices page addresses:

ALTERNATIVE CHAPTER PROJECT—SURVEY A YOUTH WILDERNESS BASE CAMP

TEACHER MATERIALS

GOALS: To use trigonometry skills to survey a model of a property that will be used for a youth wilderness base camp, and to use measuring and scaling to calculate elevations, distances around the boundary, and area of the property.

OUTCOME: Students will construct a three-dimensional terrain out of modelling clay, plaster, or papier maché, define the boundary of the youth camp, determine survey points, and take measurements of elevations, angles, and lengths on their property. To do this they will need to develop simple measurement aids: a level, a plumb-bob, and survey rods. They will convert their measurements on the scale model to 'actual' measurements using a scale of 1:500. They will then complete calculations of elevations and distances between survey points around the boundary, and calculate the area of the property. To do this, they will need to use trigonometric functions applied to the horizontal and vertical planes.

PREREQUISITES: Students will need to:

- understand scaling;
- use a magnetic compass.

ABOUT THIS PROJECT: This project allows hands-on construction of a terrain and physical measurements. It encourages students to think about the practical limitations of making their measurements. Once they have made their measurements, they then apply trigonometry in 3-D calculations that they can visualize on their physical model.

Have students work in small groups to complete this project.

1. Start to plan

Many of the world's early navigators and this country's first explorers were surveyors. To begin, have students discuss the role of land surveyors in history and today. Have them look at different types of maps (plan views, topographical maps, terrain cross-sections) and discuss what kinds of measurements are needed for each one. How can three dimensions be represented on a flat map? Contour lines on a topographical map, colours on atlas maps. Review the use of scaling in mapmaking. At a scale of 1:500, how big would a typical person of 165 cm tall be? (3.3 mm) How tall would a 30-metre-tall tree be? (60 mm)

Student groups will now construct their terrain. Start with a square base at least 50 cm × 50 cm (representing 250 metres × 250 metres at a scale of 1:500). Using modelling clay, plaster, or papier maché, the students will add topographical features such as small hills, valleys, and bodies of water. They should keep in mind that this property should be reasonably suitable for a camp. It should not consist entirely of cliffs, or be covered mostly by water. Students should also choose and mark a direction that will be North on their terrain.

In order to survey around their property, the students will now define survey points on a boundary. These points, connected by straight lines, will define the closed boundary of their base camp. Each section should have a reasonably constant slope. If they are measuring up or down hills, they may need to break a straight line segment over a hill into several line segments with constant slope. The students should mark each survey point and label it with a letter. They should also draw straight lines connecting the points on their terrain. Encourage the students to use a minimum of five boundary segments at different bearings.

2. Construct surveying instruments and measure around the property

PART A: Construct surveying instruments

First have the students construct their measuring aids. These instruments—level, plumb-bob, rod, measuring tape, and compass—are the simplest complete set of instruments that can be used to perform a survey. The instruments that they will make are on a scale appropriate to the project scale. Consider that, by making the water level, plumb-bob, and rod longer, students can perform real survey measurements at full scale.

WATER LEVEL: A level is a device that indicates whether a line or surface is horizontal.

EQUIPMENT NEEDED: 1 piece of clear plastic flexible tubing approx. 30 cm long.

METHOD: Fill the tubing with water for approximately $\frac{2}{3}$ of its length.

THEORY: When the tubing is bent into a u-shape, the water level on both ends will be the same. A line connecting the water levels between the ends of the tube will be horizontal.

How to use the level: The diagram on the right shows how the level is used to measure differences in elevation.

SURVEY RODS: The survey rods act as the vertical reference against which elevations are measured and between which the horizontal is measured.

EQUIPMENT NEEDED: 2 bamboo skewers, at least 30 cm long. Two small strips of paper (1 cm × 2 cm) and tape.

METHOD: On each rod, measure and mark with a felt marker a point 10 cm from the pointed end of the skewer. Wrap one strip of paper tightly around each skewer and tape the end. The paper collar should be free to slide up and down on the skewer, but should be snug enough that it doesn't slide freely on its own.

HOW TO USE THE RODS: The diagram on the right shows how to use the survey rods with the level to measure a difference in elevation and the horizontal distance between two points. The 10 cm corresponds to the length ℓ in the diagram.

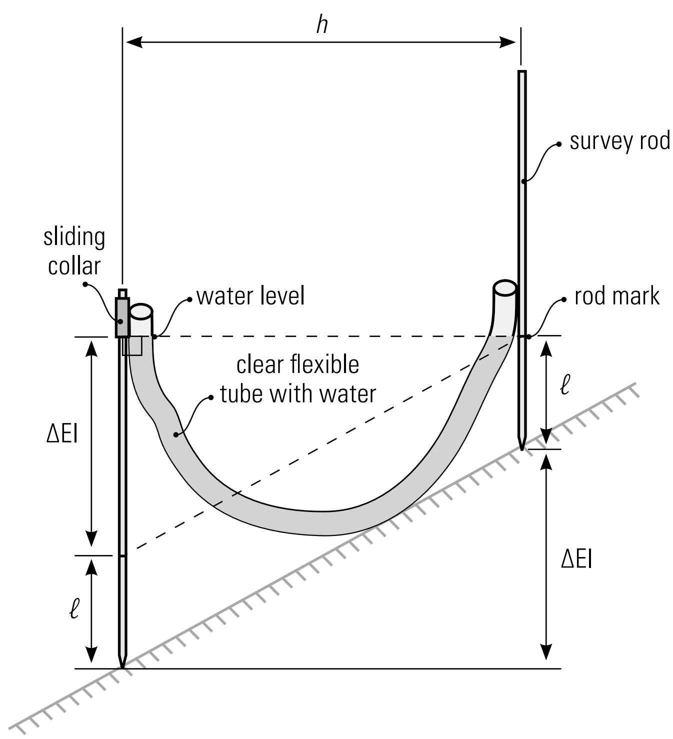
PLUMB-BOB: A plumb-bob is a device that indicates whether a line or surface is vertical.

EQUIPMENT NEEDED: 1 piece of thread approximately 30 cm long; one small weight (this can be anything that is not too bulky and can be easily tied to the end of the string; examples of objects to use are small buttons, paper clips, rings, beads).

METHOD: Tie the weight to the end of the thread.

THEORY: A weight on a string will be attracted by gravity, resulting in the string being vertical.

How to use the plumb-bob: When making the elevation and distance measurements using the rods, the rods must be vertical to reduce measurement error. Hold the plumb-bob next to the rod. Align the rod visually to be parallel with the plumb line. Move around the rod and align it with the plumb line from several (at least two) directions.



PART B: Measuring the property

Students will now do the measurements around the property using the simple surveying instruments that they have made. Give the students the table shown in Blackline Master 4.1C (p. 283) to help them record the measurements.

THE SURVEY CREW: Each student group can assign tasks to its members. Students should rotate through the tasks, so that each has the opportunity to do each task.

PERSON 1: Use the plumb-bob to work with Person 2 and 3 in aligning the rods. Person 1 will also mark, measure and record the elevation difference $\Delta E\ell$, and the horizontal distance, d.

PERSON 2: Hold rod 1 and align as instructed by Person 1.

PERSON 3: Hold rod 2 and align as instructed by Person 1. If students are working in groups of 3, Person 2 may also be Person 3; they must take care not to misalign the first rod while aligning the second.

PERSON 4: Hold the level so that the water level in the tube is at the height of the mark on the higher rod. Hold the other end of the tube next to the lower rod.

Procedure for Measuring Elevation and Horizontal Distance

1. Persons 2 and 3 hold the rods with the pointed tips on the survey points that are the end points of the segment to be measured.
2. Person 1 uses plumb-bob to help Persons 2 and 3 align the rods so they are vertical.
3. Person 4 puts the water level in place.
4. Person 1 marks the level of the water on the lower rod by sliding the paper collar to the level of the water.
5. Person 4 removes the water level.
6. Person 1 measures between the two rods at the water level marks, and records the horizontal distance, d, between the reference mark on the higher rod and the collar on the lower rod.
7. Person 1 lastly measures the distance $\Delta E\ell$ between the lower rod reference mark and the sliding collar.

Procedure for Measuring Bearing

1. Use a compass to line up your terrain north with the compass magnetic north by rotating the whole terrain.
2. Place the compass on the line segment to be measured. Align the compass reference with the line segment that you drew between the start and end points.
3. Read the angle between the compass reference and the compass needle (magnetic north). Read clockwise from north. Record the bearing angle.

3. Complete the survey calculations and prepare for presentation

At this point of the project, the students will have worked with the trigonometric functions in 2 and 3 dimensions, and completed examples and problems with calculations similar to those required for this part of the project.

Have the students complete the calculations to fill in their survey table. They should check their survey by verifying that the changes in elevation, distance north, and distance east around the property add up to zero. Because of measurement limitations (measuring to the nearest millimetre would be equivalent to measuring to the nearest half-metre), the calculations might not close exactly. Students should assess whether they are "close enough," or if not, they may need to redo some of their measurements.

On graph paper, have the students make a scale drawing of their property. Show distances in millimetres and scaled to metres.

Using the drawing, students now calculate the area of the property in square metres. They should use the principle of dividing the area into simple rectangles and right triangles. Remind them that they can subtract areas as well as add (drawing a simpler, larger shape to enclose their property, and then subtracting areas is sometimes easier than splitting up the interior of the shape).

To prepare for the final presentation, remind the students of the final presentation checklist. Have them share the presentation tasks.

ASSESSING THE PROJECT

1. Start to plan

- Remind students that this is a group project and that they must share the work, using each person in the way that they can be most useful.
- Advise them of assessment methods and how you will assign grades. With a group project like this, where there is a distribution of tasks, self and peer evaluations are of use. You may use Blackline Master 4.1A (p. 281). Inform the students how you will be using this self/peer assessment and also how you will use the Project Assessment Rubric, which follows on the next page.
- Encourage students to incorporate a reasonable level of complexity in their model terrains. The models should have significant three-dimensionality (shouldn't be too flat). The boundary that they choose should have five or more distinct segments of reasonably constant slope.

2. Construct surveying instruments and measure around the property

- Students should understand the theory and use of the simple survey instruments.
- Each student should have the opportunity to do each task on the survey crew.

3. Complete the survey calculations

- Students can use tables such as those in Blackline Masters 4.1C (p. 283) to complete the calculations. Encourage them to make sketches of geometries where the right triangle interior angles must be calculated from the bearing.
- Students should understand that, because the boundary starts and ends at the same point, the total change in elevation, change in distance north and change in distance east must add up to zero. Have them discuss why their own calculations might not add up to zero. They should make a judgment as to whether they are close enough, or need to remeasure, and be prepared to justify their judgement.

4. Making a presentation

- Once students have finished their project, allow class time for presentation. You might display the models and area plans around the classroom for viewing by the whole class. During the presentation, students should be able to discuss the use of the survey instruments, the limitations and accuracy of their measurements, how they used trigonometric functions in their calculations, and what strategies they used to make the area calculations.

PROJECT ASSESSMENT RUBRIC: SURVEY A YOUTH WILDERNESS BASE CAMP				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Explanations show an understanding of three-dimensional measurement	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are often incomplete or somewhat confusing	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and thorough
Procedural Understanding				
Accurately: • created camp features to scale • defined survey points • constructed survey instruments • used survey instruments to obtain measurements • converted measurements to scale • calculated angle of elevation/depression • prepared a scale drawing of the camp property • calculated the area of the camp	limited accuracy; major errors or omissions For example: • camp features are not to scale • survey points are missing or inaccurate • survey instruments were not constructed • measurements missing or not converted to scale • scale drawing is missing • calculations are missing or incorrect • project is incomplete	partially accurate; some errors or omissions For example: • some camp features are to scale • most survey points are correct • survey instruments constructed and used • most measurements are converted to scale • scale drawing is complete • most calculations are correct • project could use extra attention	generally accurate; few errors or omissions For example: • all camp features are to scale • survey points are correct • survey instruments constructed and used • measurements converted to scale • scale drawing is complete • calculations are correct • project is completed and correct	accurate and precise; very few or no errors For example: • all camp features are to scale and are creative • survey points are correct • survey instruments constructed and used • measurements converted to scale • scale drawing is complete • calculations are correct • adds some extra creativity to the project
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

ALTERNATIVE CHAPTER PROJECT — SURVEY A YOUTH WILDERNESS BASE CAMP

STUDENT MATERIALS

PROJECT OVERVIEW

The profession of land surveying dates back almost 5000 years. Early civilizations recognized the importance of accurate measurement of lands and buildings, the establishment of boundaries, and the usefulness of accurate maps. The ancient Egyptians used simple geometry in building the Great Pyramid at Giza 2700 BCE, and there is historical evidence that the Romans were the first civilization to employ professional surveyors. Today, land surveyors are often the first on the ground in any activity involving land, be it laying out of subdivisions and private properties, or development of large-scale public projects. Although today most surveying is done using sophisticated electronic devices, the basic trigonometric principles still apply: land surveyors measure distances and angles, and use trigonometry to calculate precise locations of points on the surface of the earth.

In this project, you will create a model terrain for a Youth Wilderness Base Camp. You will then make a set of simple survey tools—level, plumb-bob, survey rod—which, together with a ruler and compass, you will use to survey the boundary of your property and calculate its area.

GET STARTED

To start your project, think about the type of terrain that you would like to create. You might want some hills and valleys, streams, or a pond, etc. You will make your terrain at a scale of 1:500. At this scale, how tall would a 165-cm tall person be? How tall would a 30-m tree be? Start with a base around 60 cm by 60 cm cut from thin plywood or other material that will be strong enough to support the weight of a clay model. To create your terrain, use modelling clay, plaster, or papier maché. Once your terrain is dry, you might want to paint it in realistic colours, add in small trees, etc. (keeping in mind the scale of your model).

Once your model is complete, define a boundary for your base camp. To do this, choose a set of points that will define corners. Connect the points by drawing straight lines between them. Try to use more than five line segments, and include at least half of your terrain inside the boundary.

In order to survey your boundary, you must now define survey points that will approximate it with straight lines. The corners of your boundary will be survey points. However, you may want to break up the line segments into shorter pieces where the elevation is changing, such as going over a hill. Label the survey points with letters A, B, C,and so on. Label the line segments AB, BC.

Finally, define a north direction for your terrain. Draw a straight line arrow on the terrain and label it North.

CONSTRUCT YOUR SURVEY INSTRUMENTS

Your survey crew will need a set of basic survey instruments. These are: tape measure, level, plumb-bob, survey rods, and compass. As a tape measure, you will use a ruler marked in millimetres. A level is a device that indicates whether a line or surface is horizontal. A plumb-bob is a device that indicates whether a line or surface is vertical. The survey rods act as the vertical reference against which elevations are measured and between which the horizontal lengths are measured.

Your teacher will give you the instructions for making and using the level, plumb-bob, and survey rods.

MAKE YOUR MEASUREMENTS

Now send your survey crew out into the field. Your teacher will give you a crew task list and an outline of the measurement procedures. Assign tasks to each member of the crew (you may want to rotate through the tasks, so that each person has the opportunity to do each task). Review the procedures for making the measurements. Your first survey point will be the reference point (0,0). Survey your way around the boundary, taking measurements and recording them for each line segment. Use the tables provided. Record lengths to the nearest millimetre. Measure bearings to the nearest degree. Record an increase in elevation as positive and a decrease in elevation as negative.

When you have arrived back at your starting point, add up the changes in elevation. What is the total that you should expect? Why? Are you close?

When you have completed your measurements, convert them to metres using your terrain scale. Calculate the angles of elevation/depression and the ΔEast and ΔNorth distances as given in the tables using the appropriate trigonometric functions. Pay close attention to the angles that you use; it may be helpful to do a sketch of each line segment, identify the appropriate right triangle, and determine how the bearing relates to the internal angles. Record a distance south as a negative distance north, and a distance west as a negative distance east.

DRAW YOUR PLAN AND CALCULATE YOUR PROPERTY AREA

On graph paper, prepare a scale drawing of your property. You can use either the bearings and horizontal lengths that you measured, or the ΔEast and ΔNorth distances that you calculated in Table 2. Label each distance in millimetres and in scaled metres.

As a group, decide on the best way to calculate the area of your property. You can either divide the interior into simple rectangles and right triangles, enclose the whole property in a larger, simpler shape and subtract areas, or a combination of these two. Make a table to calculate the area of each section, and record the method that you used to calculate it. Finally, calculate the total area in square metres.

COMPILE YOUR WORK AND PREPARE YOUR PRESENTATION

You now have a model of your Youth Wilderness Base Camp terrain, your table of measurements and calculations, a plan of the camp, and a calculation of the area of the camp. For your presentation, you will discuss how you used the basic survey instruments (be prepared to explain how each one works), which measurements you made using the instruments and which ones you calculated, and which trigonometric functions you used. Discuss how you verified your measurements, and why the changes in elevation and distance should add to zero. Discuss why your calculations might not have added to exactly zero. Present your area plan, and show how you calculated the area. Compare your calculated area with those calculated by other groups. Are they comparable?

FINAL PRESENTATION CHECKLIST

You will present your final project to the class. This will include the following items:

- model of the Youth Wilderness Base Camp terrain, showing survey points, lines connecting the survey points, and the north arrow;
- table of measurements and calculated values, including the angle of elevation/depression, ΔEast and ΔNorth distances for each segment;
- scale drawing of the area plan showing the boundary and illustrating the strategy used to calculate the area; and
- total area calculation.

BLACKLINE MASTER 4.1A

ALTERNATIVE CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness with which you designed your camp and recorded your measurements;
- the accuracy of your calculations;
- the effectiveness of your uses of technology for completing measurements;
- the creativity you brought to planning and presenting; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

Were you able to complete all aspects of the project? If not, why? Did you allot your time effectively?

In what areas did you excel? ______________________

Are there areas in which you could improve? ______________________

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 4.1B

ALTERNATIVE CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PROJECT CHECKLIST	
❒ Is your camp model ready and did you create your camp features to scale? Does your model include connected survey points and an arrow pointing north?	
❒ Did you make accurate measurements with your survey instruments and record these on a table?	
❒ Are your measurements converted to scale?	
❒ Did you calculate the angle of elevation and depression?	
❒ Have you prepared a scale drawing of the camp property that includes property boundaries?	
❒ Did you calculate the area of your camp? Did you record the strategy you used to calculate the area?	

BLACKLINE MASTER 4.1C

ALTERNATIVE CHAPTER PROJECT TABLE

Name: ______________________ Date: ______________________

CHAPTER PROJECT TABLE

Line Segment	d_m *(mm) measure*	$\Delta E\ell_m$ *(mm) measure (positive if increasing, negative if decreasing)*	d *(m) calculate using scale*	$\Delta E\ell$ *(m) calculate using scale (positive if increasing, negative if decreasing)*	*Angle of elevation/depression* α *(degrees) calculate using trigonometric formula* $\frac{d}{E\ell} = \tan\alpha$
AB					
BC					
CD					
DE					
	Total $\Delta E\ell_m$:		Total $\Delta E\ell$:		

BLACKLINE MASTER 4.9

REVIEWING PRIOR CONCEPTS

Name: ______________________ Date: ______________________

The Sine Ratio

1. An angle of 43° is opposite a side of 10 cm. What is the length of the hypotenuse?
2. An angle is opposite a side that is 4 in. and the hypotenuse is 14 in. What is the sine ratio?
3. An angle is opposite a side that is 4 in. and the hypotenuse is 14 in. What is the angle?
4. An angle of 16° is next to a hypotenuse of 24 cm. What is the opposite side of this triangle?
5. A triangle has sides of 12 cm, 5 cm, and 13 cm. Use the sine ratio to calculate the angles in the triangle.

The Cosine Ratio

6. An angle of 23° is adjacent to a side of 8 cm. What is the length of the hypotenuse?
7. An angle is adjacent to a side that is 3.5 in. and the hypotenuse is 6 in. What is the cosine ratio?
8. An angle is adjacent to a side that is 3.5 in. and the hypotenuse is 6 in. What is the angle?
9. An angle of 11° is next to a hypotenuse of 85 cm. What is the adjacent side of this triangle?
10. A triangle has sides of 21 cm, 20 cm, and 29 cm. Use the cosine ratio to calculate the angles in the triangle.

The Tangent Ratio

11. An angle of 51° is opposite a side of 9.9 cm. What is the length of the adjacent side?
12. An angle is opposite a side that is 13 in. and adjacent to a side of 16 in. What is the tangent ratio?
13. An angle is opposite a side that is 13 in. and adjacent to a side of 16 in. What is the angle?
14. An angle of 34° is adjacent to a side of 5 cm. What is the opposite side of this triangle?
15. A triangle has sides of 8 cm, 15 cm, and 17 cm. Use the tangent ratio to calculate the angles in the triangle.

The Pythagorean Theorem

Find the missing side (a = adjacent, b = opposite, and c = the hypotenuse).

16. a = 28 in. b = 45 in. c = ?
17. a = ? b = 9 cm c = 41 cm
18. a = 12 b = ? c = 37
19. a = 17 ft b = 144 ft c = ?
20. a = ? b = 7 c = 25

BLACKLINE MASTER 4.9: SOLUTIONS

1. $$\sin 43° = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 43° = \frac{10}{x}$$

$$x \sin 43° = 10$$

$$x = \frac{10}{\sin 43°}$$

$$x \approx 15 \text{ cm}$$

2. $$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin \theta = \frac{4}{14}$$

$$\sin \theta \approx 0.2857$$

3. (including the solution of #2)

$$\theta = \sin^{-1}(0.2857)$$

$$\theta \approx 17°$$

4. $$\sin 16° = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 16° = \frac{x}{24}$$

$$24 \times \sin 16° = x$$

$$6.6 \text{ cm} \approx x$$

5. $$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{12}{13}$$

$$A = \sin^{-1}\left(\frac{12}{13}\right)$$

$$A \approx 67°$$

Reassign the opposite and adjacent sides to use the sine ratio to find the other angle.

$$\sin B = \frac{\text{opp}}{\text{hyp}}$$

$$\sin B = \frac{5}{13}$$

$$B = \sin^{-1}\left(\frac{5}{13}\right)$$

$$B \approx 23°$$

6. $$\cos 23° = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 23° = \frac{8}{x}$$

$$x \cos 23° = 8$$

$$x = \frac{8}{\cos 23°}$$

$$x \approx 8.7 \text{ cm}$$

7. $$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos \theta = \frac{3.5}{6}$$

$$\cos \theta \approx 0.5833$$

8. $$\theta = \cos^{-1}(0.5833)$$

$$\theta \approx 54°$$

9. $$\cos 11° = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 11° = \frac{x}{85}$$

$$85 \times \cos 11° = x$$

$$83 \text{ cm} \approx x$$

10. $$\cos a = \frac{\text{adj}}{\text{hyp}}$$

$$\cos a = \frac{21}{29}$$

$$\cos a = 0.7241$$

$$a = \cos^{-1}(0.7241)$$

$$a \approx 44°$$

Reassign the opposite and adjacent sides to use the cosine ratio to find the other angle.

$$\cos b = \frac{\text{adj}}{\text{hyp}}$$

$$\cos b = \frac{20}{29}$$

$$\cos b \approx 0.6897$$

$$b = \cos^{-1}(0.6897)$$

$$b \approx 46°$$

11. $$\tan 51° = \frac{\text{opp}}{\text{adj}}$$

$$\tan 51° = \frac{9.9}{x}$$

$$x \tan 51° = 9.9$$

$$x = \frac{9.9}{\tan 51°}$$

$$x \approx 8 \text{ cm}$$

12. $$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{13}{16}$$

$$\tan \theta = 0.8125$$

13. (including the solution of #12)

$$\theta = \tan^{-1}(0.8125)$$

$$\theta \approx 39°$$

14. $$\tan 34° = \frac{\text{opp}}{\text{adj}}$$

$$\tan 34° = \frac{x}{5}$$

$$5 \times \tan 34° = x$$

$$3.4 \text{ cm} \approx x$$

15. $$\tan a = \frac{\text{opp}}{\text{adj}}$$

$$\tan a = \frac{8}{15}$$

$$\tan a \approx 0.5333$$

$$a = \tan^{-1}(0.5333)$$

$$a \approx 28°$$

Reassign the opposite and adjacent sides to use the tan ratio to find the other angle.

$$\tan b = \frac{\text{opp}}{\text{adj}}$$

$$\tan b = \frac{15}{8}$$

$$\tan b = 1.875$$

$$b = \tan^{-1}(1.875)$$

$$b \approx 62°$$

16. $\sqrt{28^2 + 45^2} = 53$ in

17. $\sqrt{41^2 - 9^2} = 40$ cm

18. $\sqrt{37^2 - 12^2} = 35$

19. $\sqrt{17^2 + 144^2} = 145$ ft

20. $\sqrt{25^2 - 7^2} = 24$

BLACKLINE MASTER 4.10

PRACTISE SOLVING PROBLEMS WITH MULTIPLE TRIANGLES

Name: ______________________________ Date: ______________________________

You can use the sine, cosine, and tangent ratios to solve for right-angled triangles. But what can you do about triangles with no right angles? You can break them up into right-angled triangles and solve from there.

The tools you can use to solve triangle problems with multiple triangles include:

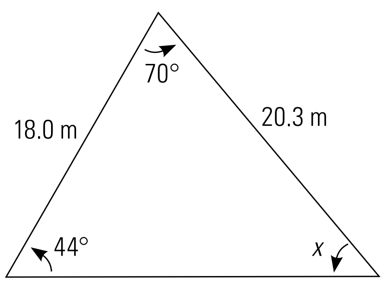
1. the Pythagorean Theorem ($a^2 + b^2 = c^2$)
2. breaking a problem into parts (for example, dividing a large triangle or shape into smaller triangles)
3. sine ratio
4. cosine ratio
5. tangent ratio
6. drawing imaginary lines (for example, from a height to the ground) to create right-angled triangles

All of these strategies are used in this chapter.

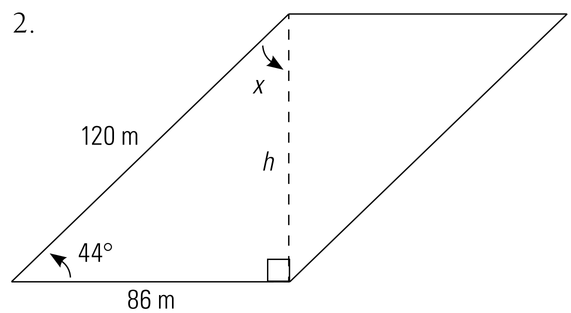
Part 1

Write the steps that you would use to break the following triangles into right-angled triangles that you could solve. Then follow the steps to find all the labelled angles and sides.

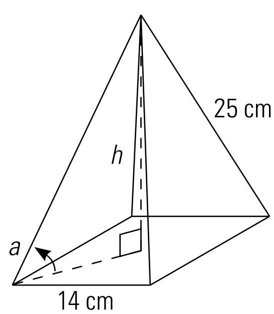
1.



2.



3.



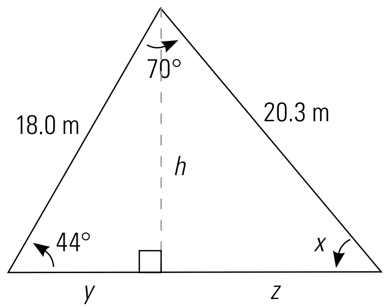
Part 2

Create three problems that could each be solved by one of the tools above (use a different tool for each problem). Draw one or more triangles that are missing measurements. Trade with a classmate to solve, or show the worked solution yourself.

BLACKLINE MASTER 4.10: SOLUTION

Part 1

1.



Step 1: Divide into two right triangles.

Step 2: Solve for h using the sine ratio.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 44° = \frac{h}{18}$$

$$18 \times \sin 44° = \frac{h}{18} \times 18$$

$$18 \times \sin 44° = h$$

$$12.5 \approx h$$

Step 3: Use 18.0 m and h to solve for the base of the left triangle using the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

$$hyp^2 = y^2 + h^2$$

$$18^2 = y^2 + 12.5^2$$

$$18^2 - 12.5^2 = y^2$$

$$\sqrt{324 - 156.25} = y$$

$$\sqrt{167.75} = y$$

$$13 \approx y$$

Step 4: Use the Pythagorean theorem to solve for z.

$$c^2 = a^2 + b^2$$

$$hyp^2 = h^2 + z^2$$

$$20.3^2 = 12.5^2 + z^2$$

$$20.3^2 - 12.5^2 = z^2$$

$$\sqrt{412.09 - 156.25} = z$$

$$\sqrt{255.84} = z$$

$$16 \approx z$$

Step 5: Use the sine ratio to solve for angle x.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

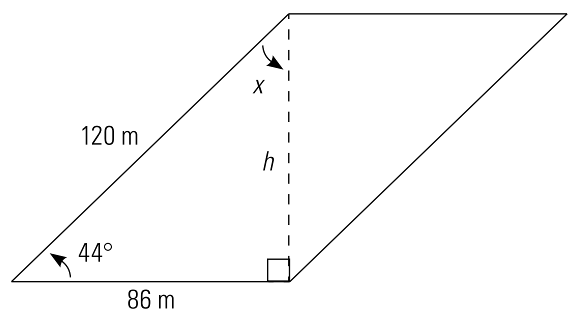
$$\sin x = \frac{h}{\text{hyp}}$$

$$\sin x = \frac{12.5}{20.3}$$

$$x = \sin^{-1}\left(\frac{12.5}{20.3}\right)$$

$$x \approx 38°$$

2.



Step 1: Solve for h using the Pythagoream theorem.

$$c^2 = a^2 + b^2$$

$$hyp^2 = base^2 + h^2$$

$$120^2 = 86^2 + h^2$$

$$\sqrt{120^2 - 86^2} = h^2$$

$$\sqrt{14\,400 - 7396} = h$$

$$\sqrt{7004} = h$$

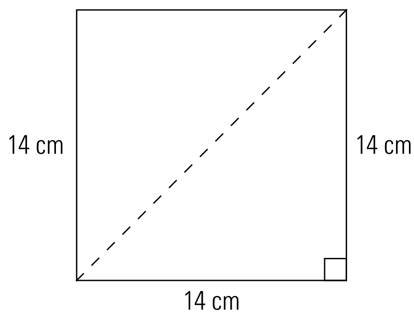
$$83.7 \approx h$$

Step 2: Solve for angle x.

$$x = 180° - 90° - 44°$$

$$x = 46°$$

3. Step 1: Look at the square base of the pyramid.



Solve for the distance between opposite corners using the Pythagorean theorem.

$$d^2 = \text{side}^2 + \text{side}^2$$

$$d^2 = 14^2 + 14^2$$

$$d = \sqrt{196 + 196}$$

$$d = \sqrt{392}$$

$$d \approx 20$$

The length of d is 20 cm.

The distance from the middle of the square to the corner is half this.

$20 \div 2 = 10$

Step 2: Find the height of the pyramid using the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

$$25^2 = 10^2 + h^2$$

$$25^2 - 10^2 = h^2$$

$$\sqrt{25^2 - 10^2} = h$$

$$\sqrt{625 - 100} = h$$

$$\sqrt{525} = h$$

$$23 \approx h$$

The height of the pyramid is 23 cm.

Step 3: Solve for angle a using the tangent ratio.

$$\tan a = \frac{\text{opp}}{\text{adj}}$$

$$\tan a = \frac{23}{10}$$

$$a = \tan^{-1}\left(\frac{23}{10}\right)$$

$$a \approx 67°$$

Angle a is 67°.

Chapter 5

Scale Representations

INTRODUCTION

STUDENT BOOK, pp. 208–251

In this chapter, students learn the features of different styles of technical drawings. Students have the opportunity to use and enhance their skill and comfort level working with scale.

This chapter addresses the outcomes of the Geometry strand in the Workplace and Apprenticeship Mathematics Common Curriculum Framework. The chart below locates this chapter within the curriculum.

GEOMETRY, GRADES 10–12		
This chart illustrates the development of the Geometry strand in the Workplace and Apprenticeship Mathematics pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 5 addresses.		
Grade 10	Grade 11	Grade 12
General Outcome Develop spatial sense.	**General Outcome** Develop spatial sense.	**General Outcome** Develop spatial sense.
Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:	**Specific Outcome** It is expected that students will:
Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.	Solve problems that involve two and three right triangles.	Solve problems by using the sine law and cosine law, excluding the ambiguous case.
Demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.	Solve problems that involve scale.	Solve problems that involve triangles, quadrilaterals, and regular polygons.
Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	Model and draw 3-D objects and their views.	Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including translations, rotations, reflections, and dilations.
Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.	Draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects.	
Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.		
Demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex by drawing, replicating and constructing, bisecting, and solving problems.		

CURRICULUM AND CHAPTER OVERVIEW

General Outcome:
Develop spatial sense.

Specific Outcome:
Solve problems that involve scale.

Specific Outcome:
Draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects.

Specific Outcome:
Model and draw 3-D objects and their views.

Section 5.1: Scale Drawings and Models

- Math on the Job
- Explore the Math
- Mental Math and Estimation
- Discuss the Ideas: The Impact of Scale
- Activity 5.1: Draft a Floor Plan and Build a Model
- Build Your Skills

Section 5.2: Two-Dimensional Representations

- Math on the Job
- Explore the Math
- Mental Math and Estimation
- Discuss the Ideas: Views
- Activity 5.2: Japanese Cushion Designs
- Activity 5.3: Design and Build a Greenhouse Frame to Scale
- Puzzle It Out: Shape Shifting
- Build Your Skills

Section 5.3: Three-Dimensional Representations

- Math on the Job
- Explore the Math
- Discuss the Ideas: Diagrams at Work
- Mental Math and Estimation
- Activity 5.4: Draw a Qamutik
- The Roots of Math: Technical Illustrations
- Build Your Skills

Chapter Project: Produce a Set of Drawings and a Scale Model

Reflect on Your Learning

Practise Your New Skills

THE MATHEMATICAL IDEAS

SCALE, PROPORTIONAL REASONING, AND 2-D AND 3-D DRAWINGS

This chapter provides students with an opportunity to apply their prior knowledge of scale. They will use this knowledge to create different styles of technical drawings and to illustrate objects from different points of view. Such drawings are used in a wide range of careers.

It is expected that students will create diagrams that are neat, have crisp lines and corners, and are labelled with the required details. Reviewing the following concepts will help students successfully complete this chapter.

Scale factor: Use guided questions to help students realize that a scale factor is a multiplicative relationship. Explain to students that scale factor is the number by which all dimensions of the original object or image are multiplied to find the dimensions of the enlargement or reduction. Let students know that people sometimes mistakenly conclude that scale factor is an additive relationship instead.

Proportional reasoning: Students have used proportional reasoning in a variety of contexts. Here, they will apply it to concrete situations (3-D objects) and use it to calculate how to transfer the information to a scale model or drawing using proportional reasoning.

Scale and units: You can review how to apply a scale factor to both SI and imperial units. If necessary, you can hand out a review sheet that asks students to convert different units, or provide them with conversion tables.

The following concepts will be new to most students. They will benefit from careful and thorough explanations. Providing students with time to practise making the new types of diagrams covered in this chapter will also increase understanding.

Views: A view is a 2-D representation of one face, or plane, of an object. To accurately represent an object with view drawings, it is necessary to draw each plane that has different dimensions or features. If two or more planes of an object have the same dimensions, this view need only be drawn once. As a class, you can examine a 3-D object, identify its different faces, and draw its top, front, and side views.

Component parts diagrams: These diagrams show the different parts of an object. They are two-dimensional drawings that are made to scale and labelled with their component name and dimensions. If parts of an object are the same, they are usually only drawn once. For example, if a bookshelf has three shelves that have the same dimensions, only one shelf would appear in a component parts diagram.

Isometric diagrams: Isometric diagrams are commonly drawn on isometric dot paper. The dots on the SI version of this paper are spaced 1 centimetre apart. When drawing isometric diagrams on dot paper, the lines of the diagram should always pass through or run parallel to the dots.

These diagrams are used to represent a 3-D object's true dimensions, since isometric drawings use the same scale to draw an object's length, width, and height. Parallel lines do not become distorted, but are drawn to be parallel.

One-point perspective diagrams: A one-point perspective drawing is a 2-D representation of a 3-D object. This type of drawing is used to create the impression of depth and space. Lines that are parallel in reality appear to converge at a vanishing point and objects in the foreground are represented as larger than those in the background.

Exploded diagrams: Exploded diagrams are representations of 3-D objects that show how the parts of an object fit together to form the whole. Exploded diagrams show the different parts of an object drawn separately and set apart. Dashed lines are used to show where the parts will connect together. These diagrams can be seen as a visual explanation of the relationship between the component parts.

If students need practise completing the different diagrams covered in this chapter, you can provide them with Blackline Masters 5.5 (p. 335) and 5.6 (p. 336).

WHY ARE THESE CONCEPTS IMPORTANT?

- Many careers require the ability to read, interpret, and follow diagrams.
- As consumers, students will encounter situations where they must be able to read, interpret, and follow diagrams. For example, they may need to assemble a piece of furniture, using illustrated instructions.
- Diagrams are important when communicating ideas at work and in our personal lives. For example, some students may use blueprints to build their own homes, or leave illustrated instructions for a co-worker to follow.

PRIOR SKILLS AND KNOWLEDGE

Students will apply their knowledge of scale and their ability to convert between SI and imperial units to make different diagrams. Students might have made some of the diagrams covered in this chapter, although they may not know them by name.

Because of past experiences with drawing scale diagrams, students may feel overwhelmed or start with a feeling "that they will never be able to draw the diagram." By breaking the process down into individual steps and with some strategies, they will be successful and gain confidence. This process requires students to take their time and create neat and accurate drawings.

1. Concepts
   a) proportional reasoning;
   b) scale factor;
   c) scale statement;
   d) enlargement and reduction; and
   e) representing the same object in two and three dimensions.
2. Mathematics Skills
   a) using a fractional equation to solve for an unknown; and
   b) multiplication and division.
3. T Technology
   a) computer-aided design (CAD) software (optional);
   b) basic calculator functions;
   c) internet research skills; and
   d) geometry software.

REVIEWING PRIOR CONCEPTS

You may want to assist some students to review the following concepts and processes:

- scale;
- ratios and proportions;
- converting between imperial and SI units;
- reading, understanding, and using a ruler for whole and partial measurements, using both SI and imperial units; and
- trigonometry.

Blackline Master 5.8 contains review questions and solutions. It is found on p. 340 of this teacher resource.

PLANNING CHAPTER 5

This chapter will take about three weeks of class time to complete. Class period estimates are based on a class length ranging from 60 to 75 minutes. Actual time may vary depending on individual classroom needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
		Introduce the Chapter Project: Produce a Set of Drawings and a Scale Model	20 minutes	Blackline Master 5.4 (p. 334)
5.1	210	Math on the Job: Aerial Photographer	15 minutes	
5.1	210 211	Explore the Math Examples 1, 2	20 minutes	
5.1	213	Mental Math and Estimation	5 minutes	
5.1	213	Discuss the Ideas: The Impact of Scale	15 minutes	
5.1	214	Activity 5.1: Draft a Floor Plan and Build a Model	40 minutes for part 1 and 1 class for part 2	paper, pencils, eraser, ruler, materials for scale model
5.1	215	Build Your Skills	40 minutes	
5.2	218	Math on the Job: Metal Fabricator	30 minutes	
5.2	219	Explore the Math	30 minutes	
5.2	220 222	Examples 1, 2 Mental Math and Estimation	1 class	Blackline Master 5.1 (p. 331) (isometric dot paper) if giving students items to practise drawing
5.2	222	Discuss the Ideas: Views	15 minutes	
5.2	223	Activity 5.2: Japanese Cushion Designs	45 minutes	pencils, paper, rulers, calculators
5.2	224	Activity 5.3: Design and Build a Greenhouse Frame to Scale	45 minutes	pencils, paper, rulers, calculators, set square
5.2	225	Puzzle It Out: Shape Shifting	30 minutes	graph paper, scissors, pencils, or manipulatives
5.2	226	Build Your Skills	40 minutes	paper for drawings
	231	Chapter Project: Draw Your Object	1 class	access to computer lab, paper, extra rulers, pencils, and erasers
5.3	232	Math on the Job: Illustrator and Animator	20 minutes	
5.3	232	Explore the Math	30 minutes	
5.3	234	Examples 1, 2	60 minutes	Blackline Master 5.1 (p. 331) (isometric dot paper) if giving students drawing practise
5.3	240 240	Discuss the Ideas: Diagrams at Work Mental Math and Estimation	30 minutes	
5.3	241 242	Example 3 Activity 5.4: Draw a Qamutik	1 class	Blackline Master 5.1 (p. 331) (isometric dot paper)
 5.3	243 244	The Roots of Math: Technical Illustrations Build Your Skills	1 class	Blackline Masters 5.1 and 5.2 (pp. 331–332)

PLANNING FOR INSTRUCTION				
Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	247	Chapter Project: Complete Your Drawings and Build a Scale Model	2 classes	
	247	Reflect on Your Learning		
	248	Practise Your New Skills	1 class	Blackline Masters 5.1 and 5.2 (pp. 331–332)
		Chapter Test (p. 321 of this resource)	1 class	

PLANNING FOR ASSESSMENT		
Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Discuss progress of chapter project to determine if students need clarification or extra help. • Ask students to explain the process they used to complete their diagram. • Display a checklist of features for each type of diagram on the board or poster paper for easy reference for the students. • Math on the Job • Examples • Discuss the Ideas • Explore the Math	• Observe student drawings in class and provide immediate feedback and suggestions as they create their drawings. • Allow students to self-monitor their work by checking it against a checklist of features for each type of drawing. • Allow students to provide feedback to fellow classmates on their drawings and assembly instructions. • Observe interactions among students.
Assessment as Learning	• Ask students to identify the correct and incorrect features of sample diagrams (teacher-generated). • Explore the Math • Activities • Project	• Check homework. • Listen to ideas, build on ideas, and encourage each student to participate in class discussions. • Discuss the usefulness of scale and the different diagrams.
Assessment of Learning	• Conference with students to provide feedback on their drawings (can be formal or informal). • Allow students as many times as needed to revise/redraw diagrams. • Activities • Chapter Project	• Record notes if students are assisting others. • Record notes on student understanding of necessary features of each diagram. • Quizzes and tests are not necessary to assess student learning.
Learning Skills/ Mathematical Disposition	• Classroom interactions and use of technology	• Check to see that students are able to create appropriate and correct scale drawings using technology.

CHAPTER PROJECT — PRODUCE A SET OF DRAWINGS AND A SCALE MODEL

GOALS: In this chapter project, students apply their knowledge of scale and technical drawings to support their instructions on how to assemble an object in a workplace.

OUTCOME: Students will prepare 2-D, 3-D, one-point perspective or isometric, component parts, and exploded diagrams to aid the communication of their instructions.

PREREQUISITES: Students need an understanding of proportions and scale. They will need to convert between units, visualize objects from different views, and do reasonable estimations.

ABOUT THE PROJECT: Brainstorming will help students who cannot decide on an object for their project. Approve their object before they proceed.

Some objects to base a project on include: hockey net, coffee table, entertainment unit, wagon, bedframe, outdoor swing, jewellery box, desk, spice rack, exercise equipment, crib, child's toy, plant shelf, lawn furniture, chair, t-shirt, costume, wedding cake, jewellery, kite.

An alternative project, "Design and Build a Birdhouse," is included on pp. 346–352.

1. Start to plan

STUDENT BOOK, p. 209

Although the project is placed at the beginning of the chapter, you can introduce it after the first type of drawing has been learned. Students will have a better understanding as to what type of drawings are necessary and thus make the most appropriate choice.

Most students will pick objects typically made from wood, plastic, metal, or fabric. Students will need to think about the various types of drawings that are required for the project. You will have to let students know where certain drawings of their object would not be suitable or possible.

After students have had their object approved, they can sketch their object. If they submit their sketch at this point, you can provide feedback.

Tactile learners can benefit from the hands-on experience of creating and assembling the components of their scale model.

2. Draw your object

STUDENT BOOK, p. 231

T Prior to this second phase of the project, book a computer lab so students can use the internet to research their object. As they research, ask students if the measurements for all components of their object are stated in the same units and same system of measurement.

Encourage students to use their research to reassess their sketch and to make any necessary modifications. Students should choose a scale that will allow their drawing to fit on a 8.5″ × 11″ sheet of paper. Ask students to clearly state the scale on their drawing and to include the units with the object's actual measurements.

Students will also need to choose materials from which to make their scale models. You will need to decide whether to provide some or all of these materials, or to ask students to find them independently.

3. Complete your drawings and build a scale model

STUDENT BOOK, p. 247

You can divide this stage of the project over two class periods, one for completing drawings and one for building models. Remind students to bring the materials to class the day before the models are built, if required. You may want to have students give presentations showing how their objects evolved from their initial sketch to the final model. They should also share discoveries they made about the construction process. The diagrams and models could also be displayed in a classroom.

Before students begin working on their drawings, review the characteristics of each type of drawing. By writing these on paper posted in the room, students can use them for reference.

As students work, circulate and provide encouragement and feedback. For those students who are struggling, one of these suggestions may help: draw lightly and have a good white eraser, tape the edges of the paper to the table to stop the paper from moving, use a protractor to create accurate angles, invert the object and draw it upside down, place a sheet of graph paper underneath the drawing paper (to act as a guide for making straight lines with a ruler and to help with 45° angles), or have a sample of the object (or something similar) that they can hold, turn and look at.

This will provide tactile and visual learners with the opportunity to manipulate and to look at the object at eye level which will in turn aid them in "seeing" what they need to draw. Some students may benefit from looking at the object they need to draw as individual shapes (squares, rectangles, circles, etc.). Then they can draw the object one shape at a time. By identifying the shapes and not thinking of the whole object, students may find it easier and less overwhelming.

Encourage the students to add any other information they feel would help the reader to interpret their diagrams. It could be things like colour coding, numbered steps, written explanations, a listing of tools needed, or photos or a video on how to assemble the object.

Students should compile their project so that it is complete and looks professional.

ASSESSING THE PROJECT

1. Start to plan

STUDENT BOOK, p. 209

Decide what the assessment tools will be and explain them to the students before they begin. Decide on the mark value for each component of the project.

If students choose an object that is too difficult or too simple, encourage them to add to or scale down their object. This is an opportunity for you to assess how well the student understands and is applying the curricular outcomes.

Provide the students with a rubric, expectations, and mark allocations so that they may reference this information as they prepare their project.

2. Draw your object

STUDENT BOOK, p. 231

As students are working on their drawings, keep notes on how the student utilizes class time, if they work but struggle and/or it takes them longer to complete the drawings, if this area of math is a strength, and/or if students assist their classmates.

3. Complete your drawings and build a scale model

STUDENT BOOK, p. 247

Before students submit their final project, have them do an assessment of their project based on the same rubric you will use (p. 298). You may consider having the students justify the mark they have given themselves using reference to the characteristics their drawings should have.

If possible, find a location in your school to display your students' work. It is a great opportunity to promote the course and showcase student learning.

PROJECT ASSESSMENT RUBRIC				
	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
CONCEPTUAL UNDERSTANDING				
• Explanations show an understanding of scale, exploded, component parts, isometric, and perspective diagrams	Shows very limited understanding; explanations are incorrect	Shows partial understanding; explanations are often incomplete or somewhat confusing	Shows understanding; explanations are appropriate	Shows thorough understanding; explanations are effective and thorough
PROCEDURAL KNOWLEDGE				
• Accurately: ▪ draws perspective and scale diagrams ▪ chooses an appropriate scale for a model ▪ uses a scale to build a scale model ▪ calculates the dimensions of an object, given a scale ▪ draws angles in the correct direction for 3-D representations ▪ shows how to assemble an object using an exploded diagram ▪ breaks down an object into its correct component parts	limited accuracy; major errors or omissions For example: • not all drawings are drawn correctly and to scale • scale is acceptable for type of object • most dimensions of the scale object were not calculated correctly • angles in 3-D representations do not go in correct direction • does not show how to assemble an object using an exploded diagram • does not break down an object into its correct component parts • scale model is missing or incomplete	partially accurate; some errors or omissions For example: • some drawings are drawn correctly and to scale • scale is acceptable for type of object • more than half the dimensions of the scale object were calculated correctly • angles in 3-D representations go in correct direction • shows how to assemble an object using an exploded diagram • breaks down an object into most of its correct component parts • scale model is incomplete and has some incorrect measurements	generally accurate; few errors or omissions For example: • drawings are drawn correctly and to scale • scale is acceptable for type of object • all the dimensions of the scale object were calculated correctly • angles in 3-D representations go in correct direction • shows how to assemble an object using an exploded diagram • breaks down an object into its correct component parts • scale model is complete and has correct measurements	accurate and precise; very few or no errors For example: • drawings are precise and to scale • scale is acceptable • all the dimensions of the scale object were calculated correctly • angles in 3-D representations go in correct direction • shows how to assemble an object using an exploded diagram • breaks down an object into its correct component parts • scale model is creative, complete, and has correct measurements
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; refuses to try suggestions on how to create diagrams	uses some appropriate strategies, with partial success, to solve problems; reluctantly tries suggestions on how to create diagrams	uses appropriate strategies to successfully solve most problems and explain solutions; willingly tries suggestions on how to create diagrams	uses effective and often innovative strategies to successfully solve problems; successfully creates diagrams on own
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

5.1 Scale Drawings and Models

TIME REQUIRED FOR THIS SECTION: 5 CLASSES

STUDENT BOOK, pp. 210–217

Some students may feel insecure about their drawing ability. Emphasize that they are developing technical drawing skills, using measuring and drafting tools, and artistic skills are not required. You can review use of graph paper, a straight edge, and other drawing tools. Working in groups that include a student with more confidence in their drawing ability will help those that are less so. This is a chapter where the artists in the group may shine in math class (especially when perspective drawings are introduced).

MATH *ON THE JOB*

STUDENT BOOK, p. 210

The entrance point to the section can be made through an existing scale statement. You can circulate or project a map or aerial photo that includes a scale and elicit what the scale is used for, and what information it conveys. In the scale statement, what does the unit on the left of the colon represent? The one on the right? Ask students to recall other situations where scale is used. Students have studied scale in grade 10, and this is an opportunity to activate their prior learning.

SOLUTIONS

1. Multiply to find the length and width.

   Length: 3.2 × 10 000 = 32 000

   Width: 4.1 × 10 000 = 41 000

   The fenced area is 32 000 cm long and 41 000 cm wide, or 320 m long and 410 m wide.

2. One hectare is 10 000 square metres. The field is therefore 100 m long and 100 m wide.

   Convert to centimetres.

   100 × 100 = 10 000

   The field is 10 000 cm long and 10 000 cm wide.

   Divide the length and width by the scale factor, 10 000.

   10 000 ÷ 10 000 = 1

   In the photograph, the pasture would measure 1 cm × 1 cm.

EXPLORE THE MATH

STUDENT BOOK, p. 210

Students will likely be familiar with scale statements but they may not understand exactly what they mean. Guide students to realize that scale can be used for reductions as well as enlargements. Students who have experience sewing from commercial patterns or building models of airplanes, boats, or trains may be able to help their fellow students to build understanding of scale statements from the point of view of a consumer. Invite them to think about where these scale statements come from and how they are worked out before they are printed on a box or set of instructions.

You may need to remind students that a ratio can be expressed as a fraction ($\frac{1}{350}$) or a decimal (0.00285714). The form of the ratio that is chosen depends on the calculations that need to be done.

Ask students to consider why a map scale statement is expressed with units, for example, 1 cm equals 1000 km, whereas other scale statements, such as those on model kits, do not contain units. Students could work with a partner or in a small group to research examples of each and develop statements that describe the situations, such as these:

- Sometimes metric diagrams had scales that did not include units.
- If a map included a scale that was drawn as a bar and marked with numbers, the scale did not include units.
- The units used to express the scale of the item might not be metric or imperial units. They might be specific to the diagram, so units are not included in the scale.

One idea that students would likely be familiar with is the projection of learning materials on an overhead projector or onto a white board. Teachers could do such a projection to introduce the idea of scale. They could project a photograph, for example. Note that this is an enlargement not a reduction but is still a scaled enlargement.

When introducing the concept of scale, some students find it easier to understand when you demonstrate the enlargement or reduction of an object. Have a 2 cm × 3 cm rectangle cut out of a thicker material for each student (or just one for demonstration purposes). For example, cut it out of the side of a cereal box. Ask the student(s) to measure the rectangle and record its measurement on paper. Project the rectangle using an overhead. Ask the students if the rectangle has changed shape. Ask for a volunteer to measure the rectangle that is projected on the whiteboard. Have the students record the enlarged measurements. Demonstrate how to calculate the scale. For example:

The sample rectangle measures 2 cm × 3 cm and the enlarged rectangle measures 20″ × 30″.

$$\frac{2}{20} = \frac{1}{10}$$

$$\frac{3}{30} = \frac{1}{10}$$

The scale of the enlargement is 10 in:1 cm, which means that 10″ on the enlargement is equivalent to 1 cm of the actual object.

You may want to preset the distance the overhead is from the wall to ensure a nice scale factor for the students to discover. Ask students when might a scale diagram be an enlargement of an actual object.

Mental Math and Estimation

STUDENT BOOK, p. 213

This question provides an opportunity for students to apply their understanding of measurement to ratio and proportion estimations. You can bring in scale maps of your area and adapt this question to the roads around your community.

SOLUTION

By looking at the scale on the map, students can judge that the scale is 1 cm:40 km. By looking at the map, students can estimate that the distance between Fort Qu'Appelle and Nipawin is about 7 cm. Mentally, students can multiply 7 by 40 to determine that the distance between these towns is about 280, or 300 km. A person driving at 100 km/h would take about 3 hours to cover this distance, so students should estimate that a person driving at 90 km/h would take a little under 3 hours.

DISCUSS THE IDEAS

THE IMPACT OF SCALE

STUDENT BOOK, p. 213

This discussion provides an opportunity for students to observe the impact of changing scale factors in a concrete way, by looking at differing map scales and observing the changing amount of detail that can be included. Encourage students to make a connection between the scale factor on maps and the scale factor on others kinds of drawings or models, and the level of detail that the scale allows. Also discuss the fact that the format (print or web-based in the case of a map) in which a scale drawing is going to be made will be a controlling factor in the scale selected.

Students may need some assistance navigating around this site. It may be helpful to have students work with a partner or in a small group to research the maps and share their observations.

If you do not have access to a computer lab or feel your students would benefit from having a map in their hands, have a class set of maps (that you could borrow from the social studies/geography teacher), or make a class set of copies of maps (that can be re-used).

SOLUTIONS

1. Maps selected will vary. The national maps have a scale of 1 cm:590 km.
2. Students will find that when they zoom in on their home province, the scale changes, allowing more detail to be included on the provincial map than on the national.

3. The scale controls the amount of information that can reasonably be included on a map.
4. The size of the paper that it is printed on or the size of the usual web page could affect the scale of the map. Other factors that could affect the scale at which a map is drawn could include the diversity of data that the map needs to express, the amount of geographic detail the map needs to express, or the area that needs to be covered by the map.

ACTIVITY 5.1
DRAFT A FLOOR PLAN AND BUILD A MODEL

STUDENT BOOK, p. 214

This activity can be used as an alternative chapter project.

PART 1

This activity provides students with an opportunity to apply their problem-solving skills by using their knowledge of scale to create a floor plan following specific guidelines. If you allow students to create the floor plan with computer software, most printers will not print to 11″ × 17″ paper. Check the software the students will be using and choose an appropriate size.

The website http://floorplanner.com allows users to easily create floor plans and it can also convert them into 3-D views.

PART 2

It is recommended that you create a scale model. This will allow you to work through any problems that the students may encounter.

Before assigning part 2, you will need to decide how much class time you will provide for the students to create the scale model, if you will provide different materials for the students, or if they will they need to supply their own. You will want to decide whether students should take into account the thickness of the material they use for the walls in order to ensure that the rooms are the correct size. As well, you may either set the scale for the model or allow the students to choose their own scale.

Some suggested materials may include: paper, foam, cardboard, scrap pieces of thin wood, glue, tape, scissors, sandpaper, paint, markers.

You may want to provide students with some time to plan the look they desire for the outside of the cabin and the material they would like to use to build the scale model.

SOLUTIONS

Students will choose different scales. Their answers will be different. This sample answer uses the scale 1 inch:3 feet. Divide each measurement by 3. The drawing dimensions will be as follows.

PART 1

The cabin is 5″ wide and 7″ long.

The great room at the front of the cabin measures 5″ × 4″.

The bedroom measures 3″ × 3″.

The bathroom measures 2″ × 3″.

The windows measure $\frac{5}{6}$″ wide.

The front door is 1″ wide.

The deck measures 2″ × 2″.

The bedroom and bathroom doors measure $\frac{5}{6}$″ wide.

PART 2

Using the same scale, the cabin's walls will be approximately $2\frac{3}{4}$ inches high and the roof's peak will be approximately $3\frac{5}{16}$ inches from the cabin's floor. The other components will have the measurements expressed in part 1.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 215

1. a) Scale models are used by architects, salespeople at trade shows, biology teachers, and commercial builders.
   b) Scale drawings are used by surveyors, seamstresses, landscapers, interior designers, engineers, and architects.

2. a) A diagram would need to be reduced to fit on the page it is printed on, or to be proportionate to other diagrams.

   A model would need to be reduced in size if its scale were reduced, or if the actual item it is based on was reduced in size.

   b) A diagram would need to be enlarged if it needed to show more detail, or if its scale changed from, for example, 1:5 to 1:2.

   A model would need to be enlarged to show more detail, or for advertising displays.

3. Measure to find that the scale sari is $\frac{12}{16}$″ wide. The scale is $\frac{3}{4}:44$.

   Measure to find that the scale sari is 3 inches long. Use proportional reasoning to find out how long the sari will be. Convert $\frac{3}{4}$ to a decimal.

   $$\frac{0.75}{3} = \frac{44}{x}$$

   Multiply by the product of the denominators.

   $$3x \times \left(\frac{0.75}{3}\right) = \left(\frac{44}{x}\right) \times 3x$$

   $$0.75x = 132$$

   Divide by 0.75.

   $$x = 176$$

   Alli will need a piece of fabric that is 176 inches long or 14 feet and 8 inches long.

4. a) You can calculate the scale using the given length of the actual doghouse. It will be 5 feet long.

   Convert to inches by multiplying by 12.

   $5 \times 12 = 60$

   In the diagram, the length measures 2 inches. Using the scale length and the actual length, the scale is 2:60, which can be simplified to 1:30.

   b) Measure the scale parts of the doghouse.

   Roof:

   Roof width: $\frac{7}{8}$ in

   Multiply by the scale factor of 30 to find the actual width.

   $30 \times \frac{7}{8} = 26\frac{1}{4}$ in

   Roof length: $2\frac{1}{4}$ in

   Actual length: $30 \times 2\frac{1}{4} = 67\frac{1}{2}$ in

   Roof depth: $\frac{1}{16}$ in

   Actual depth: $30 \times \frac{1}{16} = 1\frac{7}{8}$ in

   Side wall:

   Side wall length: 2 in

   Actual length: $30 \times 2 = 60$ in

   Side wall height: $\frac{15}{16}$ in

   Actual height: $30 \times \frac{1}{16} = 28\frac{1}{8}$ in

   Front wall:

   Front wall height (at peak): $1\frac{3}{8}$ in

   Actual height: $30 \times 1\frac{3}{8} = 41\frac{1}{2}$ in

   Front wall width: $1\frac{1}{4}$ in

   Actual width: $30 \times 1\frac{1}{4} = 37\frac{1}{2}$ in

   Door:

   Door height: $\frac{3}{4}$ in

   Actual height: $30 \times \frac{3}{4} = 22\frac{1}{2}$ in

   Door width: $\frac{1}{2}$ in

   Actual width: $30 \times \frac{1}{2} = 15$ in

c) **Door height:**

The door should be at least $2\frac{1}{2}$ feet or 30 inches tall. In the diagram, it is $\frac{3}{4}$ of an inch tall, or $22\frac{1}{2}$ inches tall in reality. It is probably not tall enough.

The door is 15 inches, or $1\frac{1}{4}$ feet wide. It should probably be over 2 feet wide for the dog to comfortably exit and enter.

5. a) Set up a proportion to find the new width of the ad.

$$\frac{2.5}{4.5} = \frac{2}{x}$$
$$4.5x \times \frac{2.5}{4.5} = \frac{2}{x} \times 4.5x$$
$$2.5x = 9$$
$$x = 3.6$$

The new ad width is 3.6 cm.

b) Set up a proportion to find the new height of the ad.

$$\frac{2.5}{4.5} = \frac{x}{6}$$
$$27 \times \frac{2.5}{4.5} = \frac{x}{6} \times 27$$
$$15 = \frac{27x}{6}$$
$$6 \times 15 = \frac{27x}{6} \times 6$$
$$90 = 27x$$
$$3.3 \approx x$$

The new ad height is about 3.3 cm.

6. Convert Reiko's height to inches.

$5 \times 12 = 60$

$60 + 7 = 67$

Convert Reiko's height to centimetres.

$67 \times 2.54 \approx 170.2$

Use proportional reasoning to solve.

$$\frac{3.9}{8.1} = \frac{170.2}{x}$$

Multiply by the product of the denominators.

$$8.1x \times \frac{3.9}{8.1} = \frac{170.2}{x} \times 8.1x$$
$$3.9x \approx 1378.6$$

Divide to isolate x.

$$x \approx 353.5$$

The statue is 353.5 cm tall, or about 3.5 m tall.

7. a) Set up a proportion to solve for x, the length of the actual mosquito.

$$\frac{\text{actual visible length}}{\text{diagram length}} = \frac{\text{actual length A to B}}{\text{diagram length A to B}}$$
$$\frac{12\text{ mm}}{107\text{ mm}} = \frac{x}{65\text{ mm}}$$
$$65 \times \frac{12}{107} = \frac{x}{65} \times 65$$
$$7.3 \approx x$$

The length of the actual mosquito from point A to point B is about 7.3 mm.

b) $$\text{magnification} = \frac{\text{drawing size}}{\text{object size}}$$
$$\text{magnification} = \frac{107}{12}$$
$$\text{magnification} \approx 8.9$$

The drawing is a magnification of about 8.9.

Extend Your Thinking

8. a) Multiply by the scale factor of 1.25 to find the dimensions for the new beam.

Depth: $5 \times 1.25 = 6.25$

Width: $20 \times 1.25 = 25$

Length: $120 \times 1.25 = 150$

The second scale beam would have a depth of 6.25 cm, a width of 25 cm, and a length of 150 cm.

b) The scale drawing of the beam has been reduced in size. The original length was 120 cm. The new length is 3 cm. Set up a proportion using the form of new measurement:old measurement.

3:120

Simplify by dividing by each number by 3.

The scale used was 1:40.

5.2 Two-Dimensional Representations

TIME REQUIRED FOR THIS SECTION: 5 CLASSES

STUDENT BOOK, pp. 218–231

MATH *ON THE JOB*

STUDENT BOOK, p. 218

This Math on the Job is an opportunity to explore two-dimensional representations in relation to recreational activities students enjoy. Surfboards, snowboards, and bikes can all be designed using preliminary 2-D representations. If any students have used drawings to design objects, ask them pass on tips to the rest of the class.

SOLUTION

1. Multiply the dimensions in the diagram by the scale factor of 13.

   Top tube: $3.5 \times 13 = 45.5$ cm

   Seat tube: $3.5 \times 13 = 45.5$ cm

   Down tube: $4.3 \times 13 = 55.9$ cm

   Head tube: $1.5 \times 13 = 19.5$ cm

2. Divide by the scale factor to find the new lengths of the components.

   Top tube: $45.5 \div 7 = 6.5$ cm

   Seat tube: $45.5 \div 7 = 6.5$ cm

   Down tube: $55.9 \div 7 \approx 8$ cm

   Head tube: $19.5 \div 7 \approx 2.8$ cm

3. You would need to know the diameter of the tube and the thickness of its wall.

EXPLORE THE MATH

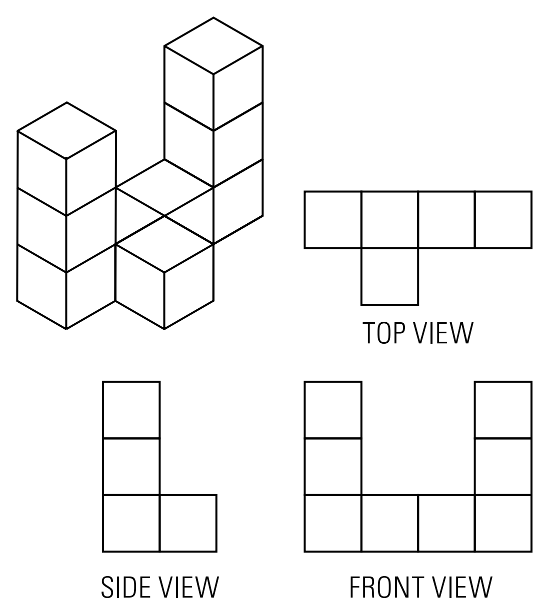
STUDENT BOOK, p. 219

Begin with a discussion of the views of objects (top, left, right, bottom, front). Ask students when it might be useful to have a top or bottom view. Examples could include a construction worker using a floor plan (top view) or a mechanic referring to the diagram of an undercarriage (bottom view).

Have students suggest when approximate diagrams provide enough information and when scale diagrams are needed. In the workplace, approximate diagrams can convey information, but scale diagrams are required for accuracy. An interior designer, for example, might make sketches during the design stage, but scale diagrams would be needed to complete a renovation. Ask students what would happen if a land surveyor had an approximate map or a furniture manufacturer was given an approximate drawing of an object (in either case, this could lead to serious errors).

Ask students if they have ever used a CAD program. If they have, find out what they used it for. Ask the class if they can think of jobs where people would use CAD programs to create views. Examples could include outdoor gear designers, electronics designers, and automobile engineers.

Have students draw different views of an object. You will find that some will be able to draw views quickly and easily, while others will struggle. Providing cubes can help different types of learners draw views successfully. You may want to begin by walking your class through how to draw views of a simple combination of cubes, such as this one.



Mental Math and Estimation

STUDENT BOOK, p. 222

Seven feet of the 12-foot wall are taken, so there are 5 feet of space available for the couch. Students will need to mentally convert feet to inches by dividing 30 by 12. Twelve divides into 30 twice, with 6 left over. Six is half of 12, so 12 divides into 30 2.5 times. This will let students determine that 1 inch equals 2.5 feet of actual space. By looking at the diagram, students should see that the couch is over 2 inches, or 5 feet long. Therefore, the couch will not fit in the living room.

DISCUSS THE IDEAS

VIEWS

STUDENT BOOK, p. 222

This section is designed to start students thinking about how symmetrical and asymmetrical objects can be represented using views. This is an opportunity to discuss with students when bottom views might be useful, for example, when building or assembling legs on a table.

SOLUTIONS

1. Objects that would need fewer than six views to accurately represent them could be a garden shed, book, chair, or road sign.
2. Objects that would need six views to accurately represent them could include a house with an addition on the back, a couch that narrows at one end, or an asymmetrical piece of clothing.
3. If the jewellery box had interior compartments, you would need a top, front, side, and back view. You would need to see the component parts for the shorts. These would include the front and back views, as well as a side view. Assuming the pipe is symmetrical, you would need to see a front or top view and a side view.

ACTIVITY 5.2

JAPANESE CUSHION DESIGNS

STUDENT BOOK, p. 223

This activity will allow students to use their prior knowledge of geometry, specifically formulas for finding diameter and circumference. They will need to draw on this knowledge to design the *zafu* and find its measurements. If students initially think that they do not have enough information to find the *zafu*'s measurements, you can guide them to realize that using the formula for circumference will provide them with the information they need. It may be helpful for students to work in pairs.

During this activity, explain to students that a seam allowance is the area between the edge of the fabric and the stitching line. Pieces are usually sewed together front to front and when turned to the right side, enclose the seam allowance. The seam allowance may also be turned over and stitched to finish a raw edge tidily. Note that the views would not show the seam allowance.

The link below explains how the cushions are sewn.

www.urbandecline.com/articles/2006/03/13/how-to-make-a-zafu-and-zabuton

SOLUTIONS

PART 1: THE SQUARE CUSHION

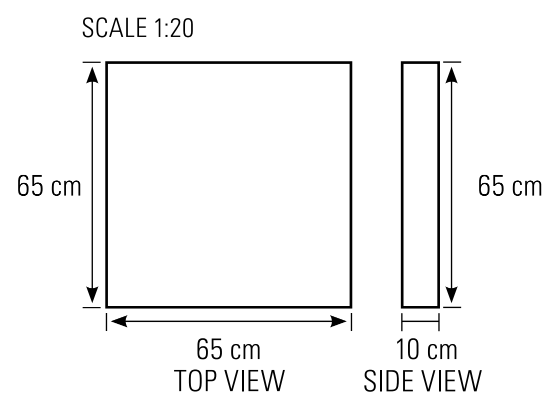
Two views are needed to represent the cushion, the top or bottom and the side.

1. To find the scale measurements for the square cushion views, divide by 20.

   Width: 65 ÷ 20 = 3.25 cm

   Length: 65 ÷ 20 = 3.25 cm

   Height: 10 ÷ 20 = 0.5 cm



2. To find the measures of the three cushion components, add the seam allowance and divide by the scale factor of 20.

   Top/bottom piece width:

   $65 + 1.5 + 1.5 = 68$ cm

   $68 \div 20 = 3.4$ cm

   Top/bottom piece length:

   $65 + 1.5 + 1.5 = 68$ cm

   $68 \div 20 = 3.4$ cm

   Middle piece width:

   $10 + 1.5 + 1.5 = 13$ cm

   $13 \div 20 = 0.65$ cm

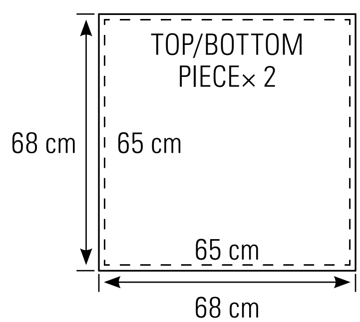
   Middle piece length:

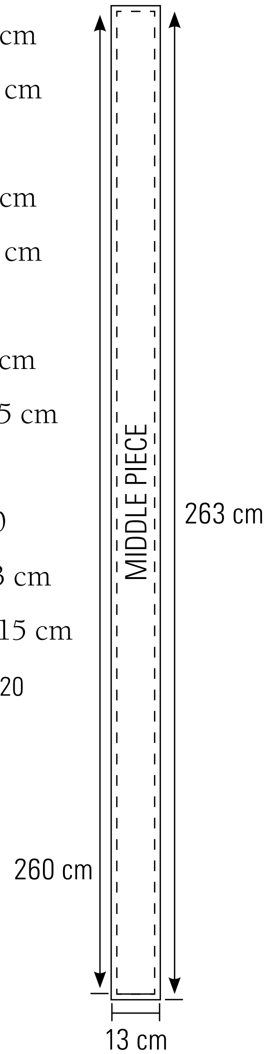
   $65 \times 4 = 260$

   $260 + 1.5 + 1.5 = 263$ cm

   $263 \div 20 = 13.15$ cm

SCALE 1:20





PART 2: THE *ZAFU*

1. Add the seam allowance to the radius.

   $20 + 1.5 = 21.5$

   Multiply by 2 to find the diameter.

   $21.5 \times 2 = 43$

   Use the formula for circumference to calculate the length of the rectangular middle piece of fabric. The side piece is sewn at the seam line of the circle, *not* the outer edge.

   $C = \pi d$

   $C = \pi \times 43$

   $C \approx 135$

2. Add 30 percent to the length, for the pleats.

   $135 \times 0.30 = 40.5$

   $40.5 + 135 = 175.5$

   Add seam allowances to find the length of the rectangular piece of fabric.

   $175.5 + 1.5 + 1.5 = 178.5$

   Add seam allowances to find the width of the rectangular piece of fabric.

   $22 + 1.5 + 1.5 = 25$

   The length of the rectangular piece will be 178.5 cm and the width will be 25 cm.

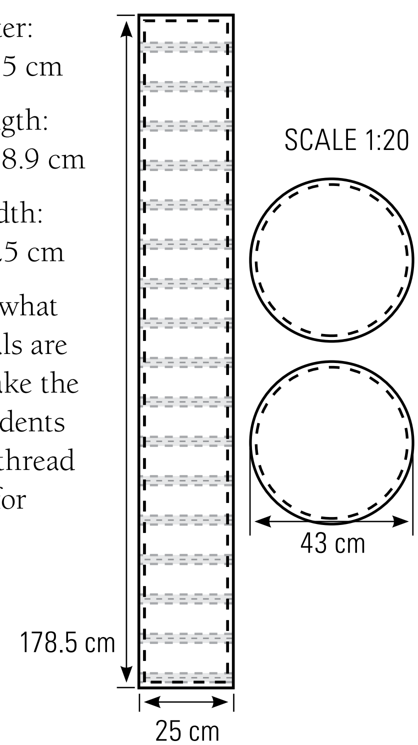
3. Find the measurements of the scaled components.

   Circle diameter:
   $43 \div 20 = 2.15$ cm

   Rectangle length:
   $178.5 \div 20 \approx 8.9$ cm

   Rectangle width:
   $25 \div 20 = 1.25$ cm

   When asked what other materials are needed to make the cushions, students may suggest thread and stuffing for its interior.



4. Calculate the measurements of the new components by multiplying by the scale factor.

   Square cushion top/bottom piece:

   Width/length: $65 \times 1.5 = 97.5$

   Add for the seam allowance.

   $97.5 + 1.5 + 1.5 = 100.5$

   The width and length of the top and bottom pieces will measure 100.5 cm.

Middle piece:

Width: $10 \times 1.5 = 15$

Add for the seam allowance.

$15 + 1.5 + 1.5 = 18$

Length: $260 \times 1.5 = 390$

Add for the seam allowance.

$390 + 1.5 + 1.5 = 393$

The top and bottom pieces will measure 100.5 cm by 100.5 cm and the middle piece will measure 393 cm by 18 cm.

Zafu **circular piece:**

Circular piece diameter:

$d = 2r$

$d = 2 \times 20$

$d = 40$

Multipy by 1.5

$40 \times 1.5 = 60$

Add the seam allowance.

$60 + 1.5 + 1.5= 63$

The circular *zafu* piece will have a diameter of 63 cm.

Calculate the circumference:

$C = \pi d$

$C = \pi \times 63$

$C \approx 198$

The circular zafo piece will have a circumference of 198 cm.

The sides piece is sewn at the finished diameter, not at the outer edge of the top piece.

Rectangular piece length: $175.5 \times 1.5 = 263.25$

Add the seam allowance.

$263.25 + 1.5 + 1.5 = 266.25$

The length will be 266.25 cm.

Rectangular piece width: $22 \times 1.5 = 33$

Add for the seam allowance.

$33 + 1.5 + 1.5 = 36$

The width will be 36 cm.

ACTIVITY 5.3

DESIGN AND BUILD A GREENHOUSE FRAME TO SCALE

STUDENT BOOK, p. 224

Supply students with a variety of materials they can use to make their greenhouses. The frame can be made of bookboard, paperboard, popsicle sticks, or straws. The frame pieces can be secured with glue strips, tape, or putty.

Asking students to sketch the greenhouse before they build it will help them to work out its dimensions. Students can label the same diagram with their model's measurements, once they have calculated them.

SOLUTIONS

PART 1

To find the length of the roof, students should realize that the front face of the greenhouse is composed of a rectangle with a triangle on top. You can ask students to draw the front view of the greenhouse and determine what shapes it is made of.

The bottom of the triangle has the same measure as the greenhouse width, or 220 cm. Divide the triangle in half, starting at the roof peak. Two right triangles are produced. Each has an angle measuring 45° (the 90° angle at the peak, divided in 2) and a leg measuring half of 220, or 110 cm.

Determine the measure of the triangle's third angle.

$180 - 90 - 45 = 45$

The triangle's third angle is 45°.

Use the cosine ratio to find the triangle's hypotenuse, or the length of the roof piece.

$$\cos\theta = \frac{a}{h}$$

$$\cos 45° = \frac{110}{h}$$

$$h \times \cos 45° = \frac{110}{h} \times h$$

$$h\cos 45° = 110$$

$$\frac{h \cos 45°}{\cos 45°} = \frac{110}{\cos 45°}$$

$$h = \frac{110}{\cos 45°}$$

$$h \approx 155.6$$

The roof piece measures about 155.6 cm long.

To find the dimensions for their scale model, students will divide by the scale factor of 25.

Width: $220 \div 25 = 8.8$ cm

Length: $350 \div 25 = 14$ cm

Height: $250 \div 25 = 10$ cm

Roof piece: $155.6 \div 25 \approx 6.2$ cm

PART 2

Student greenhouses should have the calculated dimensions. If you would like to add more room for creativity, have students research different greenhouse designs and design their own, instead of working from the model in the book.

PUZZLE IT OUT

SHAPE SHIFTING

STUDENT BOOK, p. 225

Students will need graph paper, pencils, and scissors to make their drawings to solve this puzzle. If you prefer not to have students draw the diagram to scale, you can provide them with photocopies of the puzzle to cut out and work with. If you have them, you can use blocks or cardboard shapes of the same size.

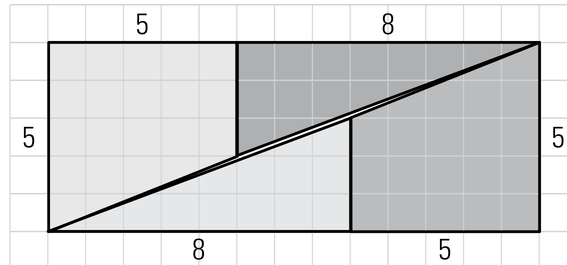
This puzzle allows students to use their spatial reasoning skills while solving a problem with a geometric connection. Students who are strong visual learners may enjoy solving this puzzle. Encourage students to use trial and error when they are constructing the rectangle, and to remember or list their unsuccessful combinations, so that they can eliminate these possibilities.

If students have difficulty solving the puzzle, you can lead them to realize that using the formula for the area of a rectangle will help them understand how to correctly arrange the shapes.

The link below provides more puzzles of this kind:

www.pleacher.com/handley/puzzles/wreck.html

SOLUTION



The rectangle will have a length of 13 units and a width of 5 units. The area of the rectangle is one square unit greater than the square because the pieces of the rectangle are not flush with each other. When students make the rectangle, they will see that there is a gap in the middle of the rectangle that accounts for the additional space.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 226

1. Johan does not have enough information to design the box because the diagram does not show the depth of the coffee machine. He would not know how deep to make the box.
2. a) The drawings of the cooking pot accurately represent it. The viewer can see the dimensions of its components, and their shapes.

   b) The drawing of the bench does not accurately represent it, because the viewer can't see the bottom.
3. To find the measurements of the views, students will have to divide by the scale factor of 6.

   Top:

   Length: $13 \div 6 \approx 2.2$ or $2\frac{3}{16}$ in

   Width: $7 \div 6 \approx 1.2$ or $1\frac{1}{8}$ in

   Side:

   Height at back: $8 \div 6 \approx 1.3$ or $1\frac{5}{16}$ in

   Height at front: $7 \div 6 \approx 1.2$ or $1\frac{1}{8}$ in

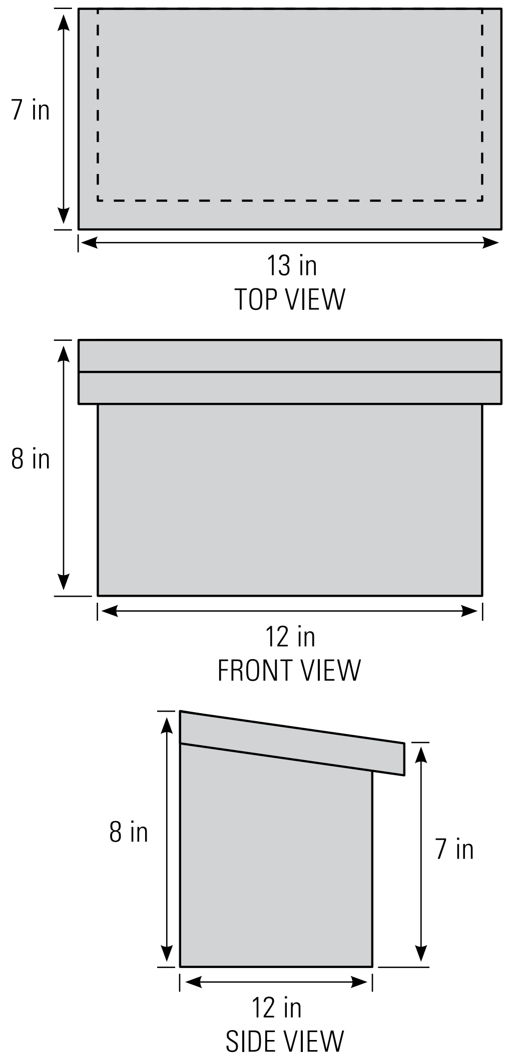
   Width: $6 \div 6 = 1$ in

Front:

Length: $12 \div 6 = 2$ in

Width: $7 \div 6 \approx 1.2$ or $1\frac{1}{8}$ in

SCALE 1:6



4. To find the measurements of the component parts, students will need to divide by the scale factor of 4.

   Components with seam allowance

   Bag length: $(1.5 + 1.5 + 40) \div 4 = 10.75$ cm

   Bag width: $(1.5 + 1.5 + 14) \div 4 = 4.25$ cm

   Bag height: $(1.5 + 1.5 + 40) \div 4 = 10.75$ cm

   Handle length:
   $(1.5 + 1.5 + 25 + 25 + 20) \div 4 = 18.25$ cm

   Handle width: $(1.5 + 1.5 + 4) \div 4 = 1.75$ cm

   Finished components

   Bag length: $40 \div 4 = 10$ cm

   Bag width: $14 \div 4 = 3.5$ cm

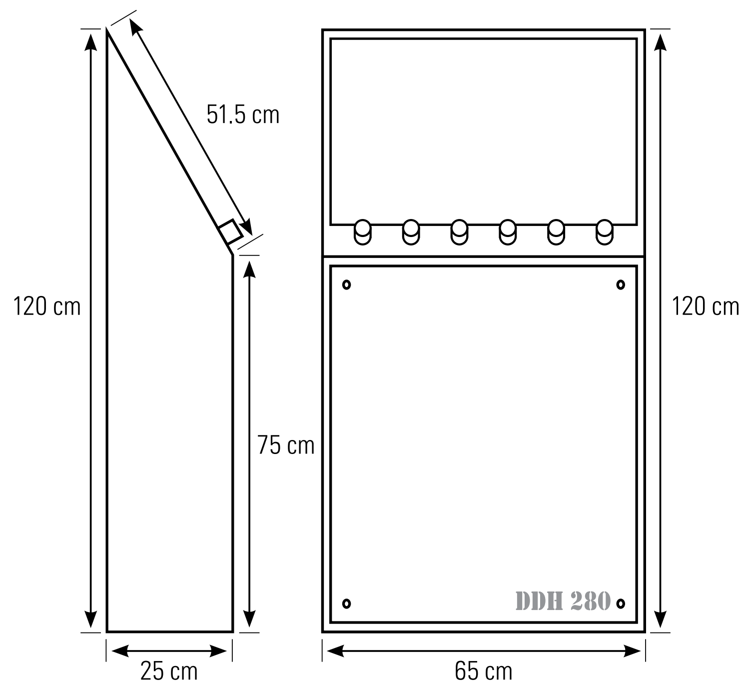
   Bag height: $40 \div 4 = 10$ cm

   Handle length: $(25 + 25 + 20) \div 4 = 17.5$

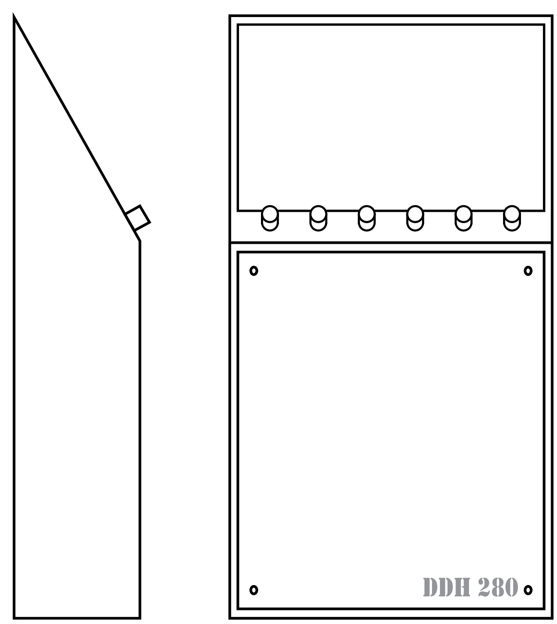
   Handle width: $4 \div 4 = 1$ cm

   See diagram on following page.

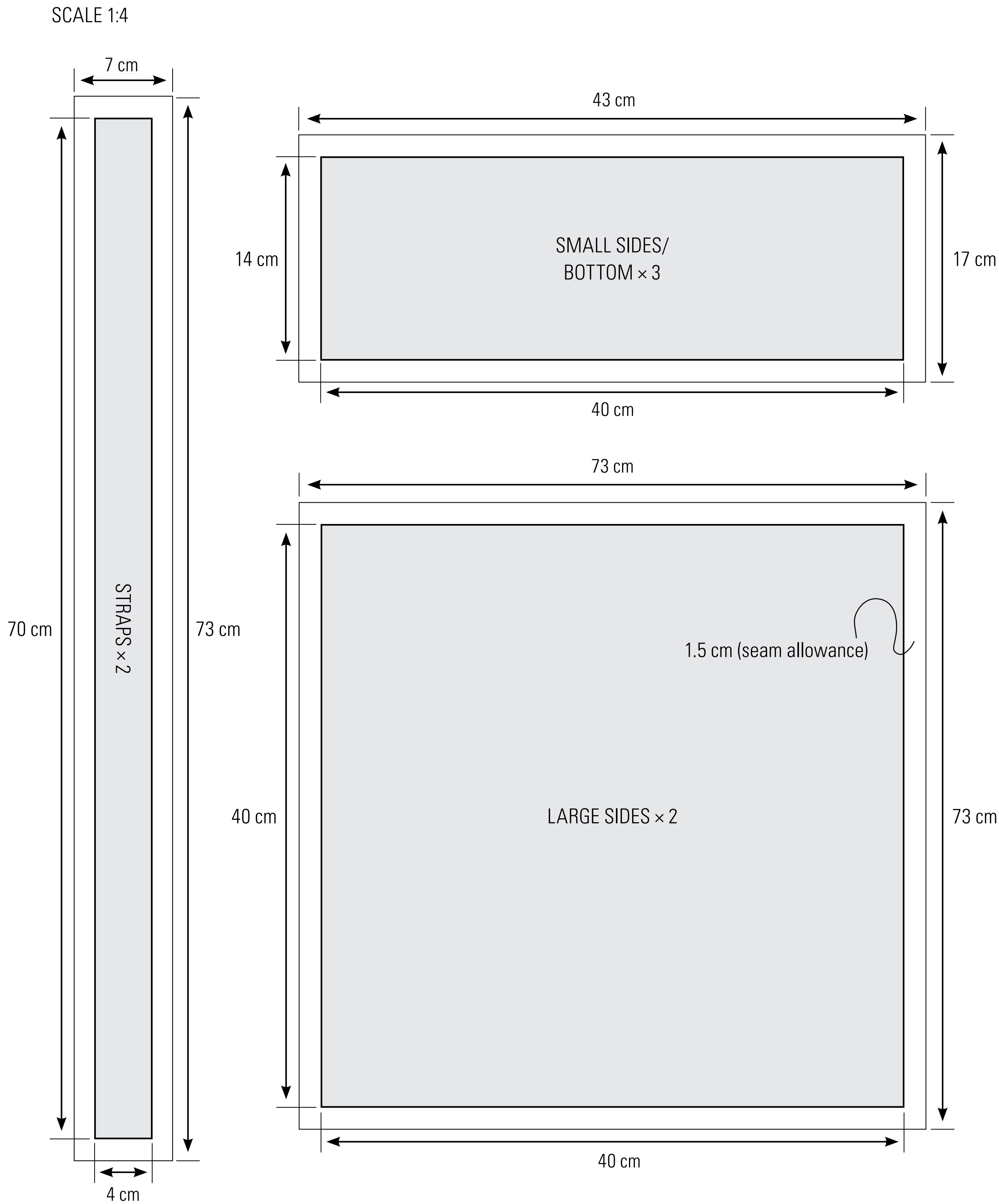
5. a) SCALE 1:15



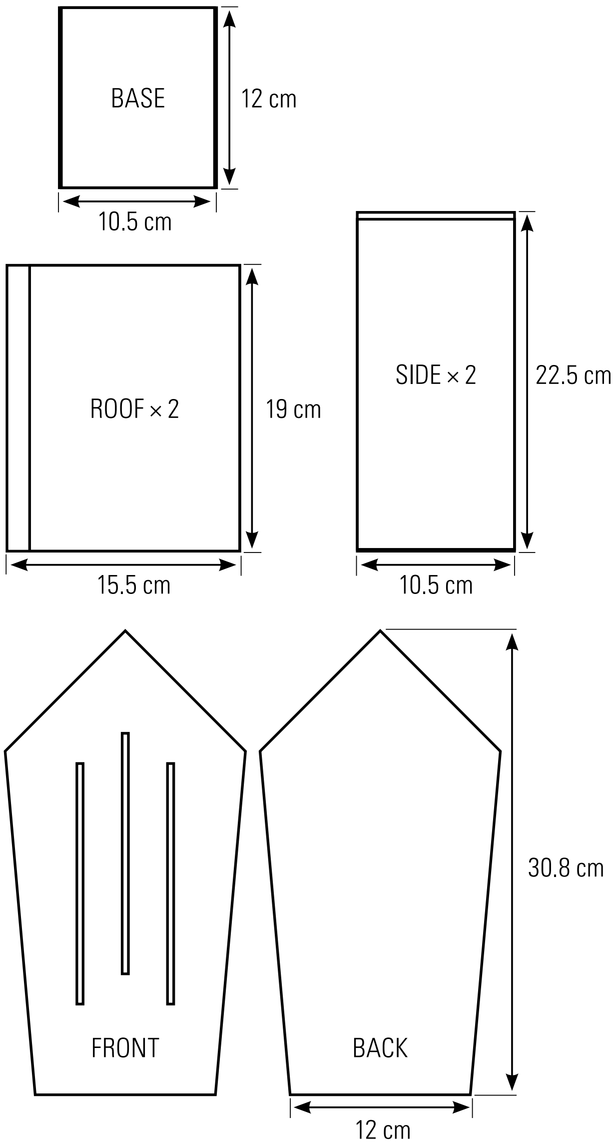
   b) You need the two views as shown below. All necessary dimensions are given in these 2 views. Top views will vary as students will get to exercise their creativity.



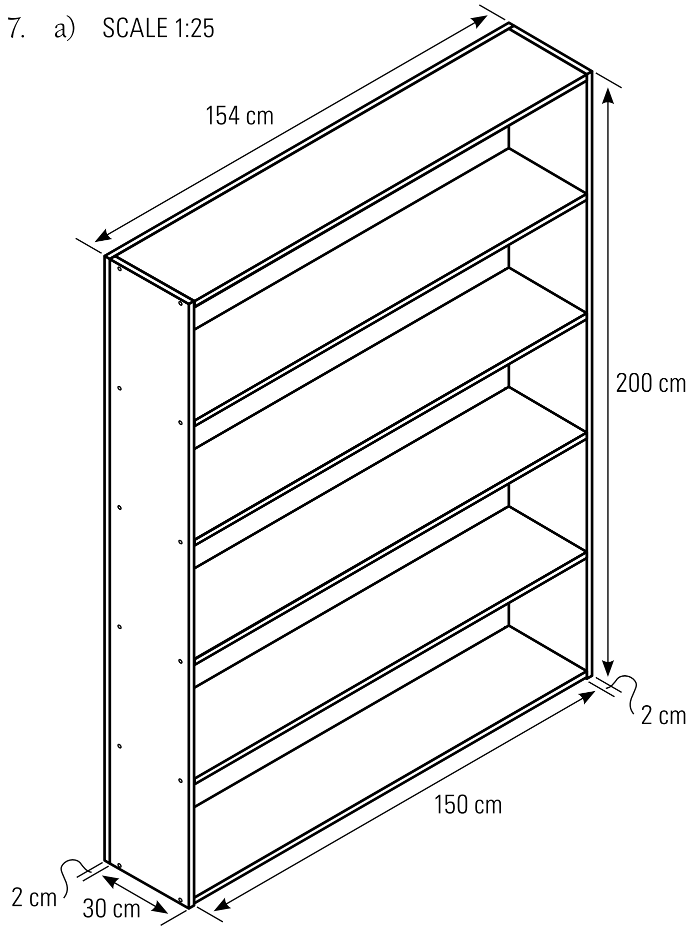
Build Your Skills, Question 4



6. SCALE 1:5



7. a) SCALE 1:25



b) Answers will vary. A sample answer is provided.

The piece used to divide the shelf will be the same length as the vertical side pieces, but its width will be smaller, since it will connect to the back piece, and not extend all the way along the depth of the shelf. You would therefore need to add a component that was 8 cm long and 1.12 cm wide to the drawing. The top, bottom, and middle pieces would be halved in length and 2 cm subtracted from their length to account for the space taken up by the thickness of the new middle component. The original component would be deleted from the drawing and a new component measuring 2.92 cm long and 0.08 cm thick would be added to the drawing. Eight of these components would be needed. The number of screws needed would increase to 48.

Extend Your Thinking

8. a) Gwen's fabric is 150 cm wide. Her paper is 15 cm wide.

   Divide to determine the scale.

   150 ÷ 15 = 10

   The scale Gwen will use will be 1:10.

   b) The main part of the apron is 85 cm wide.

   Divide to find half the width.

   85 ÷ 2 = 42.5

   Add the seam allowance.

   42.5 + 1.5 = 44

   The width will be 44 cm.

   c) Gwen will need to draw two waist straps, a neck strap, a pocket, and the main part of the apron.

   Divide to find the height of the pocket in the scale drawing.

   20 ÷ 10 = 2

   Divide to find the scale seam allowance.

   1.5 ÷ 10 = 0.15

   Add the seam allowance.

   2 + 0.15 + 0.15 = 2.3

   The pocket will be 2.3 cm high, with the seam allowance.

   d) Gwen will need to draw two waist straps, a neck strap, a pocket, and the main part of the apron.

   e) See diagram below.

   f) Calculate the measurements for the neck strap's length.

   60 + 1.5 + 1.5 = 63

   Divide by the scale factor, 10.

   63 ÷ 10 = 6.3

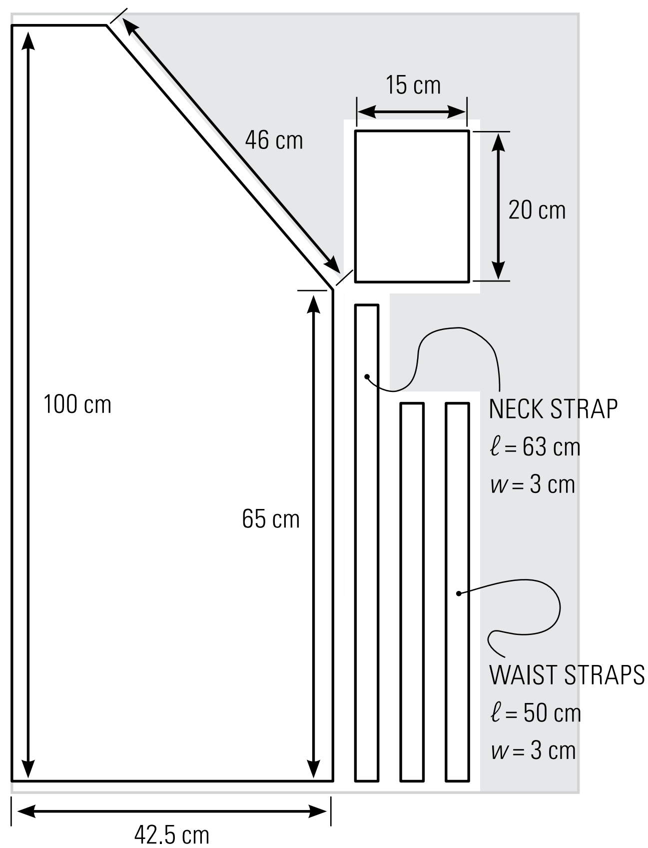
   The neck strap is 6.3 cm long.

   Calculate the neck strap's width.

   3 ÷ 10 = 0.3

   The neck strap is 0.3 cm wide.

SCALE 1:10



   g) The fabric needs to be as long as the height of the main part of the apron, plus the seam allowance.

   100 + 1.5 + 1.5 = 103 cm

   The fabric would need to be at least 103 cm long.

5.3 Three-Dimensional Representations

TIME REQUIRED FOR THIS SECTION: 7 CLASSES

STUDENT BOOK, pp. 232–246

MATH *ON THE JOB*

STUDENT BOOK, p. 232

You can begin this section by discussing how diagrams, illustrations, and animation can be useful educational tools. Ask students to think of an occasion when they have learned how something works by referring to illustrations or diagrams. Examples could be found in science textbooks, home economic textbooks, and instructions for machinery use in woodworking class.

This profile provides an opportunity to engage students who are interested in art and art-focussed jobs. You can ask students about other career opportunities they are aware of that involve illustration and design, such as video-game development. Because Natasha Bowen entered the workforce as a high school graduate, students interested in her career should be encouraged to see that after graduation, it is possible to find a fulfilling job related to their interests.

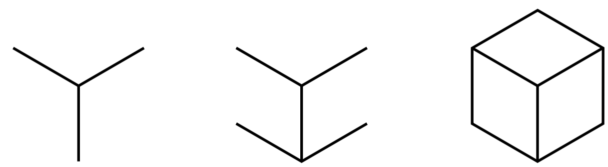
SOLUTIONS

1. Answers will vary. Students with weaker drawing skills may simply draw a stick figure. Students may also draw a table using perspective. The front edge of the table may be longer than the back edge of the table, and the back legs may be shorter than the front legs.
2. Answers will vary. A woodworker would likely not have enough information from a one-point perspective drawing of a table to build it. Unless detailed measurements were given, the dimensions of the table would be unknown and it would be difficult to figure out the proper shape of the table.

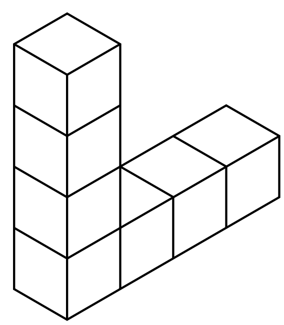
EXPLORE THE MATH

STUDENT BOOK, p. 232

Isometric drawings: Taking a few minutes to demonstrate isometric drawing with a simple object like a cube will help students draw isometric diagrams successfully. Hand out Blackline Master 5.1 (p. 331) for students to practise making isometric diagrams.



After successfully drawing one cube isometrically, ask students to draw a series of cubes that are attached to each other.



You may work through the steps outlined in Example 1 as a class as well. Encourage those students who are not having any difficulty to create an isometric drawing of their name (or the first 4 letters of their name).

Perspective drawings: As a class, you can work through the steps outlined in the example perspective drawing. You can ask students to read the steps aloud and draw each step on the blackboard until the drawing is complete. Ask the students to complete the same drawing in their notebooks.

Exploded diagrams: Gather samples of exploded diagrams and have them on hand to show the students before introducing the concept. Distribute them around the room. Ask the students what they notice about the diagrams and when these diagrams would be useful; record their answers on the board. Work through Example 3 as a class to practise drawing exploded diagrams. Discussions are a great way to assess what prior knowledge students have.

As you discuss each type of drawing, brainstorm and record situations where each could be useful. For example, one-point perspective drawings are used to represent products in advertisements and to depict scenes in paintings. Isometric drawings could be used by a metal fabricator designing a component, a person sketching a scale model, or a technical illustrator drawing the dimensions of an object. An exploded diagram could be used in an instructional manual.

DISCUSS THE IDEAS

DIAGRAMS AT WORK

STUDENT BOOK, p. 240

This is an opportunity to emphasize the purpose of each style of diagram and for students to think beyond their typical uses.

SOLUTIONS

1. Sample answers may include the following. A prop builder may use an isometric diagram when he or she needs to see more than one side of a prop.
2. A prop builder would most likely give his or her apprentice an exploded diagram so that the apprentice could easily understand how to build the oven.
3. There would need to be a scale drawing of the car with the scale measurements for the model provided and a component parts diagram.

Mental Math and Estimation

STUDENT BOOK, p. 240

Ask students to study the first diagram. The table legs on the left and right are triangular prisms. They are flush against the edges of the tabletop.

The legs of diagram a) are rectangular prisms. Their sides are not flush with the sides of the table.

Diagram b) shows the same table. The leg on the left is a triangular prism.

ACTIVITY 5.4

DRAW A QAMUTIK

STUDENT BOOK, p. 242

Provide students with isometric dot paper for their drawings.

Students may find this activity difficult. If they do, you can suggest that students do not need to draw the entire qamutik to scale. Instead, students could draw one runner and one cross-piece to scale, to complete question 2. For question 3, you could have students draw an exploded diagram of part of the qamutik. For example, students could draw an exploded diagram that contains the two runners and one cross-piece.

When introducing this activity, let students know that the Inuvialuit word qamutik sounds similar to "caw-moo-tick."

If you are interested in including other Inuktitut terms in this lesson, a link to a useful website is given below.

http://icor.ottawainuitchildrens.com/node/15

You can direct students to the following website when they are researching how qamutiks are built.

www.eenorth.com/eenorth/documents/qamutiit.html

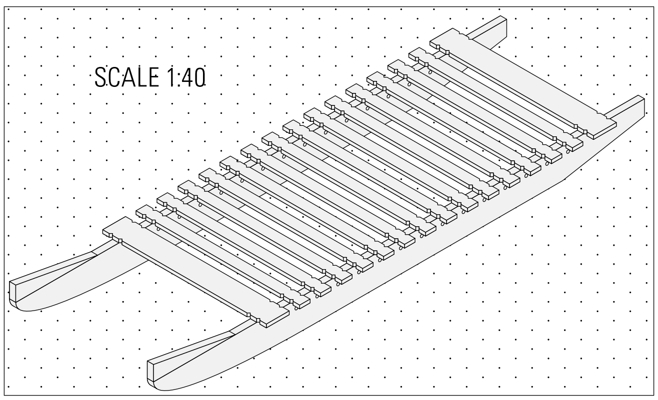
Information on qamutik dimensions is found on p. 7 of the following document.

www.umanitoba.ca/science/zoology/faculty/riewe/z338/inuit.pdf

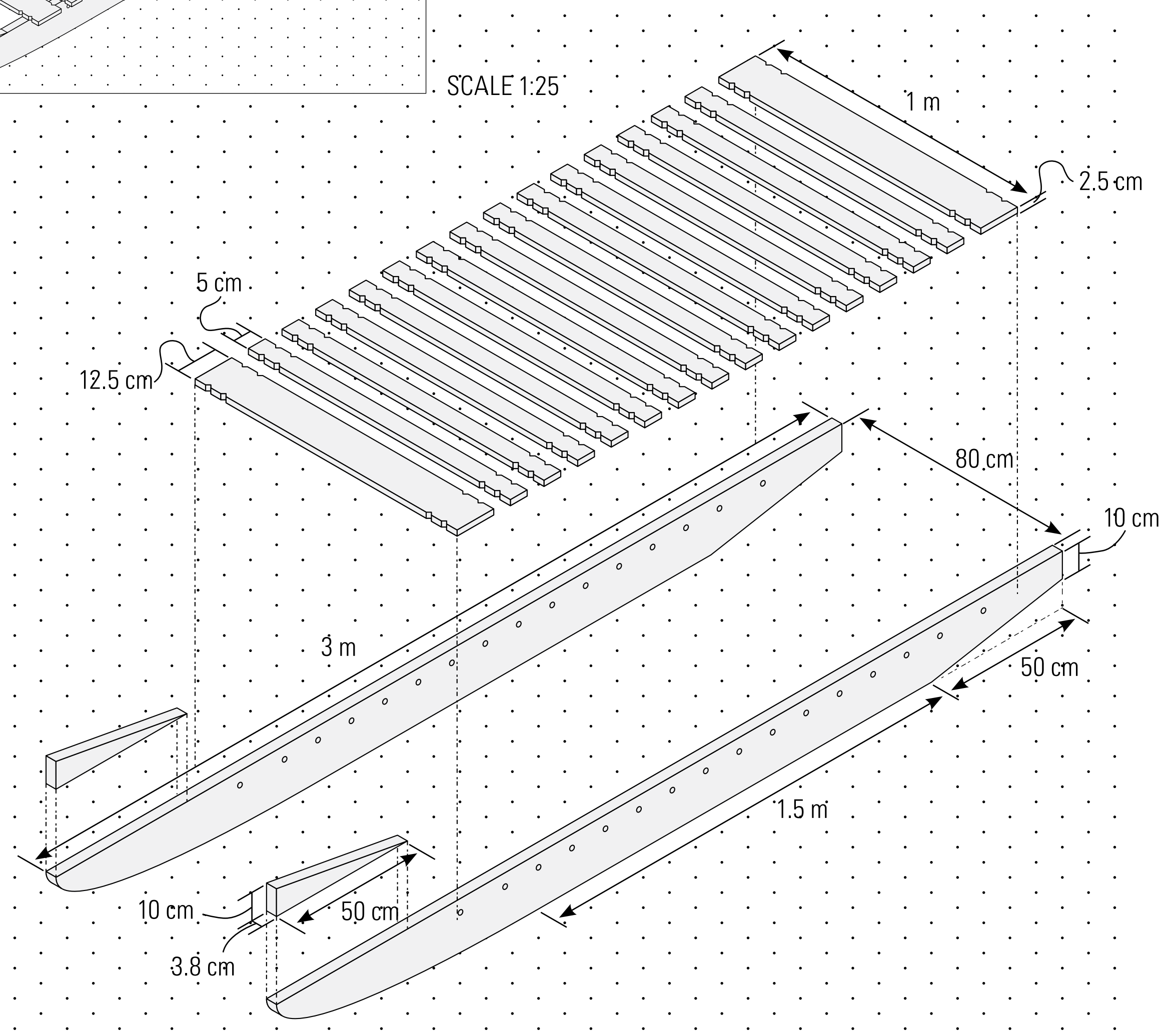
SOLUTIONS

1. Today, qamutiks are made from wood and metal using modern power tools. The bottoms of the runners are sometimes covered with metal, to make them more durable and smooth. The cross-pieces of the platform are usually tied to the runners with rope. Holes are drilled in the runners so that rope can be looped over the cross-pieces and through the holes. Sometimes a solid piece of wood can be fixed on top of the cross-pieces. Some qamutiks also have a compartment on top. Goods can be stored inside of it, or it can be used as a sheltered seating area.

2. Answers will vary. The component parts of a qamutik include two runners made of wood or wood with metal plating on the bottom; middle cross-pieces made of wood that compose the platform; two larger cross-pieces that go in the front and back of the platform; and nails or cord used to secure the cross-pieces to the runners. If cord is used to secure the cross-pieces to the platform, grooves are made at both ends of the cross-piece. The cord passes through the grooves, then through holes in the runners. Goods are usually placed at the front or middle of the qamutik.



3. Answers will vary. To assemble the qamutik, place one large cross-piece at the front of the qamutik and one at the back. Place the other cross-pieces in the middle, with even spacing between each. Attach the cross-pieces to the runners with cord or nails. An exploded diagram is shown below.



THE ROOTS OF MATH

TECHNICAL ILLUSTRATIONS

STUDENT BOOK, p. 243

You can lead into this section with a class discussion about objects students have built themselves. Did they use a set of instructions? Why or why not? If they did, were the diagrams in the instructions helpful and easy to understand? What are some features that make a diagram easy to understand? Do students find it easier to work by following instructions and diagrams, or do they prefer to figure things out independently?

SOLUTIONS

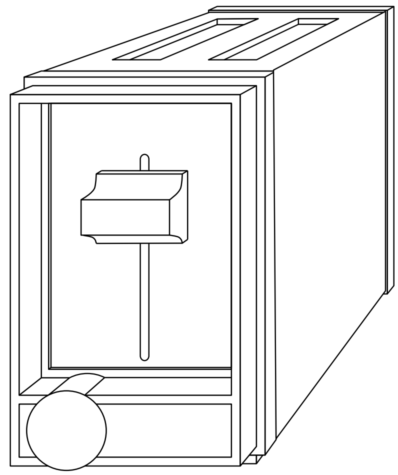
1. Student answers will vary. The components of the object will determine whether students make a 2-D or 3-D representation. If students chose a woodworking project, such as a bookshelf, they would likely produce a 3-dimensional, exploded drawing and include scale measurements. A front or side view of the shelf would best allow the viewer to see the shelf's components and how they are assembled to make the whole shelf.
2. Illustrations will vary. As students are making their illustrations, you can circulate through the class and ask them why they chose to make their drawing as they did. If you think a different view would convey an object more effectively, you can suggest it to the student and discuss with them why it could be more effective.

BUILD YOUR SKILLS: SOLUTIONS

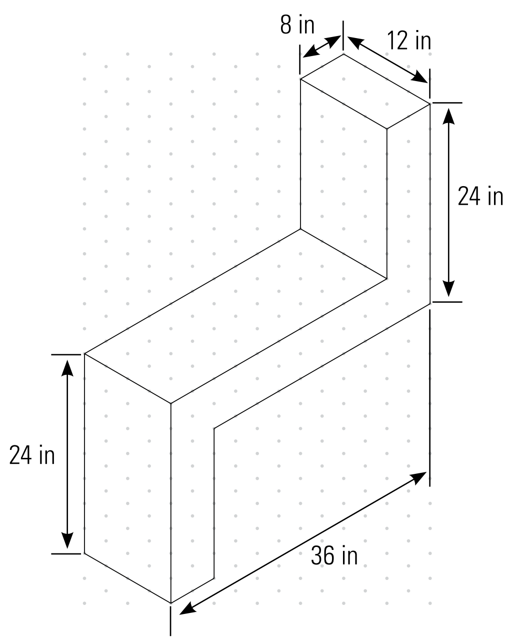
STUDENT BOOK, p. 244

1. The vanishing point is at the end of the alley and between the two buildings. It is along the horizon line.

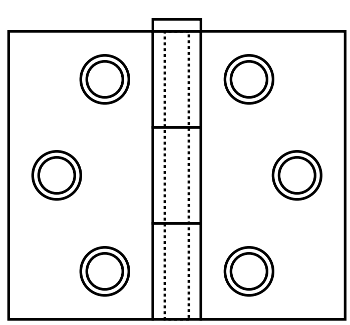
2. Student answers will vary. A one-point perspective drawing of the front of the toaster might look something like this.

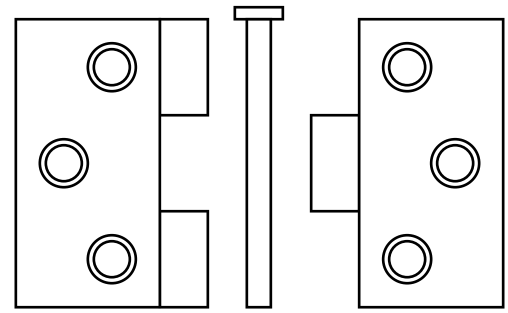


3. SCALE 1:24

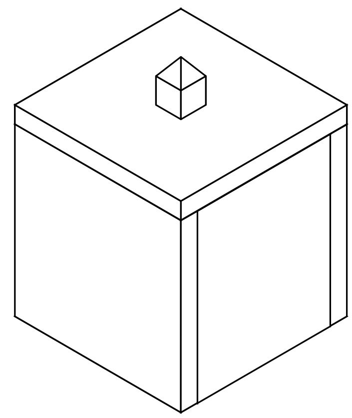


4.



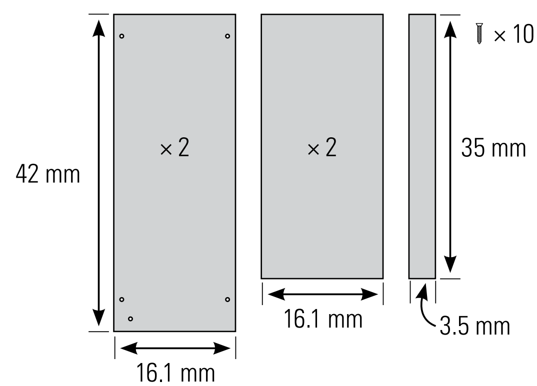


5.



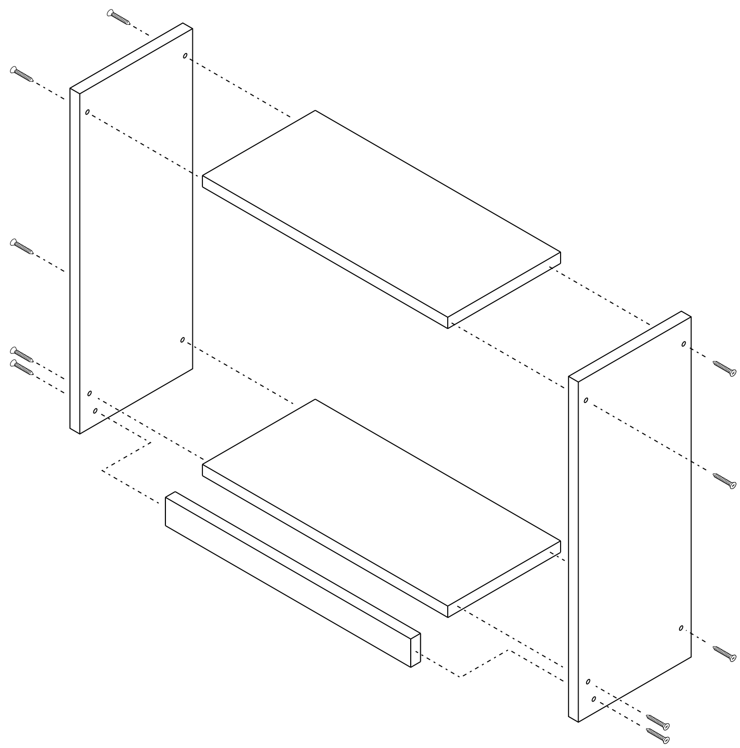
Extend Your Thinking

6. a) SCALE 1:1



b) Answers will vary. A sample answer follows. Screw the bottom shelf to the left and right side piece. Screw the top shelf to the right and left side piece. Screw the kickplate to the left and right side piece.

c)



REFLECT ON YOUR LEARNING

SCALE REPRESENTATIONS

STUDENT BOOK, p. 247

In this chapter, students have drawn on their prior knowledge of scale model building and plan view drawing as they explored different types of technical illustrations and scale models. Students have learned how to apply the above in a wide variety of workplace settings. They should now be able to create simple technical drawings, including views and component parts, and isometric, perspective, and exploded parts drawings.

Invite students to synthesize and reflect on their learning by discussing the following questions with them.

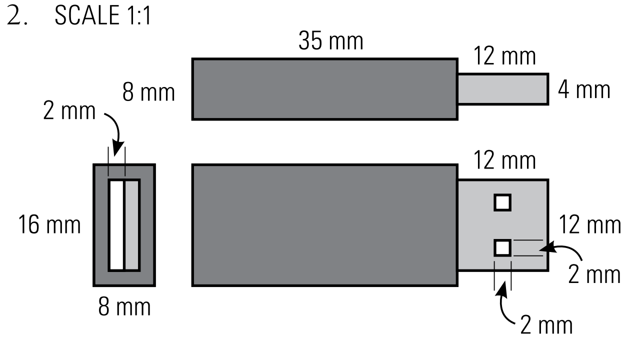
1. Describe situations where you would need to use view drawings or component parts drawings. List several workplace settings where these would be used and where they would be created.
2. Identify several workplace settings where perspective drawings would be helpful. What different situations would be best served by isometric drawings? In which jobs would you make these drawings? Use them?
3. Describe your experiences with technical drawing. Do you feel more comfortable making drawings than before you studied this material? What other aspects of technical drawing would you like to learn? (Some students might like to extend their learning to animation, illustrating graphic novels, movie set or costume design, or computer game drawing.)
4. Did you experiment with technology drawing aids in the activities and problems? Which programs did you try out? Were you able to share your knowledge of technology with other students in your class?

   There are a wealth of materials on the world wide web that students can use to find out more about technical illustration, including technical drawing programs. Encourage them to apply this new knowledge of technical illustration in other areas of their education to enhance written assignments and to communicate visually with their teachers and peers.

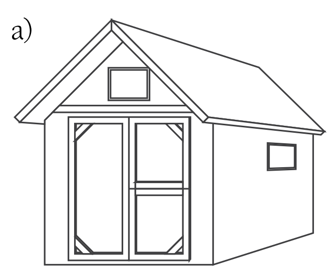
PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 248

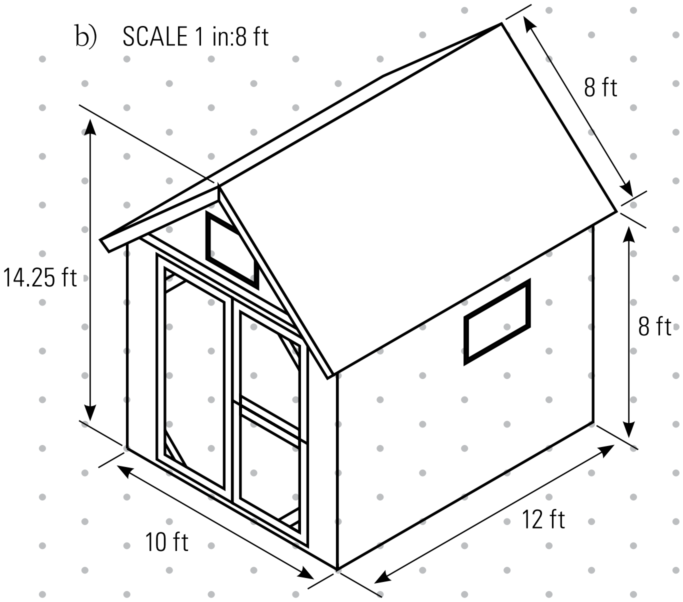
1. The vanishing point is located at the end of the railway tracks. The horizon is made by the sea.
2. SCALE 1:1



3. a)



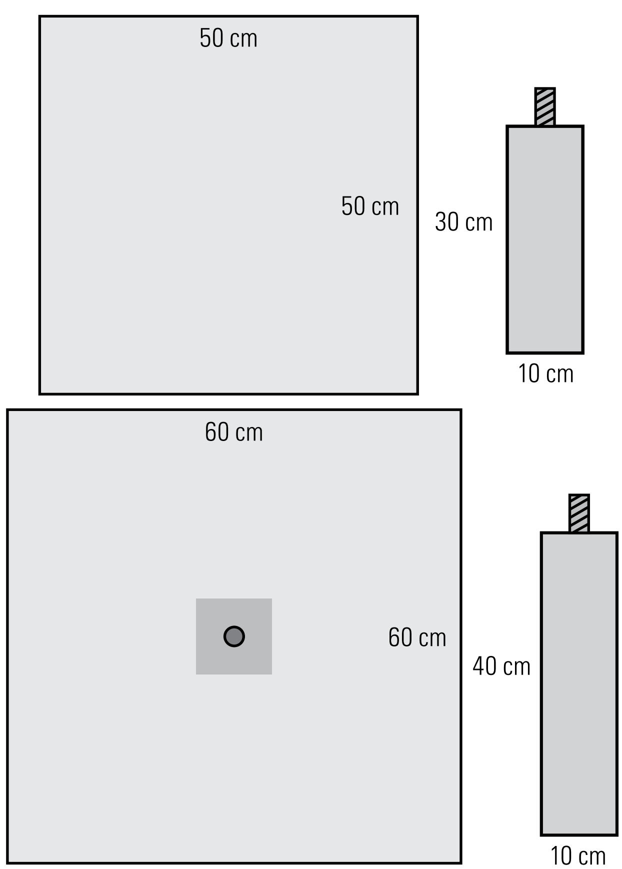
b) SCALE 1 in:8 ft

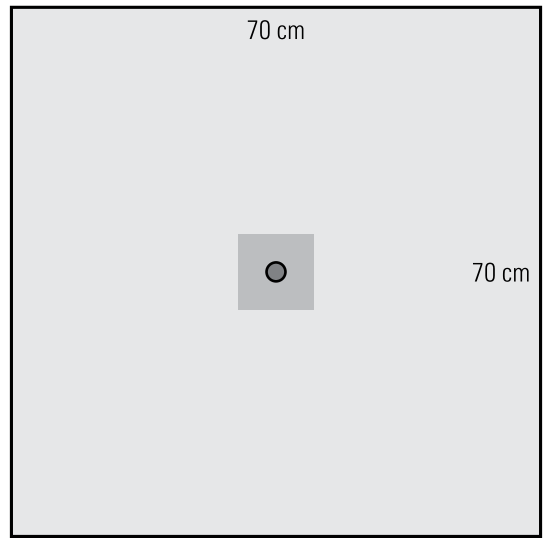


c) The isometric drawing would be more useful because it would allow you to see all of the shed's dimensions to scale and see what the completed shed might look like.

4. a) Answers and scales will vary. The post is assembled by connecting the square platforms with the rectangular posts. The posts are screwed into the centres of the platforms.

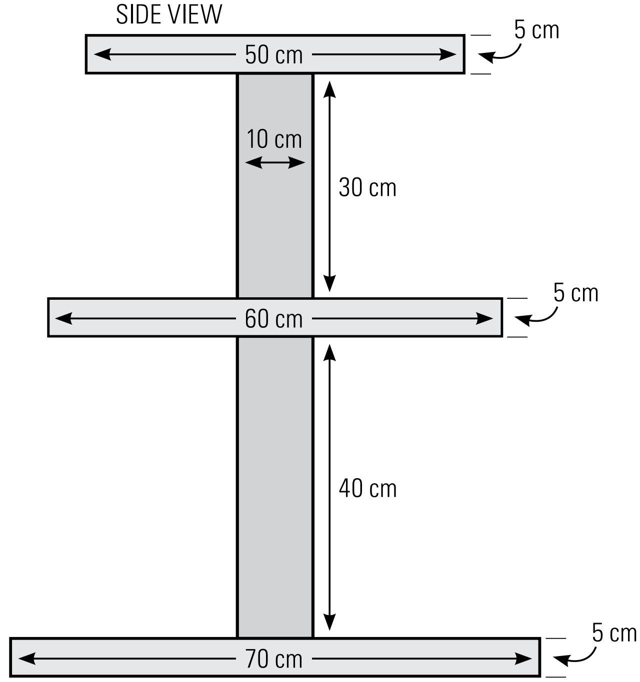
SCALE 1:10



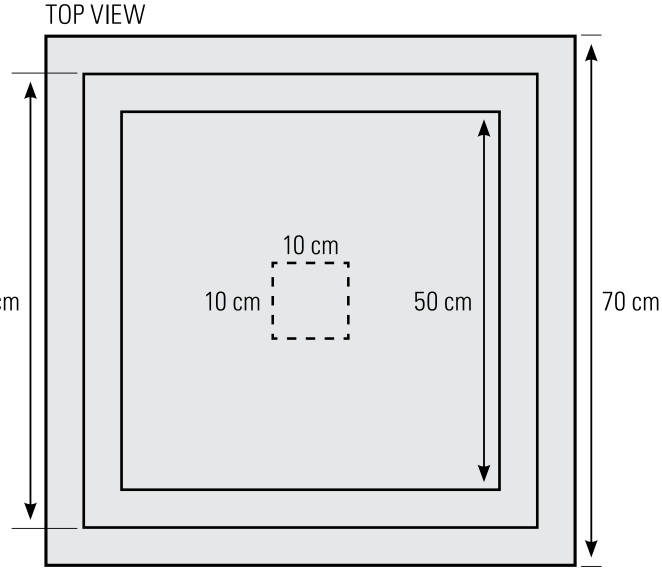


b) SCALE 1:10

SIDE VIEW



TOP VIEW



5. a) Calculate the scale factor. 35 ÷ 7 = 5

Divide the actual dimensions by the scale factor.

Back component heights:

35 ÷ 5 = 7 cm

30 ÷ 5 = 6 cm

25 ÷ 5 = 5 cm

Back component width: 25 ÷ 5 = 5 cm

Side component height: 20 ÷ 5 = 4 cm

Side component width: 6 ÷ 5 = 1.2 cm

Shelf/bottom piece length:

25 − 1 − 1 = 23

23 ÷ 5 = 4.6 cm

Shelf/bottom piece width: 6 ÷ 5 = 1.2 cm

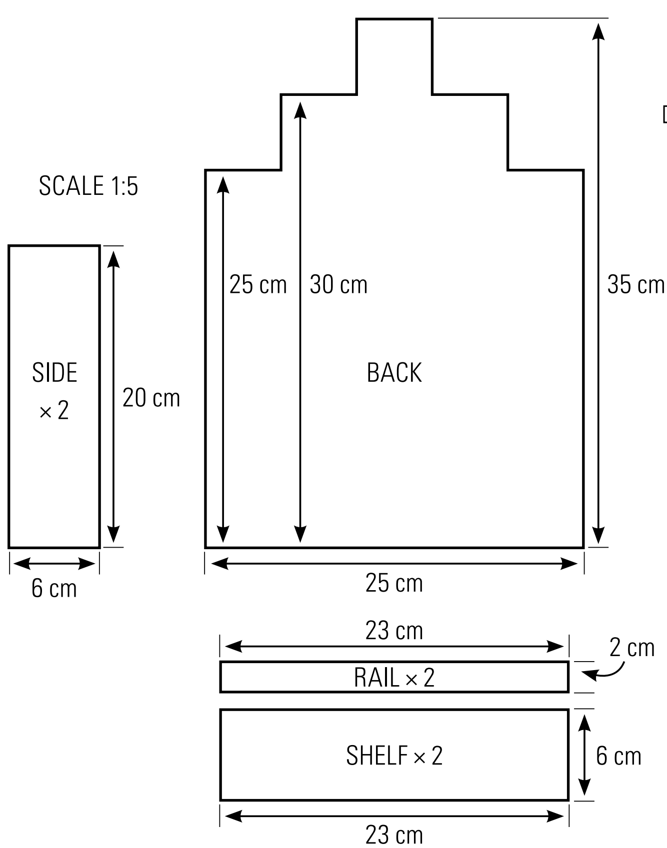
Rail length:

25 − 1 − 1 = 23

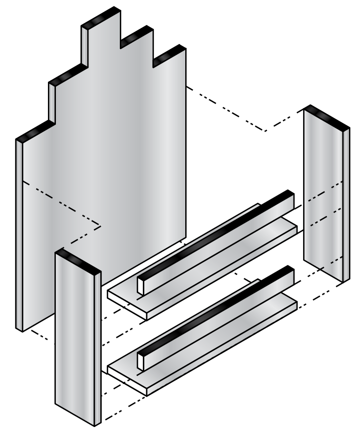
23 ÷ 5 = 4.6 cm

Rail width: 2 ÷ 5 = 0.4 cm

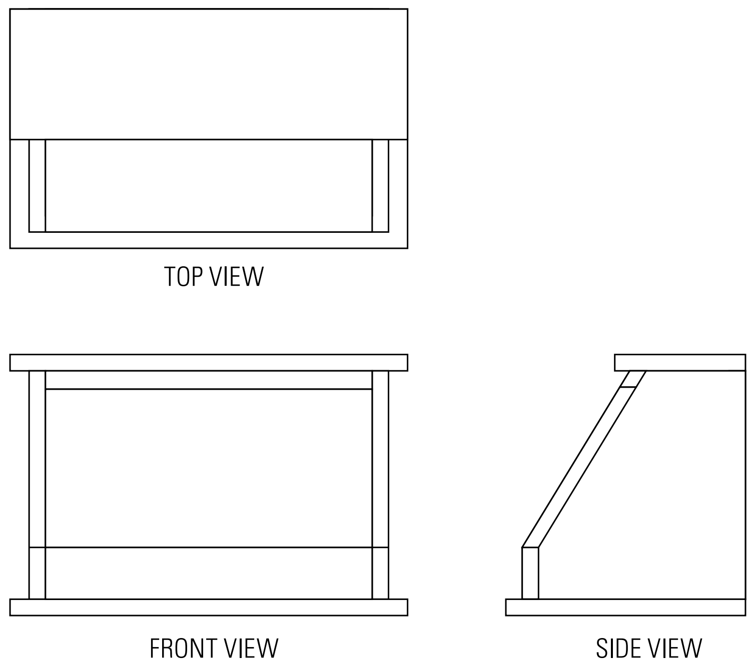
Wood thickness: 1 ÷ 5 = 0.2 cm



b)



6. a)



b) Answers will vary. A sample answer might be:

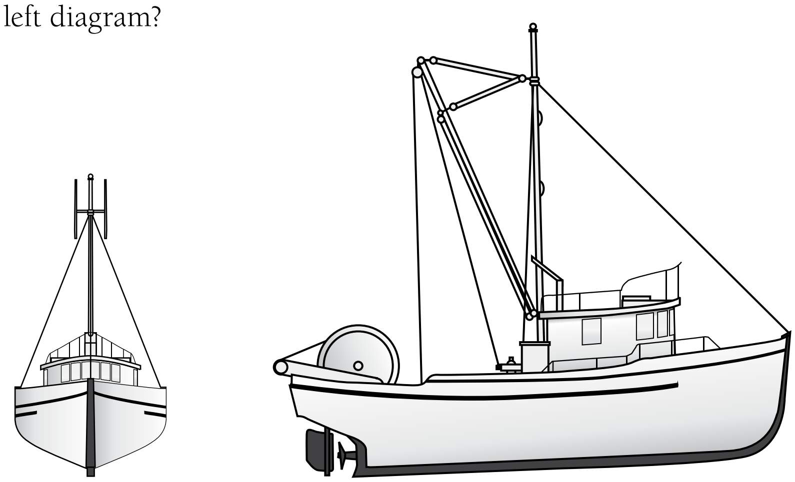
1. Start at the base and attach the back and two side pieces to it.
2. Attach the small rectangular piece to the base, and the slanted front part to the top of that.
3. Attach the small rectangular top piece to the interior of the breadbox. One end of this piece is attached to the left side piece and the other end is attached to the right side piece.
4. Attach the top piece to the sides, back, and front.

SAMPLE CHAPTER TEST

Name: ______________________ Date: ______________________

Part A: Multiple Choice

1. A drawing of a salmon drum seiner has been enlarged. By what scale was the right diagram enlarged in relation to the left diagram?



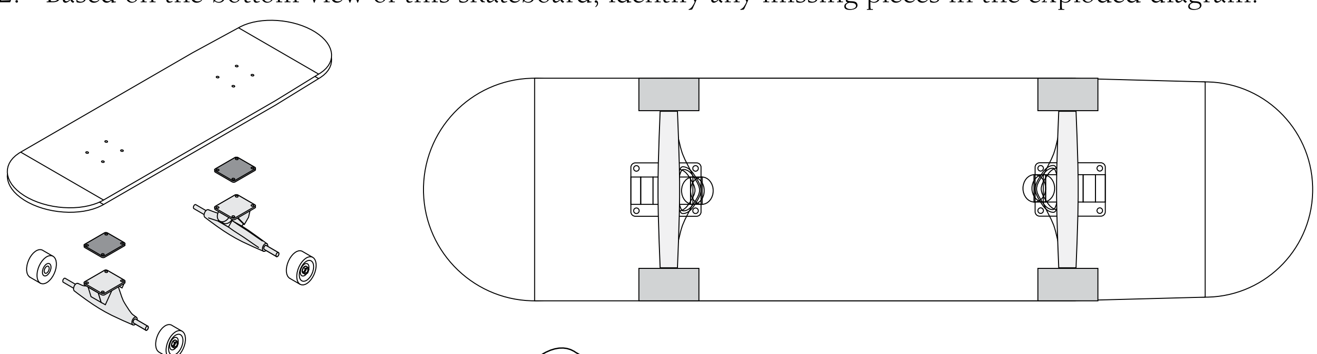
a) 1:125

b) 1:2

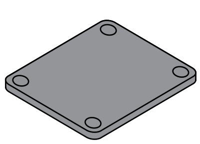
c) 1:150

d) 1:1.5

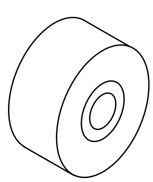
2. Based on the bottom view of this skateboard, identify any missing pieces in the exploded diagram.



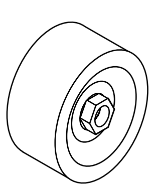
a)



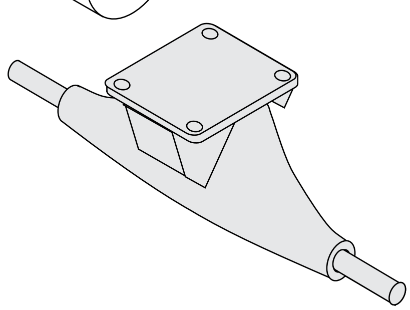
b)



c)



d)



3. The style of drawing that will provide information about all of the parts of an object is:
   a) component parts
   b) exploded view
   c) one-point perspective
   d) isometric

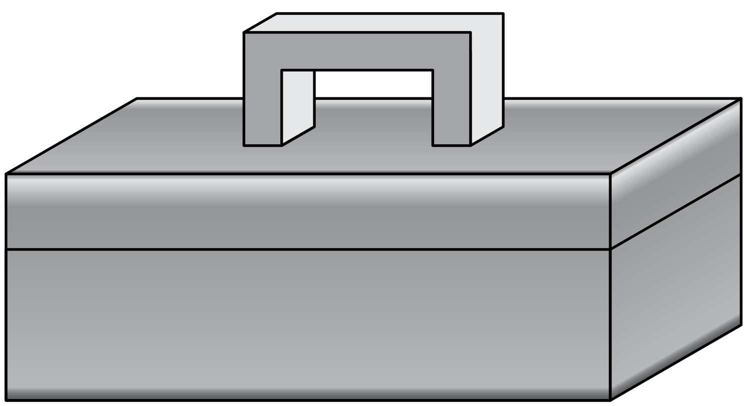
4. Ella restored her half-ton truck. She had to disassemble the truck entirely before she began the restoration. What types of technical drawings did she need to use when reassembling the truck?
   a) isometric
   b) one-point perspective
   c) component parts
   d) exploded view

Part B: Short Answer

5. In Roland, Manitoba there is a large pumpkin-shaped monument. It is a tribute to Edgar VanWyck for his contributions to the community and the large pumpkins he grew. Ty, who is 172 cm tall, has his photo taken in front of the monument. In the photo, Ty is 3.5 cm tall and the monument is 9.2 cm tall. How tall is the monument in real life?

6. Jayden buys a flat-packed coffee table to assemble at home. The packaging has a drawing of the coffee table labeled with the scale 1:7. Jaden assembles the coffee table, which has a rectangular top that measures 1.2 m by 1.5 m. It also has legs that are 55 cm long. Calculate the dimensions of the scale drawing on the package.

7. The following diagram of a toolbox needs to be reduced in size by a scale factor of 1:2.5, so that it will fit on a brochure. The handle is 3 cm long, 1.5 cm high, and 0.5 cm wide. What are the dimensions of the handle in the brochure diagram?



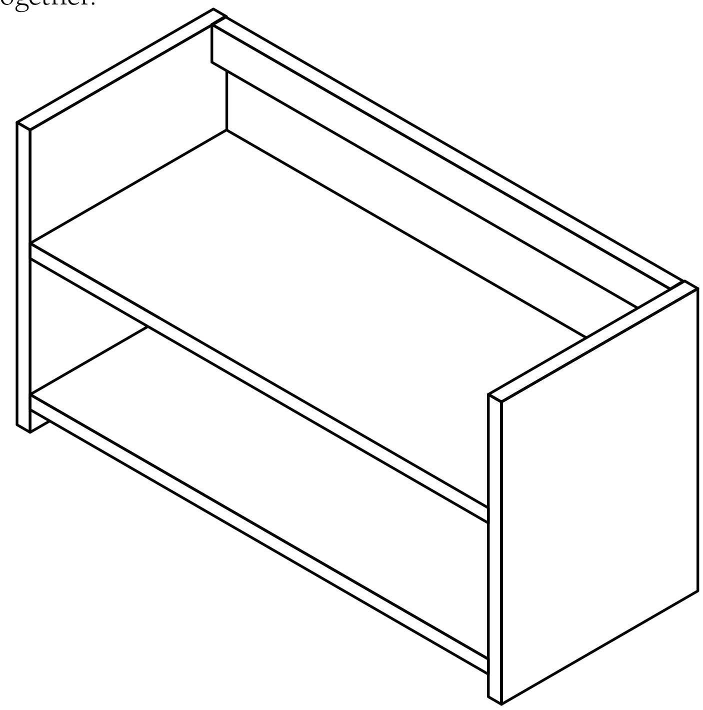
Part C: Extended Answer

8. You volunteer at the local senior centre. You are designing a new built-in shelf for the centre, which will cover one complete wall in the common room. You have gathered the following information.

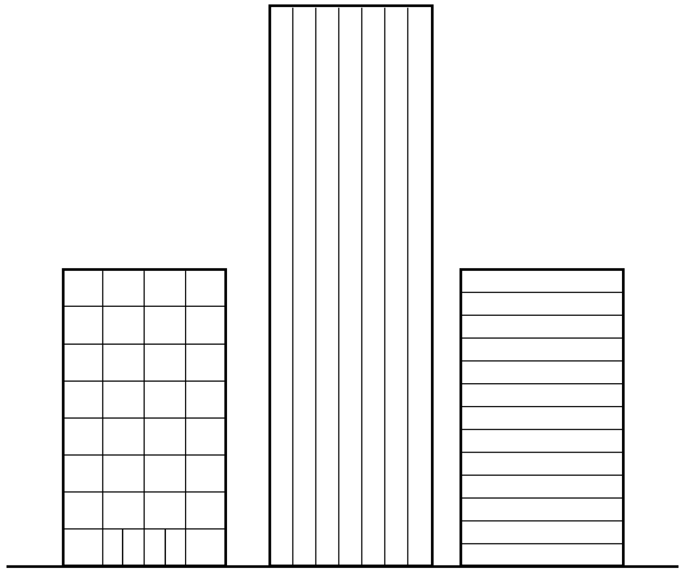
   - The centre members like to play cards, board games, do puzzles, read books, watch TV and movies, listen to music, and visit.
   - They have a 56″ TV.
   - The wall the shelf will go on is 9′ high and 13′ wide.

   Create a scale diagram of a shelf that could accommodate the materials mentioned above. Your diagram should cover most of a 8.5″ by 11″ sheet of paper. Include your scale on the diagram.

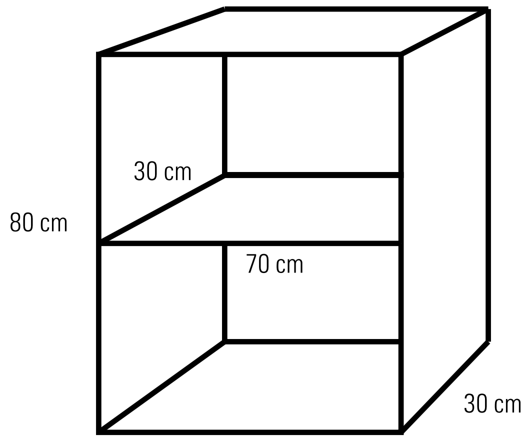
9. Your brother is assembling a shoe rack. The instructions were missing and he needs your help to put it together. The only information you have to work with is the picture on the box. Create an exploded diagram showing how all the components fit together.



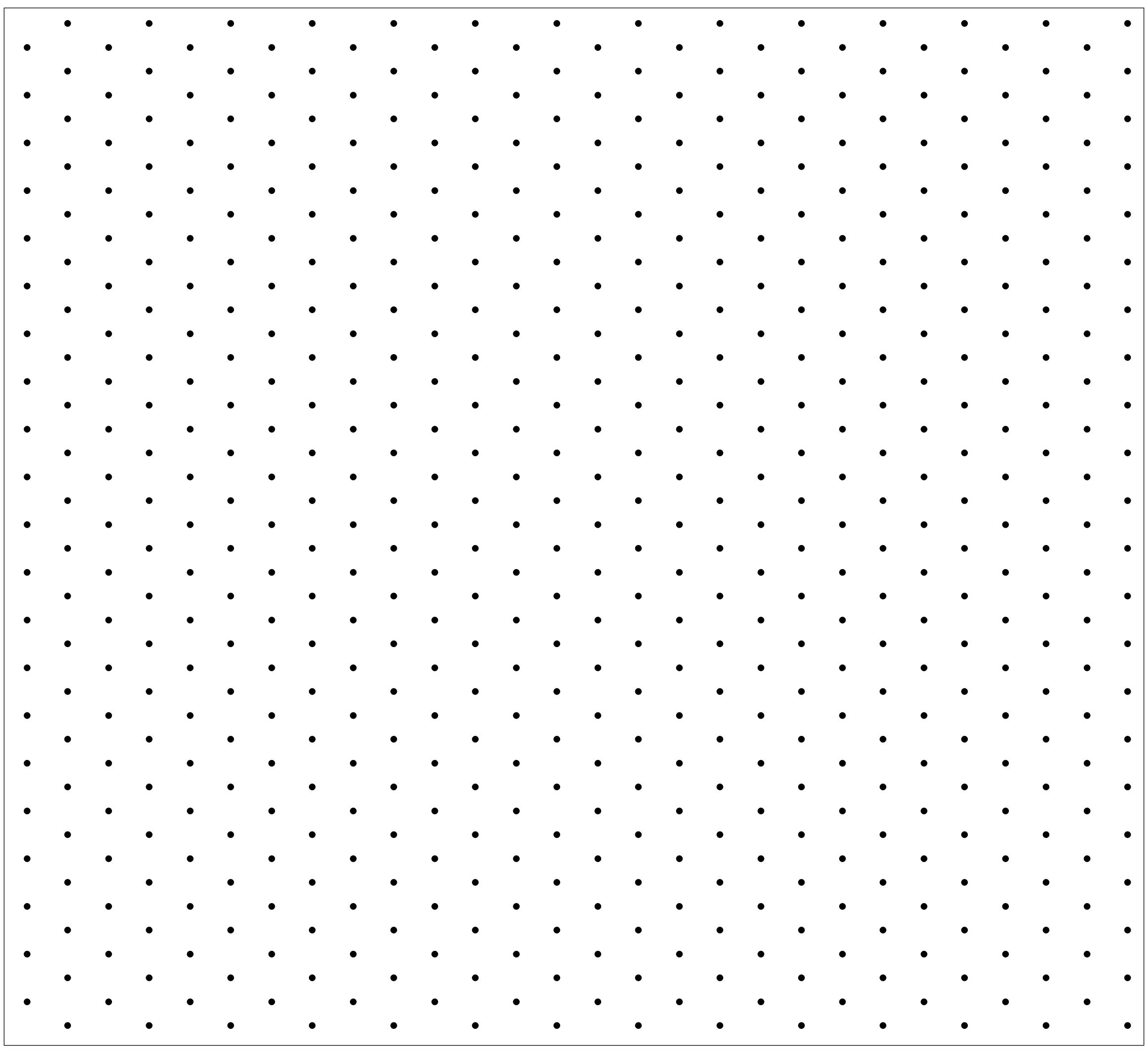
10. As the art director of the new hit series "ALIENS," you have asked one of the set artists to draw these downtown buildings in one-point perspective, so that it may be used to create the scene that will be filmed from above.



11. Draw an isometric diagram of this display case on the isometric dot paper provided.

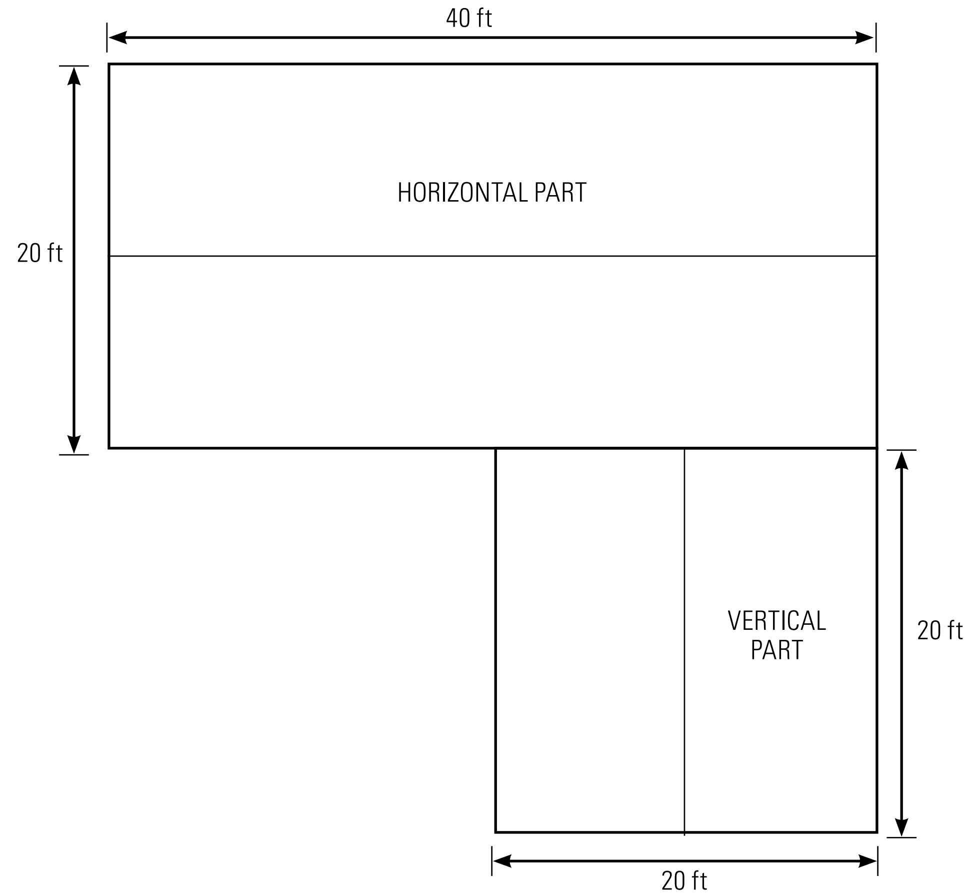


SCALE 1:20



12. Suk owns a roofing company. He is preparing a quote for putting new shingles on a customer's cabin which has a triangular, or gable, roof. Suk has sketched the top view of the house to scale.

SCALE 1 in:10 ft



a) Calculate the area of each section of roof to determine how many bundles of shingles are needed to cover the entire roof. Add 10% to the overall area to allow for wastage and errors. Each bundle of shingles covers 25 square feet.

b) What is the total cost of shingles if a package costs $25.99 (tax included)?

SAMPLE CHAPTER TEST: SOLUTIONS

Part A: Multiple Choice

1. d) 1:1.5
2. b) is the missing part
3. a) component parts
4. c) and d)

Part B: Short Answer

5.
$$\frac{3.5}{9.2} = \frac{172}{x}$$
$$9.2x \times \frac{3.5}{9.2} = \frac{172}{x} \times 9.2x$$
$$3.5x = 1582.4$$
$$x \approx 452$$

The monument is about 452 cm or 4.5 m tall.

6. Tabletop:

$120 \div 7 \approx 17$

$150 \div 7 \approx 21.4$

Table leg

$55 \div 7 \approx 7.9$

The tabletop measures 17 cm by 21.4 cm and the table legs are 7.9 cm long.

The tabletop measures 17 by 21.4 cm and the table legs are 7.9 cm long.

7. Toolbox handle:

$3 \div 2.5 = 1.2$

$1.5 \div 2.5 = 0.6$

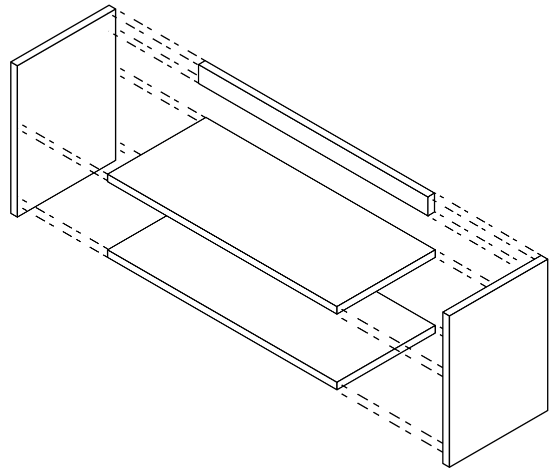
$0.5 \div 2.5 = 0.2$

The handle will measure 1.2 cm by 0.6 cm by 0.2 cm.

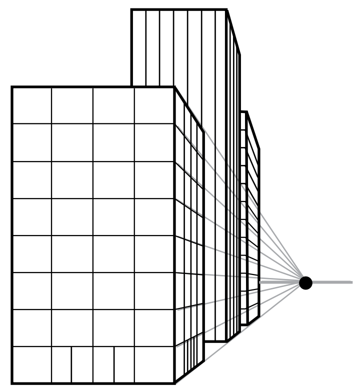
Part C: Extended Answer

8. See sample diagram on following page.

9.

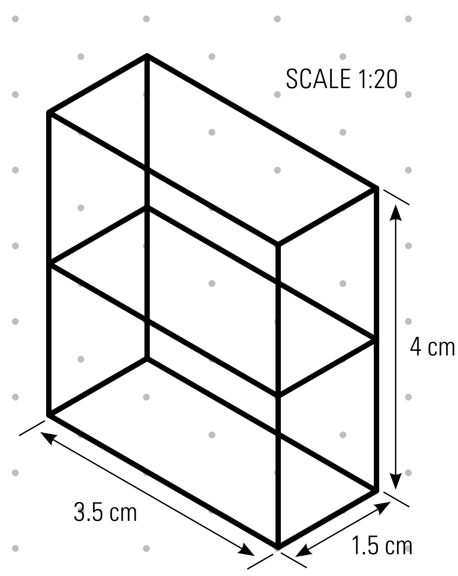


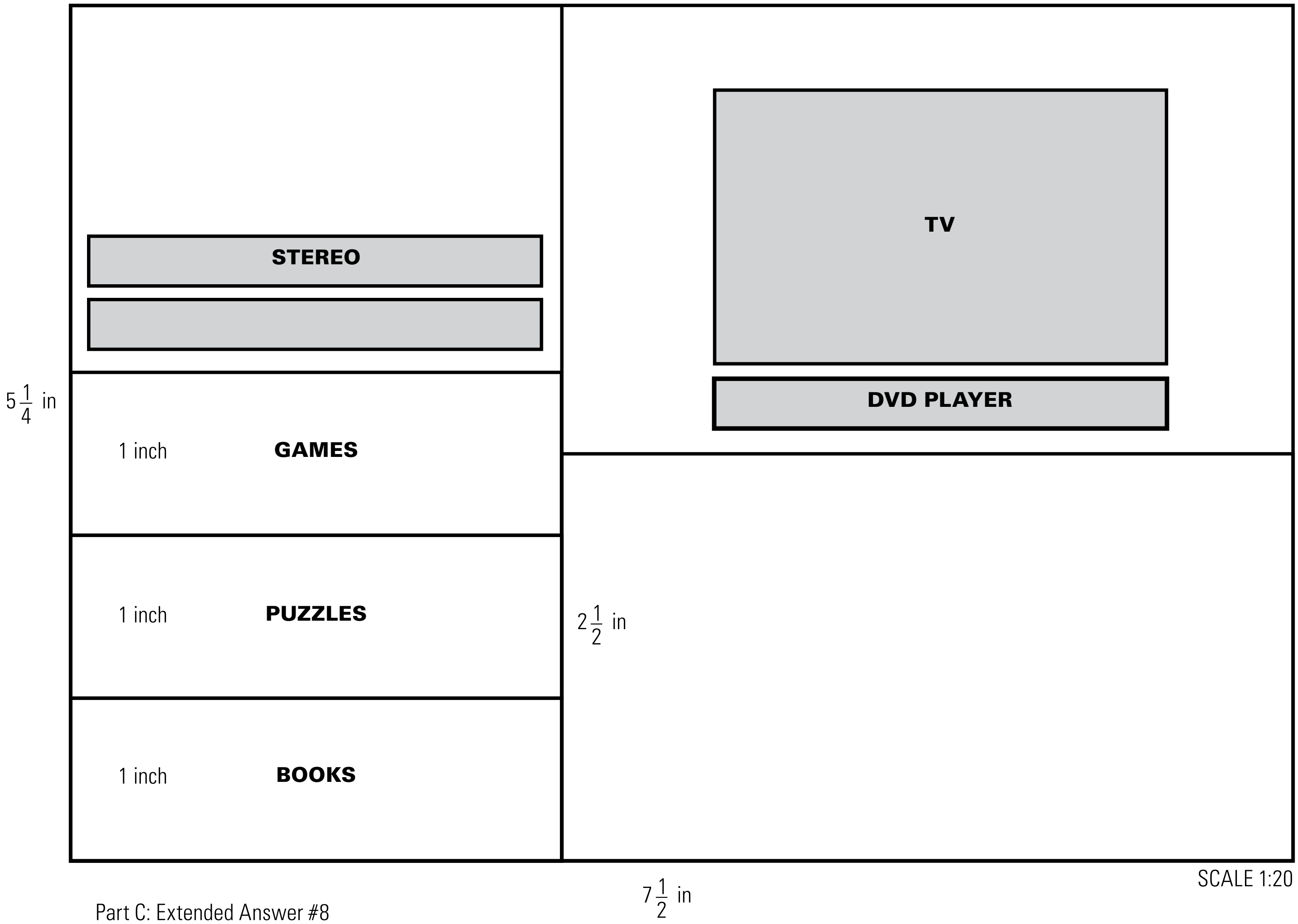
10.



Alternatively, this could be drawn from a left-hand perspective.

11.





Part C: Extended Answer #8

12. a) Horizontal part of roof:

$2 \times 10 = 20$

$4 \times 10 = 40$

The horizontal part of the roof measures 20 feet by 40 feet.

Multiply to find the area.

$20 \times 40 = 800$

The horizontal part of the roof is 800 square feet.

Add 10 percent for wastage.

$800 \times 0.10 = 80$

$880 \div 25 = 35.2$

Vertical part of roof:

$2 \times 10 = 20$

$20 \times 20 = 400$

$400 \times 0.10 = 40$

$440 \div 25 = 17.6$

$35.2 + 17.6 = 52.8$

It will take 53 bundles of shingles to cover the roof.

b) $25.99 \times 52.8 \approx 1372.27$

The shingles will cost about $1372.27.

BLACKLINE MASTER 5.1

ISOMETRIC DOT PAPER (1 CM × 1 CM)

Name: ______________________________ Date: ______________________________

BLACKLINE MASTER 5.2

ISOMETRIC DOT PAPER (0.25 IN)

Name: ______________________________ Date: ______________________________

BLACKLINE MASTER 5.3

CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research on materials used to make your object;
- the accuracy of your scale calculations on your drawings and in your model;
- the quality of your scale, exploded, component parts, and isometric or perspective drawings;
- the effectiveness of your uses of technology for research; and
- your completion of your model and drawings on time.

How do you feel you have done, given the criteria above? ______________________

____

____

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

____

____

In what areas did you excel? ______________________

____

____

Are there areas in which you could improve? ______________________

____

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

____

____

____

If you had to do the project over again, what would you do differently?

____

____

____

BLACKLINE MASTER 5.4

CHAPTER PROJECT CHECKLIST

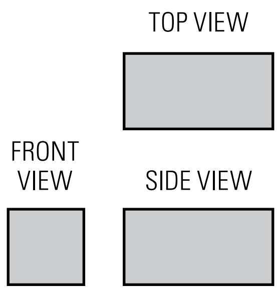
Name: ______________________ Date: ______________________

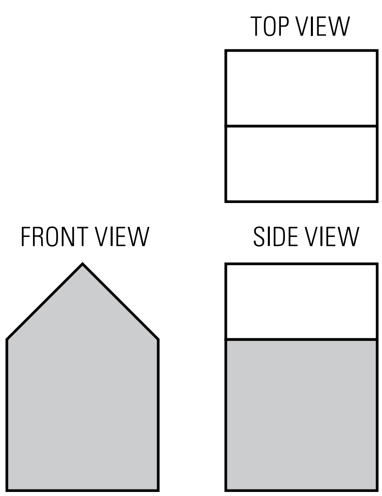
PLANNING CHECKLIST	
❒ Did you pick an appropriate scale for your model?	
❒ Are your different diagrams complete?	
❒ Did you make component parts and exploded drawings?	
❒ Have you researched materials commonly used to make your object?	
❒ Are you able to explain how you designed and built your scale model?	
❒ Did you use any special software to make your drawings?	
❒ Have you completed a scale model of the object you designed?	
❒ Other notes	

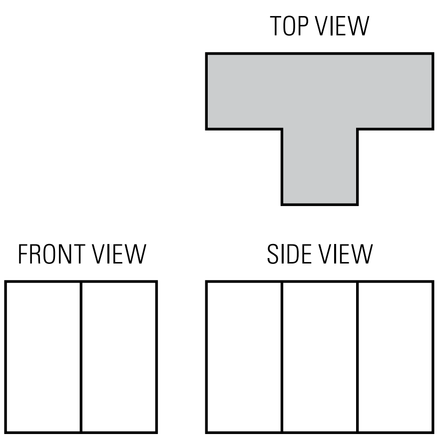
BLACKLINE MASTER 5.5

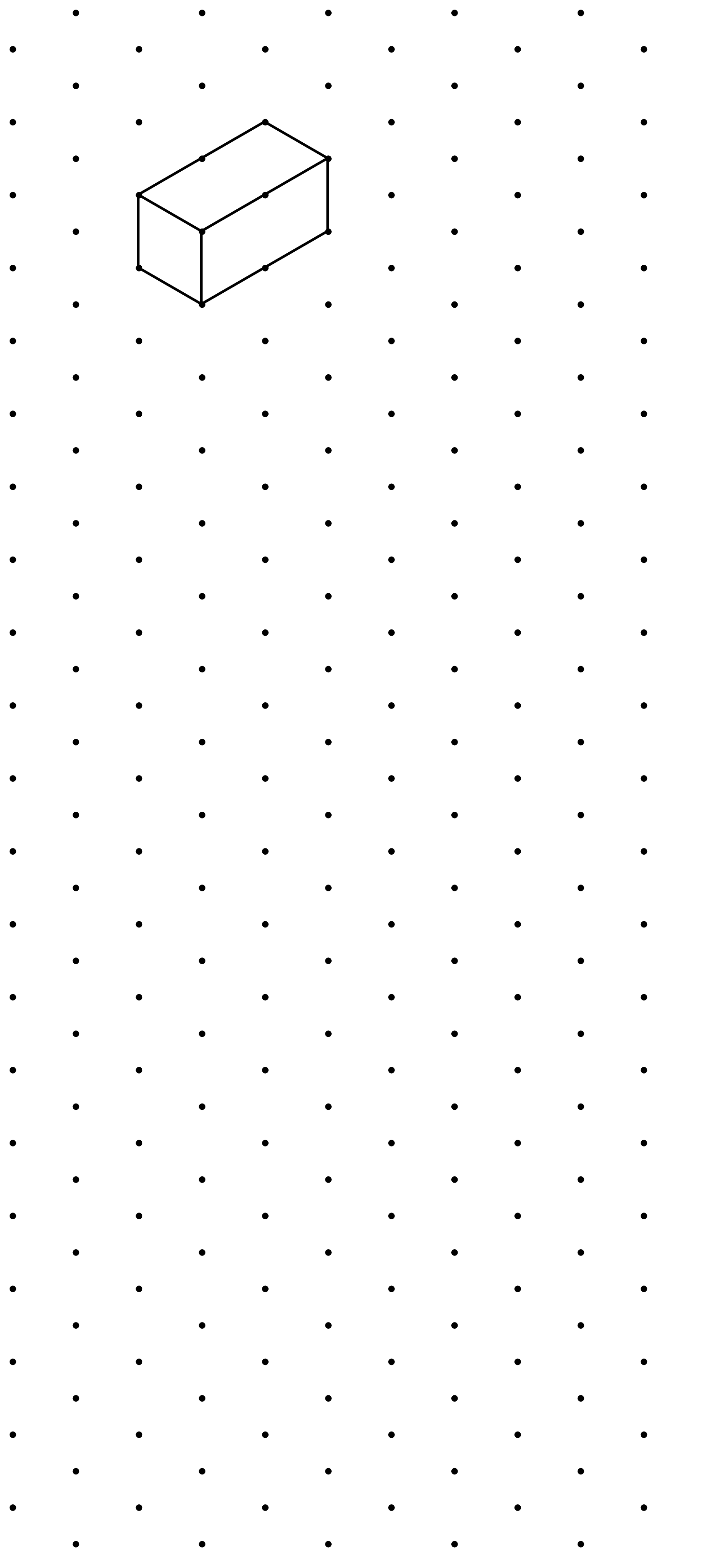
DRAWING ISOMETRIC VIEWS (ISOMETRIC DOT PAPER, 1 CM)

Name: ______________________________ Date: ______________________________





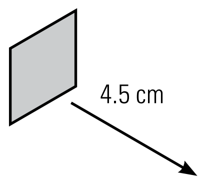


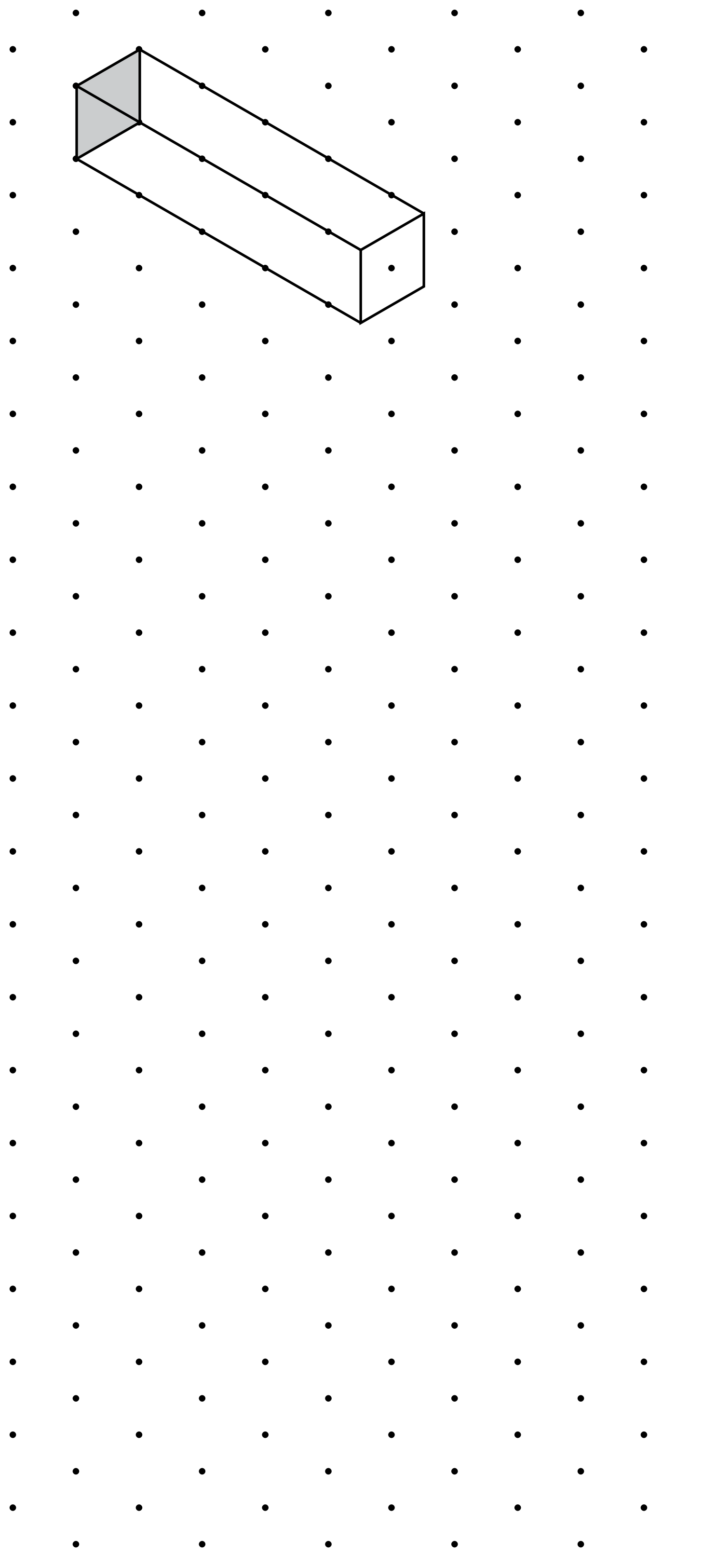


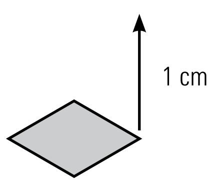
BLACKLINE MASTER 5.6

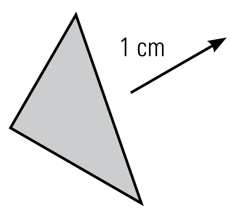
PROJECTING ISOMETRIC VIEWS (ISOMETRIC DOT PAPER, 1 CM)

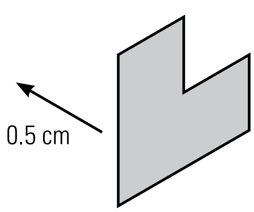
Name: ______________________________ Date: ______________________________

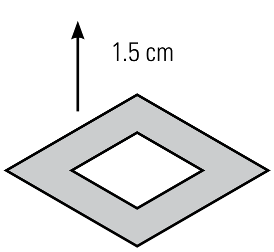








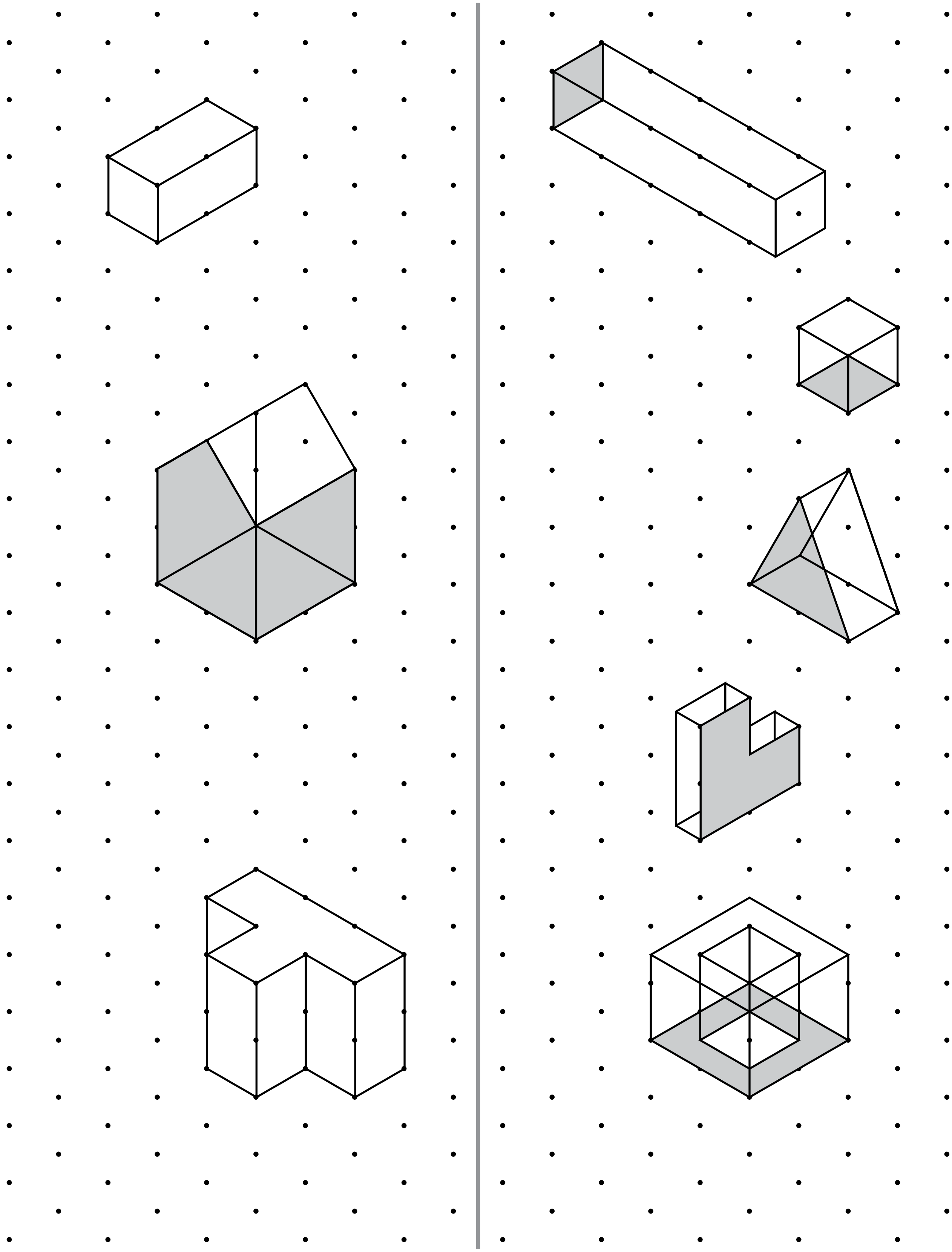




BLACKLINE MASTER 5.5 AND 5.6: SOLUTIONS

DRAWING AND PROJECTING ISOMETRIC VIEWS

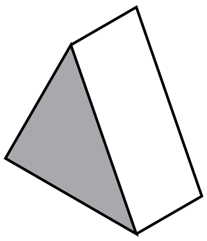
Name: ______________________________ Date: ______________________________

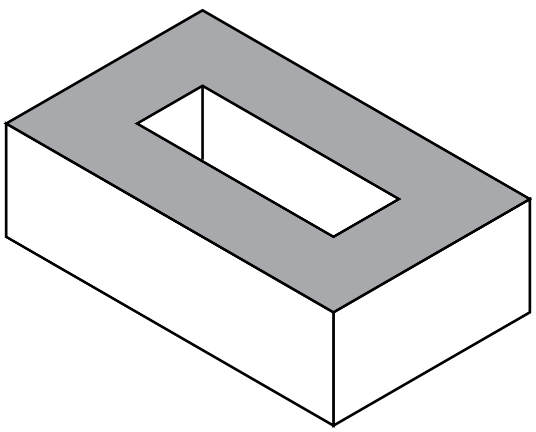


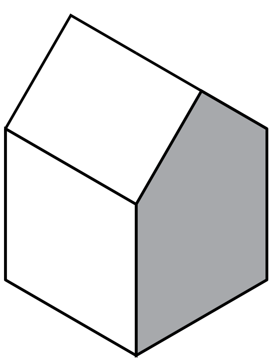
BLACKLINE MASTER 5.7

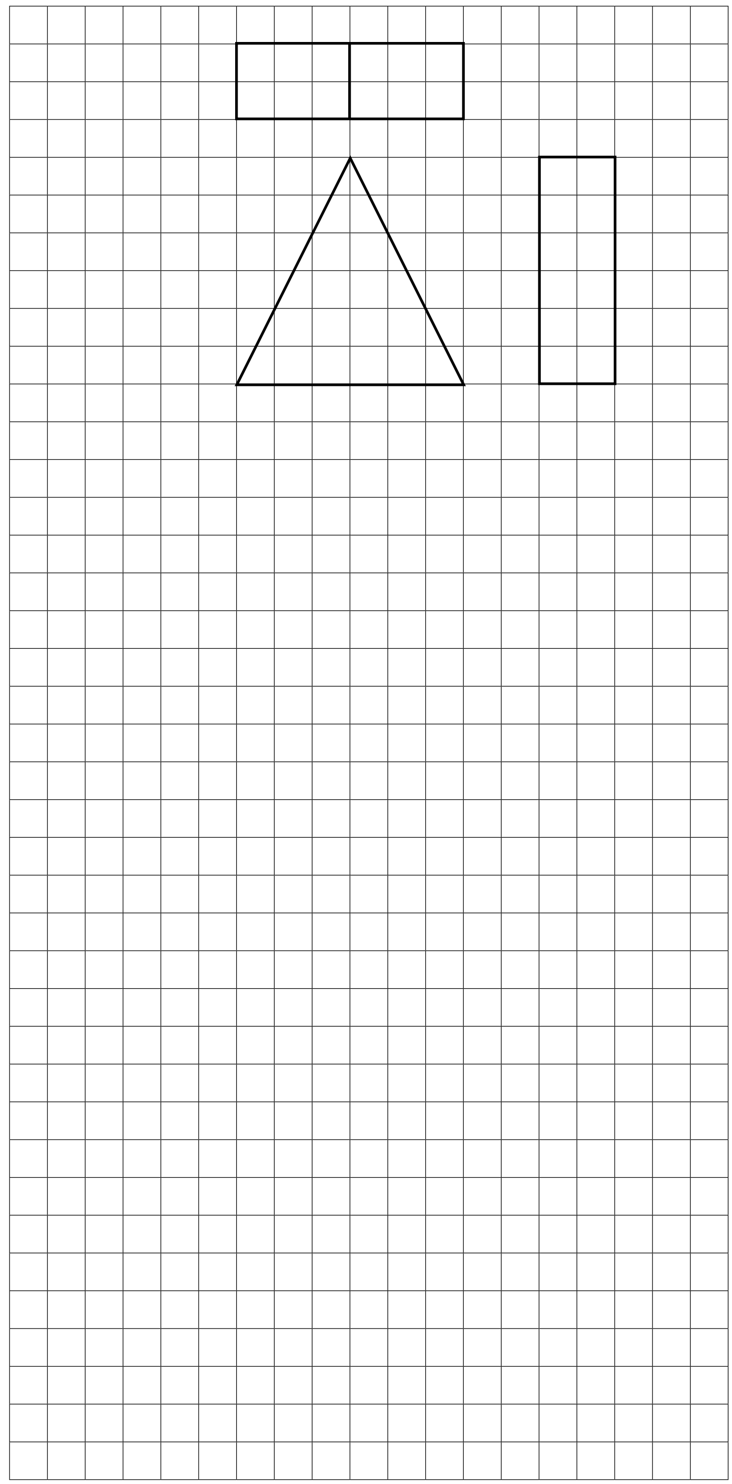
DRAWING OBJECT VIEWS—GRAPH PAPER (0.5 CM × 0.5 CM)

Name: ______________________________ Date: ______________________________





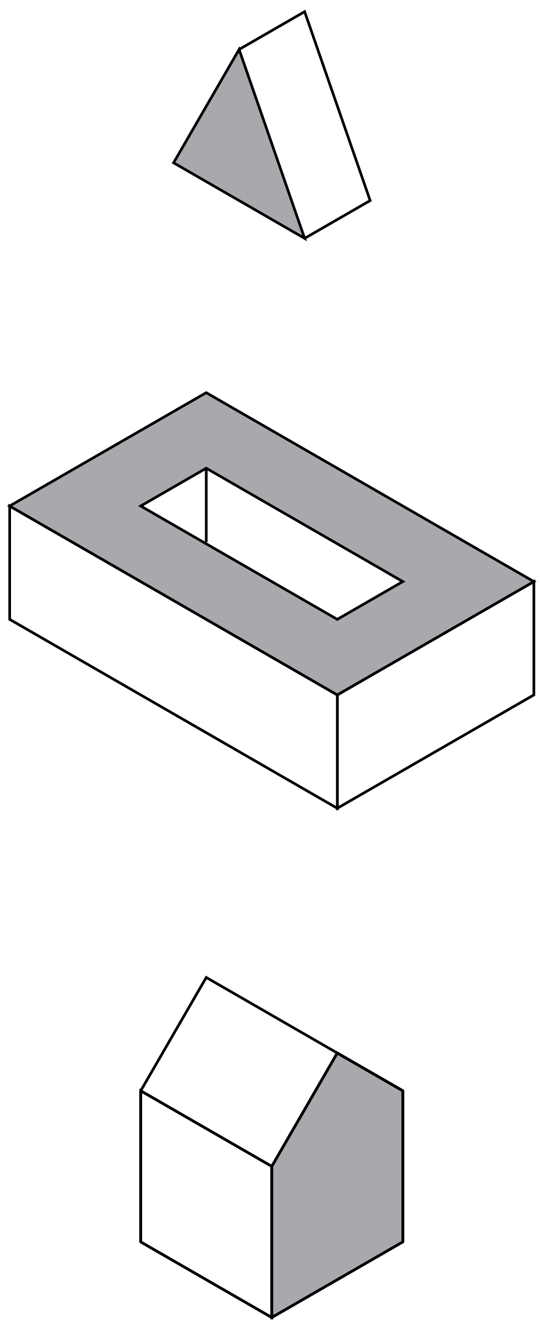


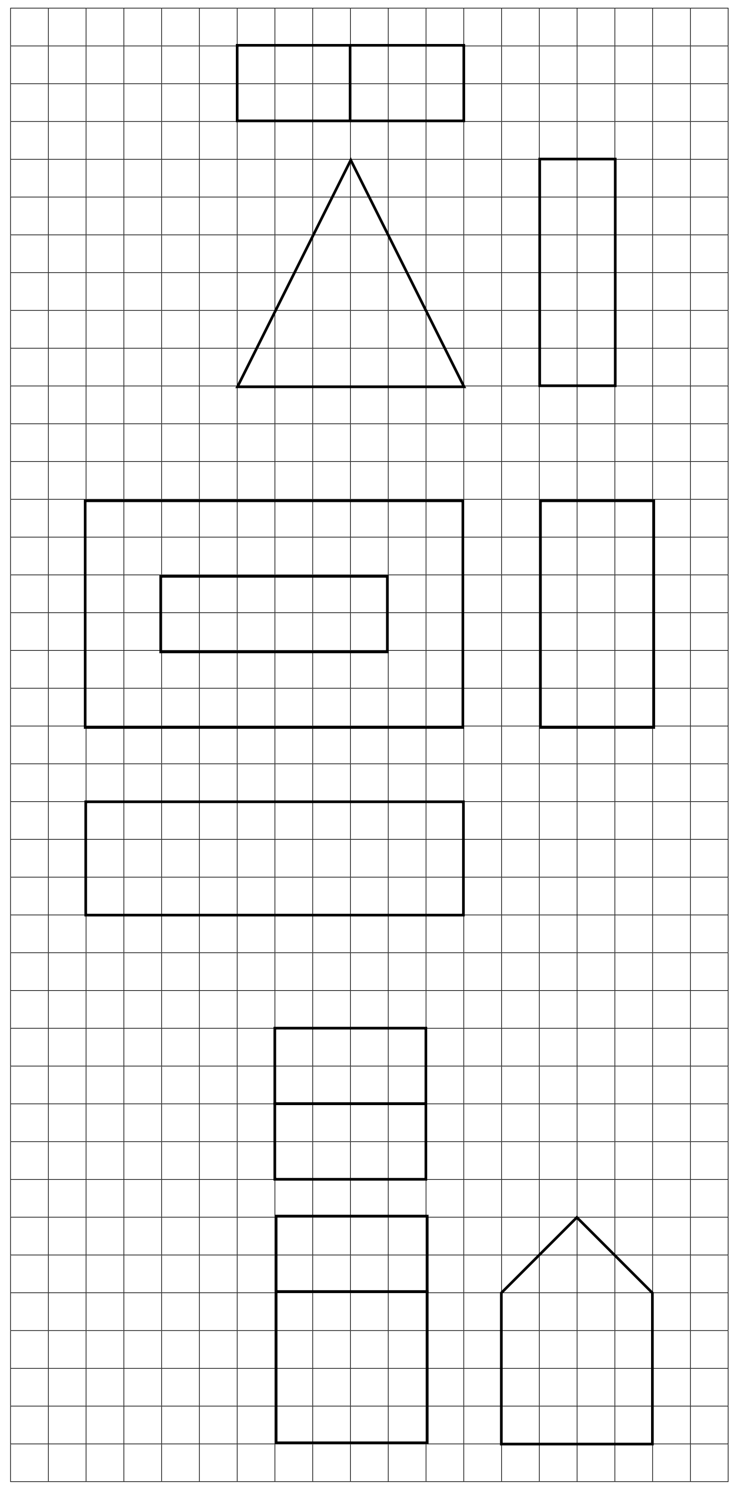


BLACKLINE MASTER 5.7: SOLUTIONS

DRAWING OBJECT VIEWS—GRAPH PAPER (0.5 CM × 0.5 CM)

Name: ______________________ Date: ______________________





BLACKLINE MASTER 5.8

REVIEWING PRIOR CONCEPTS

Name: ______________________ Date: ______________________

Working with Proportional Reasoning

1. Solve for x.

   a) $\frac{x}{10} = \frac{40}{50}$

   c) $\frac{x}{2056} = \frac{3}{4}$

   b) $\frac{3}{12} = \frac{15}{x}$

   d) $\frac{25}{x} = \frac{40}{200}$

Working With Rate and Ratio

2. Write a statement that indicates each of the following rates.

   a) 1 cm on a map represents 1000 km.

b) How much would you earn in an 8-hour day if you are paid $9.15 an hour.

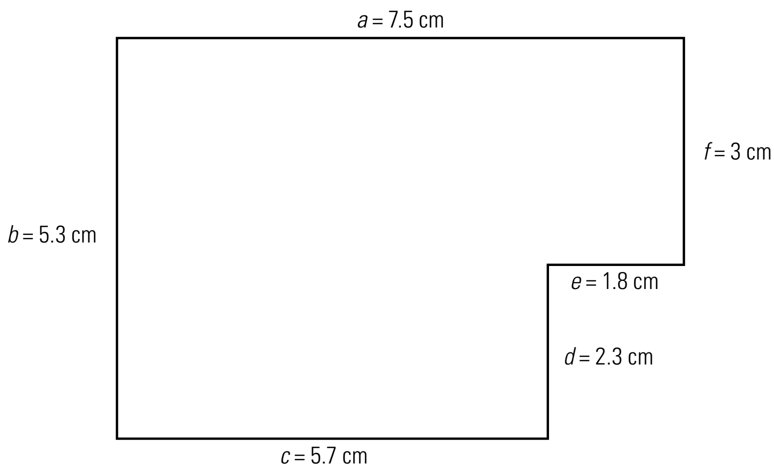
c) A car uses 7.1 L of gas for every 100 km driven.

3. If salami at the deli costs $1.99 per 100 g, how much will you pay for 350 g?

4. The ratio of flour to sugar in a recipe is 4:3. How much flour would you need if you used $1\frac{1}{2}$ cups of sugar?

Working with Similar Figures

5. Gregor is an interior designer. He has sketched a client's living room to show different arrangements of the furniture. On his diagram, the walls have the lengths shown on the diagram below. If the longest wall in the room is actually 6 m, how long are the other walls?



6. Karin is building a model of her house. She uses a scale where 10 cm represents 2 m. If one room is 6.5 m long, 4.8 m wide, and 2.8 m tall, what will its dimensions be in the model?

7. Hazuki's house is 55 ft wide and 40 ft deep. A drawing of her property shows the house as 11 in wide and 8 in deep. What scale was used on the drawing?

BLACKLINE MASTER 5.8: SOLUTIONS

Working with Proportional Reasoning

1. a)
$$\frac{x}{10} = \frac{40}{50}$$
$$50 \times \cancel{10} \times \frac{x}{\cancel{10}} = \frac{40}{\cancel{50}} \times 10 \times \cancel{50}$$
$$50x = 400$$
$$\frac{50x}{50} = \frac{400}{50}$$
$$x = 8$$

b)
$$\frac{3}{12} = \frac{15}{x}$$
$$\cancel{12} \times x \times \frac{3}{\cancel{12}} = \frac{15}{\cancel{x}} \times \cancel{x} \times 12$$
$$3x = 15 \times 12$$
$$3x = 180$$
$$\frac{3x}{3} = \frac{180}{3}$$
$$x = 60$$

c)
$$\frac{x}{2056} = \frac{3}{4}$$
$$\cancel{2056} \times \frac{x}{\cancel{2056}} = \frac{3}{4} \times 2056$$
$$x = 1542$$

d)
$$\frac{25}{x} = \frac{40}{200}$$
$$200 \times \cancel{x} \times \frac{25}{\cancel{x}} = \frac{40}{\cancel{200}} \times x \times \cancel{200}$$
$$200 \times 25 = 40x$$
$$5000 = 40x$$
$$\frac{5000}{40} = \frac{40x}{40}$$
$$125 = x$$

Working with Rate and Ratio

2. a) 1 cm:1000 km or 1 cm/1000 km.

b) Calculate how much you earn in 8 hours.

$9.15 × 8 = $73.20

The rate statement is $73.20:8-hour day or $73.20/8-hour day.

c) 7.1 L:100 km or 7.1 L/100 km.

3. Set up a proportion to solve for x, the cost of 350 g of salami.
$$\frac{\$1.99}{100\text{ g}} = \frac{x}{350\text{ g}}$$
$$\frac{1.99}{100} = \frac{x}{350}$$
$$350 \times \cancel{100} \times \frac{1.99}{\cancel{100}} = \frac{x}{\cancel{350}} \times 100 \times \cancel{350}$$
$$696.5 = 100x$$
$$\frac{696.5}{100} = \frac{100x}{100}$$
$$6.97 \approx x$$

You will pay $6.97/350 g, or about $6.97 for 350 g of salami.

4. Set up a proportion to solve for x, the amount of flour needed.
$$\frac{4}{3} = \frac{x}{1.5}$$
$$\cancel{3} \times 1.5 \times \frac{4}{\cancel{3}} = \frac{x}{\cancel{1.5}} \times \cancel{1.5} \times 3$$
$$6 = 3x$$
$$\frac{6}{3} = \frac{3x}{3}$$
$$2 = x$$

You will need 2 cups of flour.

Working with Similar Figures

5. Determine the unit scale. If 7.5 cm represents 600 cm, then calculate how many metres 1 centimetre represents.

   $600 \div 7.5 = 80$

   The scale of the drawing is 1:80.

   The remaining side lengths can be calculated by multiplying each scaled unit by 80.

$$\begin{aligned} b &= 5.3 \times 80 \\ b &= 424 \text{ cm} \\ c &= 5.7 \times 80 \\ c &= 456 \text{ cm} \\ d &= 2.3 \times 80 \\ d &= 184 \text{ cm} \\ e &= 1.8 \times 80 \\ e &= 144 \text{ cm} \\ f &= 3 \times 80 \\ f &= 240 \text{ cm} \end{aligned}$$

6. The ratio of the model to the actual room is 10 cm:2 m. Write this without units in the form of a fraction as $\frac{10}{2}$, which can be simplified to $\frac{5}{1}$.

   Represent length, width, and height with ℓ, w, and h, respectively, and set up proportions to calculate their measures.

$$\frac{5}{1} = \frac{\ell}{6.5}$$

$$\frac{5}{1} = \frac{w}{4.8}$$

$$\frac{5}{1} = \frac{h}{2.8}$$

   Solve for ℓ.

$$\begin{aligned} \frac{5}{1} &= \frac{\ell}{6.5} \\ 6.5 \times 5 &= \frac{\ell}{\cancel{6.5}} \times \cancel{6.5} \\ 6.5 \times 5 &= \ell \\ 32.5 &= \ell \end{aligned}$$

   The length of the room in the model will be 32.5 cm.

   Use the same method to solve for w and h.

$$\begin{aligned} \frac{5}{1} &= \frac{w}{4.8} \\ 4.8 \times 5 &= w \\ 24 &= w \\ \frac{5}{1} &= \frac{h}{2.8} \\ 2.8 \times 5 &= h \\ 14 &= h \end{aligned}$$

   The width in the model is 24 cm and the height is 14 cm.

7. Calculate the scale of the drawing by dividing the width on the drawing by the width of the actual house.

$$\frac{11 \text{ in}}{55 \text{ ft}} = \frac{(11 \div 11) \text{ in}}{(55 \div 11) \text{ in}}$$

$$\frac{11 \text{ in}}{55 \text{ ft}} = \frac{1 \text{ in}}{5 \text{ ft}}$$

   The scale used on the drawing was 1 in:5 ft.

ALTERNATIVE CHAPTER PROJECT—DESIGN AND BUILD A BIRDHOUSE

TEACHER MATERIALS

GOALS: To use views, one-point perspective, isometric, component parts, exploded, and scale diagrams to design and build a scale model; to calculate a suitable scale for a model; to produce a set of explanatory diagrams; to construct a 3-D model using a 2-D representation; and to synthesize learning in this chapter.

OUTCOME: Using their learning on scale models and representations, students will produce a set of explanatory diagrams that will be used to construct a scale model. Students will learn how to use diagrams to accurately convey information. They will also learn the relevance of using different diagrams to convey various kinds of information effectively.

PREREQUISITES: Students will need to know how to calculate a scale and apply it to make scale drawings. They will need to know how to take accurate measurements. They should also know how to draw three-dimensional objects.

ABOUT THIS PROJECT: In this project, students will produce a scale birdhouse that resembles their home. They will use different types of drawings to design and build the birdhouse. The drawings will be displayed on a poster that describes how students completed the project. You can engage student interest in this project by finding and circulating samples of birdhouses that have been built to resemble homes. Photos can be found at the websites below.

www.homebuying.about.com/od/buildingahome/ss/custombirdhouse.htm

www.birdfeedersdirect.com/bird-houses/decorative-bird-houses.aspx

You can assign students to find the measurements of their house as homework. If some students are unable to calculate the dimensions of their actual house, you might want to have some sample measurements of sample houses that they can use, instead. You will also need to provide students with the materials needed to build their birdhouses. Materials can include scissors, cardboard, foam board, matte board, and glue or glue strips as adhesive.

1. Start to plan

Students can complete this project in pairs and choose which house will work best as a model. A few days before introducing this project, you can ask, that as homework, students have their picture taken in front of their home.

2. Draft your birdhouse design

Students will complete view, one-point perspective, and isometric diagrams during this part of the project. You might want to assign one diagram as homework throughout the week, or set aside class time to make the diagrams. You can assess the diagrams as the project progresses and give feedback to students who need assistance to make their diagrams correctly.

3. Build your birdhouse and make your poster

This section of the project could be divided over three class periods. During the first, students can finish their isometric and component parts diagrams. During the second, they can build their birdhouses with the materials you assign or provide. The poster can be completed during a third class period, or as homework.

Extension

If your class is interested in raising money for a certain charity, challenge them to build durable, full-size, weatherproof versions of their birdhouses, suitable for use in your community. You could arrange a silent auction by donation within the school or as an outside event.

ASSESSING THE PROJECT

1. Start to plan

Ensure that each group has a suitable house on which they can base their model. You can also go over some suitable materials that students can use to build their models.

2. Draft your birdhouse design

Evaluate the initial drawings students make of their birdhouses. You can determine whether they are making their scale drawings accurately, and if they have picked an appropriate scale for the birdhouse.

3. Build your birdhouse and make your poster

You can continue to assess the accuracy of student drawings and provide constructive feedback. When grading the scale models, check that they are made to scale and have the same measurements that students used in their diagrams. You can review the posters by looking for visual clarity. Were the drawings used to clearly show how the birdhouse was designed and built?

PROJECT ASSESSMENT RUBRIC: DESIGN AND BUILD A BIRDHOUSE				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Project shows understanding of how to calculate an appropriate scale and make view, one-point perspective, isometric, exploded, and component parts diagrams	shows very limited understanding; scale is inappropriate; birdhouse is not built to scale; diagrams are not drawn correctly; poster is absent, incomplete, or contains mistakes	shows partial understanding; scale is appropriate; some parts of birdhouse are built to scale; more than half of the diagrams are drawn correctly; poster is complete	shows understanding; scale is appropriate; birdhouse is built to scale; one diagram might include mistakes; poster is complete	shows thorough understanding; scale is appropriate; birdhouse is built to scale and includes extra detail; diagrams are complete; poster is correct and detailed
Procedural Understanding				
Accurately: ▪ calculates an appropriate scale for the birdhouse ▪ calculates the real measurements of the real house ▪ uses the scale to make a scale model ▪ draws the 5 types of diagrams required ▪ uses the diagrams on a poster to show how the project was designed and completed	limited accuracy; major errors or omissions For example: • scale for birdhouse is missing or inappropriate • measurements of real house are unrealistic • model is not built to scale • diagrams are incorrect • poster is missing or incomplete	partially accurate; some errors or omissions For example: • scale for birdhouse is appropriate • measurements of real house are realistic • model is mostly built to scale • over half the diagrams are completed correctly • poster needs more work	generally accurate; few errors or omissions For example: • scale for birdhouse is appropriate • measurements of real house are realistic • model is built to scale • diagrams are completed correctly • poster is complete and easy to read	accurate and precise; very few or no errors For example: • scale for birdhouse is appropriate • measurements of real house are realistic • model is built to scale and contains detail • diagrams are completed precisely and correctly • poster is complete, creative, and easy to read
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Uses poster to clearly describe the design and building process; answers questions about the project using appropriate mathematical terminology	does not use poster to describe design and building process; has problems answering questions about the design process; does not use mathematical terminology	uses poster to partially describe the design and building process; answers some questions about the design process; uses some appropriate mathematical terminology	uses poster to clearly describe the design and building process; answers most questions about the design process; uses some appropriate mathematical terminology	uses poster to clearly and precisely describe the building and design process; answers all questions correctly; uses a range of appropriate mathematical terminology

ALTERNATIVE CHAPTER PROJECT — DESIGN AND BUILD A BIRDHOUSE

STUDENT MATERIALS

PROJECT OVERVIEW

Wrens, Purple Martins, and chickadees are some of the birds living in Canada that will use a birdhouse. This project presents the opportunity to use your math and building skills for a good cause. Using your own home as inspiration, you will design and build a birdhouse that is a scale model of your own home. The birdhouses will be presented at a mock charity fundraiser held in your community. Your presentation will include a poster that describes how you designed and built your birdhouse. It will include the drawings that you used during the design process.

START TO PLAN

T

You can complete this project with a partner. To start your project, you will need to decide on whose home you will base your birdhouse. Next, you will need to calculate and record the height, width, and length of the exterior walls of your home, as well as the dimensions of the roof. If it is too difficult to obtain these measurements, you can make a reasonable estimate. If you choose to make an estimate, include a description of your estimation method with your project.

To find your home's dimensions, you will need a photo of it or your partner's. If you have access to a digital camera, have someone take your photo in front of your house. Use proportional reasoning to determine the height of the house. For example, you may be 170 cm tall. In the photo, you may be 2 cm tall, while your house is 7.2 cm tall. You can use proportional reasoning to find the actual height of your house. Your house would be about 612 cm, or 6.12 m tall.

$$\begin{aligned} \frac{2}{7.2} &= \frac{170}{x} \\ 7.2x \times \frac{2}{7.2} &= \frac{170}{x} \times 7.2x \\ 2x &= 1224 \\ x &= 612 \end{aligned}$$

To measure the width and length of your home, use a measuring tape. Use the measurements of your home to determine a suitable scale for your birdhouse. Make a sketch of your birdhouse. Using your scale, label the sketch with the measurements your birdhouse will have. This includes the height, length, and width of the walls and roof.

List the exterior features you will include on your birdhouse, such as windows and doors. What will be their scaled measurements? Add this information to your sketch. Decide on the materials and colour scheme you will use to make the birdhouse.

PROJECT CHECKLIST

Your final project will include the following:

- view, one-point perspective, isometric, and component parts diagrams;
- a birdhouse built to scale; and
- a poster that displays your diagrams and shows how you designed and built your birdhouse.

DRAFT YOUR BIRDHOUSE DESIGN

To begin designing your birdhouse, and to get a better sense of what it will look like, draw a scale front, side, and top view of the house. Next, make a one-point perspective drawing of the house. To help you do this, you can refer to photos.

To get a sense of how much material you will use to build the house, make isometric drawings of its parts, such as the walls and roof. These drawings should be to scale.

BUILD YOUR BIRDHOUSE AND MAKE YOUR POSTER

The design process for your birdhouse is nearly finished. Next, make a components parts diagram of your birdhouse. Label the components with their scaled measurements.

Now, make an exploded diagram that shows how you will assemble your birdhouse. If the exterior includes three-dimensional items such as a landing pole, window frames, or a chimney, make components parts diagrams for these items.

Draw the parts for your birdhouse on cardboard or wood. Cut out and assemble the components. As you put your house together, use your exploded diagram as a guide. Attach any exterior features, or paint or draw them on.

To finish your project, make a poster that describes how you designed and built your birdhouse. Give your poster a title and include a scale statement. Incorporate your view, perspective, isometric, component parts, and exploded diagrams so that people can use them to understand the design process and how you built your birdhouse.

BLACKLINE MASTER 5.1A

ALTERNATIVE CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research on measurements and materials;
- the accuracy of your calculations and different types of drawings;
- the clarity and visual appeal of your poster;
- the creativity you brought to planning and completing your project; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel? ______________________

Are there areas in which you could improve? ______________________

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 5.1B

ALTERNATIVE CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ How can I calculate the measurements of my home?	
❒ What is the best scale at which to build my birdhouse ?	
❒ How will I design my poster so that people can understand how I built my birdhouse?	
❒ What features could I include that would make my birdhouse habitable for birds?	
❒ Are my diagrams accurate?	
❒ How can I design my birdhouse so that it will be a stable structure, and straightforward to build?	
❒ What decorative features can I include on my birdhouse exterior?	
❒ What questions should I be prepared to answer when I present my birdhouse?	

Chapter 6

Financial Services

INTRODUCTION

STUDENT BOOK, pp. 252–299

This is one of two chapters in the student textbook that address the outcomes of the Number strand of Workplace and Apprenticeship Mathematics 11. It also covers an outcome of the Algebra strand. In this chapter, students will develop strategies for managing their money wisely. They will be introduced to the concepts of simple and compound interest. They will also learn about the financial institutions and services available to Canadians, and will develop an understanding of the credit options available to them. The chart below locates this chapter within the curriculum.

NUMBER, GRADES 10–12

This chart illustrates the development of the Number strand in the Workplace and Apprenticeship Mathematics pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 6 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop number sense and critical thinking skills.	**General Outcome** Develop number sense and critical thinking skills.	**General Outcome** Develop number sense and critical thinking skills.
Specific Outcome *It is expected that students will:*	**Specific Outcome** *It is expected that students will:*	**Specific Outcome** *It is expected that students will:*
Solve problems that involve unit pricing and currency exchange, using proportional reasoning.	Demonstrate an understanding of compound interest.	Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.
Demonstrate an understanding of income, including wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.	Demonstrate an understanding of financial institution services used to access and manage finances.	Solve problems that involve the acquisition of a vehicle by buying, leasing, and leasing to buy.
	Demonstrate an understanding of credit options, including credit cards and loans.	Critique the viability of small business options by considering expenses, sales, and profit or loss.

ALGEBRA, GRADES 10–12

This chart illustrates the development of the Algebra strand in the Workplace and Apprenticeship Mathematics pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 6 addresses.

General Outcome Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.	**General Outcome** Develop algebraic reasoning.
Solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.	Solve problems that require the manipulation and application of formulas related to simple interest and finance charges.	Demonstrate an understanding of linear relations by recognizing patterns and trends, graphing, creating tables of values, writing equations, interpolating and extrapolating, and solving problems.

CURRICULUM AND CHAPTER OVERVIEW

General Outcome: Number
Develop number sense and critical thinking skills.

General Outcome: Algebra
Develop algebraic reasoning.

Specific outcome:
Demonstrate an understanding of financial institution services used to access and manage finances.

Specific outcome:
Demonstrate an understanding of compound interest.

Specific outcome:
Demonstrate an understanding of credit options, including credit cards and loans.

Specific outcome:
Solve problems that require the manipulation and application of formulas related to simple interest and finance charges.

Section 6.1:
Choosing an Account

- Math on the Job
- Explore the Math
- Discuss the Ideas: Choosing Your Bank Account
- Mental Math and Estimation
- Activity 6.1: Comparing Banking Services
- Activity 6.2: Using a Bank Card
- Activity 6.3: Protecting Your Personal and Financial Information
- Build Your Skills

Section 6.2:
Simple and Compound Interest

- Math on the Job
- Explore the Math
- Activity 6.4: Comparing Simple Interest and Compound Interest
- Activity 6.5: The Rule of 72
- Mental Math and Estimation
- Discuss the Ideas: Guaranteed Investment Certificates
- Activity 6.6: The Effect of Different Compounding Periods
- Build Your Skills
- The Roots of Math: The Canadian Dollar

Section 6.3:
Credit Cards and Store Promotions

- Math on the Job
- Explore the Math
- Discuss the Ideas: Using a Credit Card
- Activity 6.7: Understanding Credit Card Statements
- Activity 6.8: Understanding Debt Repayment
- Mental Math and Estimation
- Discuss the Ideas: Managing Credit and Debt
- Activity 6.9: Buy Now! No Down Payment Required!
- Build Your Skills
- Puzzle It Out: Who Has What?

Section 6.4:
Personal Loans, Lines of Credit, and Overdrafts

- Math on the Job
- Explore the Math
- Discuss the Ideas: Personal Loans
- Activity 6.10: Comparing Types of Loans
- Mental Math and Estimation
- Activity 6.11: Using Credit Effectively
- Build Your Skills

Chapter Project: Buying a Home Entertainment Centre

Reflect on Your Learning

Practise Your New Skills

THE MATHEMATICAL IDEAS

SAVING, INVESTING, AND BORROWING

The main mathematical concepts in this chapter are intended to develop student understanding and awareness of various financial services available in Canada, including those offered by banks, credit unions, and other financial institutions. Students will learn about the mathematics involved in saving, investing, and borrowing as they investigate a range of bank accounts, investments, loans, and credit cards.

Section 6.1 gives some typical bank account options offered by a fictitious bank, the Northwest Bank of Canada, and introduces students to the various types of bank fees and service charges. Students will become familiar with various way of accessing accounts, including teller services, ATMs, debit cards, and online and telephone banking. They will assess the advantages and disadvantages of each method, and determine the best banking options for real-world financial scenarios. In this section, students will also learn about the methods used to ensure the security of personal and financial information.

In section 6.2, students will learn about simple interest and the simple interest formula. They will develop skills in solving for any one of the variables, given the other three. Through a variety of activities, contextual problems, and worked examples, students will learn about simple and compound interest that they may receive on savings account balances and from simple investments such as Guaranteed Investment Certificates and Canada Savings Bonds. They will use algebraic formulas to calculate simple and compound interest, and learn about the relationship between the two. Students will approach compound interest calculations in two ways, using tables and the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$. They discover that both methods yield the same result, and that each method has its advantages and disadvantages.

T In section 6.3, students will learn how to interpret credit card statements and will look at debt repayment. Students will use an online credit card minimum payment calculator to investigate the time taken and the total interest paid on a credit card debt if the minimum payment is made each month. In the same section, students will investigate store promotions and be given scenarios where they must choose the best payment option. At the end of section 6.3 there is a logic puzzle for students to try. The puzzle is presented in the context of bank services and credit card options.

The final section, 6.4, is about personal loans. Students will learn the terminology associated with personal loans, and they will use a personal loan payment calculator that gives the monthly payment for various interest rates over different time periods. Teachers who have access to a financial application on a graphing calculator or a computer with a spreadsheet application may prefer to use those technologies rather than the table, which has limitations. Students will also study other credit options such as overdrafts, lines of credit, and payday loans.

Note that there are several questions in the student book that are based on information about the fictitious bank, the Northwest Bank of Canada. Teachers may choose to use information from actual banks to answer the questions.

WHY ARE THESE CONCEPTS IMPORTANT?

- Some students may already have part-time jobs after school or on the weekends, and many students take summer jobs. Consumer awareness about financial matters is a life skill.
- It is important for students to choose the most appropriate bank service for their needs so that they are not spending money on unnecessary bank fees.
- Students need to learn strategies for how best to manage their finances. They must learn the importance of "reading the fine print," so that they can make informed decisions.

- Although credit cards can be a very convenient way to purchase items or pay bills, it is very easy to overspend and end up only being able to afford the minimum payment. Students need to realize how expensive it can be to pay off a credit card bill by making only the minimum payment each month.
- Payday loans are an extremely expensive way to borrow money. Students need to be aware of this, and to understand the various options for obtaining loans.

PRIOR SKILLS AND KNOWLEDGE

Student work in this chapter will build on certain WNCP Common Curriculum Framework outcomes from earlier grades. Students may be aware of some of the ideas in this chapter through life skills and planning courses. Some students may have jobs and may already have bank accounts. Students will likely have been exposed to the following concepts and may have developed these skills in earlier grades, or earlier in the Workplace and Apprenticeship Mathematics pathway. If you feel that your students need a review of any of these concepts and skills, incorporate them into your lesson planning.

1. Concepts
   a) percent;
   b) rates.
2. Mathematical Skills
   a) arithmetic operations on decimals;
   b) calculating percents;
   c) solving equations;
   d) evaluating formulas.

T

3. Technology
   a) basic calculator functions including exponents;
   b) internet research;
   c) presentation software.

REVIEWING PRIOR CONCEPTS

Some students may benefit from reviewing concepts that have been covered in prior years. You may want to give some students specific review exercises in the following concepts and processes.

6.1 Choosing an Account

- converting a percentage to a decimal; and
- calculating a percentage of a given number.

6.2 Simple and Compound Interest

- converting months, days, and weeks to years; and
- converting years and partial years to months.

6.3 Credit Cards and Store Promotions

- working with formulas; and
- evaluating variables in equations.

6.4 Personal Loans, Lines of Credit, and Overdrafts

- using algebra to solve for one unknown variable.

Blackline Master 6.6 contains review questions and solutions. It is found on p. 409 of this teacher resource.

PLANNING CHAPTER 6

This chapter will take approximately 2–3 weeks of class time to complete. Class period estimates are based on a class length ranging from 60 to 75 minutes. These estimates may vary depending on individual classroom needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	253	Introduce the Chapter Project: Buying a Home Entertainment Centre	20–25 minutes	internet, electronics catalogues or advertising flyers Blackline Master 6.1 (p. 403), 6.2 (p. 404)
6.1	254 254 255 256 257	Math on the Job: Credit Union Member Services Representative Explore the Math Discuss the Ideas: Choosing Your Bank Account Example 1 Mental Math and Estimation	50 minutes	Blackline Master 6.3 (p. 405)
6.1	258 258 260	Activity 6.1: Comparing Banking Services Activity 6.2: Using a Bank Card Example 2	1 class	internet, bank account information sheets or brochures from Canadian banks and credit unions, Blackline Master 6.3 (p. 405)
6.1	262	Activity 6.3: Protecting Your Personal and Financial Information	50 minutes	internet
6.1	262	Build Your Skills	1 class	Blackline Master 6.3 (p. 405)
6.2	264 264 265 269	Math on the Job: Welder Explore the Math Examples 1, 2, 3 Activity 6.4: Comparing Simple Interest and Compound Interest	50 minutes	calculator internet
6.2	270 271	Activity 6.5: The Rule of 72 Mental Math and Estimation	50 minutes	internet, Blackline Master 6.4 (p. 406)
6.2	271 272	Discuss the Ideas: Guaranteed Investment Certificates Activity 6.6: The Effect of Different Compounding Periods	50 minutes	calculator spreadsheet software, Blackline Master 6.4 (p. 406)
6.2	272	Build Your Skills	1 class	
	274	Chapter Project: Set up a Savings Plan	20–25 minutes	internet, information on current interest rates for savings accounts, calculator
6.2	275	The Roots of Math: The Canadian Dollar	10–15 minutes	

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
6.3	276 276 277 277 279	Math on the Job: Self-employed Outdoor Outfitter Explore the Math Discuss the Ideas: Using a Credit Card Examples 1, 2 Activity 6.7: Understanding Credit Card Statements	1 class	internet, brochures on different bank credit cards
6.3	281 281	Activity 6.8: Understanding Debt Repayment Mental Math and Estimation	25–30 minutes	internet
6.3	281 282 283	Discuss the Ideas: Managing Credit and Debt Example 3 Activity 6.9: Buy Now! No Down Payment Required!	40–45 minutes	
6.3	284	Build Your Skills	1 class	
6.3	287	Puzzle It Out: Who Has What?	15–20 minutes	credit card interest information, flyers for entertainment centre equipment
	287	Chapter Project: Investigate Buying on Credit	15–20 minutes	
6.4	288 288 289 290	Math on the Job: Consumer Loans Officer Explore the Math Discuss the Ideas: Personal Loans Example 1	1 class	Blackline Master 6.5 (p. 408), calculator with graphing calculator software, or internet
6.4	291 293 294 294	Example 2 Activity 6.10: Comparing Types of Loans Mental Math and Estimation Activity 6.11: Using Credit Effectively	40–45 minutes	Blackline Master 6.5 (p. 408), calculator with graphing calculator software, or internet
	294	Chapter Project: Investigate Taking Out a Loan	40–45 minutes	
6.4	295	Build Your Skills	1 class	
	297	Chapter Project: Make a Presentation	1 class	Blackline Master 6.2 (p. 404)
	297 298	Reflect on Your Learning Practise Your New Skills	1 class	 Blackline Master 6.5 (p. 408)
		Chapter Test (p. 397 of this resource)	1 class	

PLANNING FOR ASSESSMENT

Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Chapter overview • Project discussions (ongoing) • Math on the Job • Explore the Math • Activities • Discuss the Ideas • Mental Math and Estimation • Puzzle It Out: Who Has What?	• Monitor how much work the students have done on their project activities. • Observe how students participate during discussions. • Observe how students interact during activities done as a group, in pairs, or individually.
Assessment as Learning	• Reflection and practise • Build Your Skills problems • Prompt student self-assessment • Review student work, provide feedback.	• Check homework and provide feedback on questions. • Challenge students to understand the relationship between different versions of the formulas and make connections. • Encourage reflection.
Assessment of Learning	• Chapter review • Chapter Project: Buying a Home Entertainment Centre • Assignments/homework • Quizzes • Chapter Test	• Have students present their final project to the class and allow students to give feedback to presenters. • Give short quizzes as the chapter progresses to provide as much feedback as possible. • Review assessment records and add unit results to ongoing records.
Learning Skills/ Mathematical Disposition	• Observe and record throughout the unit how students are working with new language and concepts.	• Keep a log or journal of observations to aid in reporting.

PROJECT—BUYING A HOME ENTERTAINMENT CENTRE

GOALS: To use the skills developed in this chapter—calculating monthly savings; credit card payments; the total cost of a store promotion plan; and the monthly payments, total paid, and finance charges for a loan—in order to investigate various purchase options for a home entertainment centre. Over the course of the project, students will apply their mathematical learning, develop critical thinking skills, and practise their communication skills.

OUTCOME: In this project, students will integrate what they learned about savings plans, credit options, and loans into a real-world problem—purchasing a home entertainment centre. They will investigate various payment options in order to make informed decisions. They will present their findings to their peers.

PREREQUISITES: To complete this project, students will need to understand how to calculate the sales taxes and total paid for various items they plan to purchase, which were addressed in Workplace and Apprenticeship Mathematics 10. They will need to be able to calculate the total paid for various types of in-store promotion, as well as calculate the monthly payment, total paid, and finance charges for a given loan.

Familiarity with researching online for current information on financial services, credit cards, and other relevant information will be very helpful.

ABOUT THIS PROJECT: This project is divided into five parts. Students will develop a financial plan that will allow them to purchase a home entertainment centre. As a starting point, they will imagine they have already saved $1000.00 and have $300.00 a month available from a job to save or to spend on debt payments. They will research the items they want and calculate a total purchase price including taxes. In the second part of the project, they will investigate how long it would take to save the required amount if they were going to pay cash. In the third part, students will consider paying by credit card (paying the minimum payment each month) or through a store promotion plan. In the fourth part, students will research the option of paying with a personal loan. After evaluating all these options, students will choose one, based on their findings and justify their choice. They can then present their project to their peers as a report, a poster, or using presentation software.

Students should be given class time as well as homework to complete their project. Encourage students to discuss their progress with other students or with you. You may wish to set a series of deadlines for each section of the project, and a final completion deadline.

This project can be done by individuals, pairs of students, or by small groups. The latter options will be especially helpful for students who may be struggling. Collaborating with a partner or group may help them complete the project, or they may be asked to explore fewer options.

A self-assessment rubric, Blackline Master 6.2 (p. 404), should be handed out to students early in the project. It outlines the assessment criteria and offers ways for them to reflect on their learning.

An alternative project, "Wise Money Management," is included on pp. 413–419.

1. Start to plan

STUDENT BOOK, p. 253

Introduce the project to your students as you start this chapter. Distribute the Chapter Project Planning Checklist, Blackline Master 6.1 (p. 403), to give students an idea of the scope of the project. Have them discuss the Get Started questions in the student resource and allow time for them to research the components of their entertainment system. Suggest that students use the checklist to track their progress as they complete the project activities.

SAMPLE SOLUTION

Student descriptions will vary. They will need to use the current sales tax for their province or territory in their price calculations. In the sample solution below, the tax rate is 12%.

PURCHASES	
LCD television, 37″	$1999.99
Surround sound speaker system	$788.88
DVD player with enhanced audio	$499.99
Game console	$499.99
Noise-cancelling headphones	$385.55
Total cost before tax	$4174.40
Sales tax (12%)	$500.93
TOTAL	$4675.33

2. Set up a savings plan

STUDENT BOOK, p. 274

Students will need to consider the total cost of their entertainment system to set up a savings plan. Remind them that they have $1000.00 in a savings account, and can afford up to $300.00 a month from their paycheque.

SAMPLE SOLUTION

Amount to save = $4675.33 – $1000.00

Amount to save = $3675.33

Monthly payment = $300.00

Time taken to save up = $3675.33 ÷ $300.00

Time taken to save up = 12.25 months

It will take 13 months to save up for the centre and then be able to pay by cash if I save $300.00 a month.

3. Investigate buying on credit

STUDENT BOOK, p. 287

In this section, students will need to calculate the cost of their home entertainment system using two payment options: purchasing with a credit card and purchasing on a store promotion offer. Encourage students to obtain authentic data for the interest rate and minimum payment formula for a credit card.

If students are not able to find authentic credit card data, you may wish to give them the option to use the Northwest Bank of Canada credit card, which offers an interest rate of 20.00% per annum and a minimum payment of 5% of the new balance or $10.00, whichever is greater.

T Payment option 1 involves using a credit card minimum payment calculator, available on the internet (for example, http://get-the-best-credit-cards.com/minimum-payment-calculator.html) to calculate the total interest paid on a debt, and the time taken to pay it off, if the minimum payment only is made each month. Students will also brainstorm the advantages and disadvantages of using a credit card and summarize their ideas.

Payment option 2 has students consider purchasing with a store promotion offer. Encourage students to investigate actual promotions at various stores or online. Alternatively, you may wish to give them the following store promotion information.

Golden Systems is selling a package including a 37-inch LCD TV, surround sound speaker system, DVD player, game system, and noise-cancelling headphones. Total cash value is $4174.40, with free delivery. Two payment options are offered.

Payment option 1: No payment for 12 months! After 12 months, the amount due is $4455.55 plus tax. Interest of 18.00% per annum will be charged for non-payment after 12 months. A delivery charge of $50.00 and administrative fee of $75.00 are due on day of purchase.

Payment option 2: Easy installment plan, pay no taxes! No down payment required. Payments of $372.00/month for 12 months, no tax. A delivery charge of $50.00 and administrative fee of $75.00 are due on day of purchase.

SAMPLE SOLUTION

a) I have saved $1000.00 so I only need to use my credit card for $3675.33. Using the credit card minimum payment calculator, I see that I will have to pay $1528.99 in interest charges, and it will take 8.83 years to pay it off.

The advantage of paying by a credit card is that the monthly payment is much smaller than the monthly payment on a loan or store payment plan. The disadvantages are that it takes a lot longer to pay off the credit card balance, especially if you pay the minimum payment only each month. You also pay more interest. If you pay by credit card you should try and pay as much as possible each month in order to lower your total interest payment.

b) Calculating the cost based on promotions offered above.

Option 1: No Payment for 12 Months

Cost before taxes	$4455.55
Sales Tax (12%)	$534.66
Delivery	$50.00
Admin. Charge	$75.00
TOTAL	$5015.21

Option 2: Easy Installment Plan, Pay No Taxes

Cost including taxes	$372.00 × 12 = $4464.00
Delivery	$50.00
Admin. Charge	$75.00
TOTAL	$4589.00

4. Investigate taking out a loan

STUDENT BOOK, p. 294

Encourage students to research authentic loan interest rates for various terms by visiting banks, credit unions, or by going online.

You may wish to give students the option to take a loan from the Northwest Bank of Canada. This bank loan has an annual interest rate of 6.50% per annum for loans of up to 4 years.

T Teachers may wish to use spreadsheets or graphing calculator software in place of the personal loan payment calculator table on Blackline Master 6.5 (p. 408). Have students calculate the monthly payments for terms of 1, 2, 3, and 4 years so they can decide which amortization period they wish to choose. Then calculate the total paid.

SAMPLE SOLUTION

Amount borrowed = $3675.33

Find the monthly payment for $1000.00.

Multiply by $3675.33 and divide by $1000.00.

Loan Summary

6.50% per annum	*1 year*	*2 years*	*3 years*	*4 years*
Monthly payment	$317.18	$163.74	$112.65	$87.14
Total paid on the loan	$317.18 × 12 = $3806.16	$163.74 × 24 = $3929.76	$112.65 × 36 = $4055.40	$87.14 × 48 = $4182.72
Total paid for the entertainment centre (add $1000.00)	$4806.16	$4929.76	$5055.40	$5182.72

5. Make a presentation

STUDENT BOOK, p. 297

Students will synthesize their research and calculations to choose a payment option. They will present their findings and decision in a presentation folder or using presentation software.

SAMPLE SOLUTION

I choose to take out a loan for $3675.33 from the Northwest Bank of Canada at 6.50% per annum for 2 years. I like this option because I can afford the monthly payments.

I could just about afford the store's installment plan if I pay $300.00 a month from my paycheque, and use my saved up money to add $72.00 a month so I can make the payment. This would give me a better overall price, but I prefer the lower payments each month because then I can buy DVDs and other software. If I pay by credit card and pay only the minimum payments each month, it will take nearly 9 years to pay it off, and it will cost me $1528.99 extra in interest charges.

ASSESSING THE PROJECT

1. **Start to plan**
   - Record your observations. Provide students with information on how they will be assessed, using a scheme that meets your reporting needs.
   - Check that students have successfully calculated the total cost plus taxes.
2. **Set up a savings plan**
   - Check that students first choose the amount they want to save each month, then divide the total amount they need to save by that value.
3. **Investigate buying on credit**
   - Check that calculations of interest rate and time to pay off debt are accurate.
   - Check to see that they have written a suitable paragraph.
   - Check to see that the calculations for the store promotions are correct.
4. **Investigate taking out a loan**
   - Check to see that students have calculated the monthly payments successfully.
   - Check that they are multiplying the monthly payment by the number of months of the loan to get the total paid.
5. **Make a presentation**
   - Use the Project Assessment Rubric on p. 364.
   - Ask students to self-assess their projects using Blackline Master 6.2 (p. 404).
   - Check that students have expressed the reasons for their choice in an understandable way.
   - Provide time for students to view the projects of their peers. If some students have created electronic presentations, arrange to have a projector available.

PROJECT ASSESSMENT RUBRIC

	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
CONCEPTUAL UNDERSTANDING				
• Explanations show understanding of taxes, compound interest, credit card payment, store promotions, and personal loans	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are appropriate	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and detailed
PROCEDURAL UNDERSTANDING				
• Accurately: ▪ calculates the total costs ▪ describes a savings plan including statements of monthly payment and time taken to save ▪ determines the total interest paid and the time taken to pay off a debt with minimum payments each month on a credit card ▪ analyzes the cost when the payment is through a store promotion ▪ analyzes the cost when the payment is made with a bank loan	limited accuracy; major errors or omissions For example: • costs are missing • errors in calculating tax • savings plan is missing • calculations of savings plan are inaccurate • minimum credit card payment information is missing • store promotion analysis is missing • store promotion analysis is inaccurate • bank loan analysis is missing	partially accurate; some errors or omissions For example: • costs after tax are accurate • calculations of savings plan are mostly accurate • minimum credit card payments information is missing • some errors with store promotion analysis • bank loan analysis is inaccurate • presentation is adequate	generally accurate; few errors or omissions For example: • costs after tax are accurate • calculations of savings plan are accurate • minimum credit card payments information is accurate • few errors with store promotion analysis • few errors with bank loan analysis • project is complete and correct, but there is nothing beyond what is required	accurate and precise; very few errors or omissions For example: • calculations are complete and correct • has an informative, clear presentation or handout • adds extra creativity to the project
PROBLEM-SOLVING SKILLS				
• Solves problems that involve making decisions on purchase options	does not use appropriate strategies to make decisions on purchase options	uses some appropriate strategies to make decisions on purchase options	uses correct strategies to make decisions on purchase options	uses effective and innovative strategies to make decisions on purchase options
COMMUNICATION				
• Presents work and explanations clearly using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using a few appropriate mathematical terms	presents work well and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

6.1 Choosing an Account

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 254–263

MATH *ON THE JOB*

STUDENT BOOK, p. 254

This Math on the Job presents a good opportunity to begin a discussion about banking services. You could ask students if they have a bank account, and if so, which type of bank account they have. Why did they choose this type of account? What benefits does this account offer? Do they receive a number of free transactions per month? If they have a savings account, do they know its interest rate?

This is also an opportunity to review knowledge of unit pricing and currency exchange, acquired in grade 10. Ask students if they have travelled abroad, and if so, the type of currency that was used in the country. Do they remember the exchange rate? If you think it would be helpful, provide review questions on ratios and currency exchange rates.

SOLUTION

Paul will need to convert $500.00 USD into Canadian dollars (CAD). At the time of purchase the exchange rate is $1.0526 CAD for $1.00 USD.

Calculate the cost of $500.00 in USD traveller's cheques by converting $500.00 USD to Canadian dollars.

$500.00 USD = 500 × $1.0526 CAD

$500.00 USD = $526.30 CAD

The converted cost of $500.00 USD is $526.30 CAD.

Calculate the 1% commission.

$526.30 CAD × 0.01 = $5.26 CAD

Add the commission of $5.26 and the bank fee of $1.00 to calculate the total cost for the traveller's cheques.

$526.30 + $5.26 + $1.00 = $532.56

Paul must withdraw $532.56 from the customer's account to pay for the traveller's cheques.

EXPLORE THE MATH

STUDENT BOOK, p. 254

Continue the discussion started about Math on the Job and types of bank accounts. Let the students know that informed choices about what type of account they choose can save them money each month. This section introduces students to the various financial services available to Canadians.

The various types of accounts offered by a fictitious bank, the Northwest Bank of Canada, are given in detail in the student book and as Blackline Master 6.3 (p. 405) for ease of reference. Teachers are encouraged to have students obtain authentic information online or by visiting a local bank or credit union. The Northwest Bank of Canada information is typical of what students might come across in their research.

Ensure that students understand the meaning of the following terms: monthly fees and rebates; waived; self-service and full-service transactions; minimum balance; ATM; withdrawal; and deposit.

DISCUSS THE IDEAS

CHOOSING YOUR BANK ACCOUNT

STUDENT BOOK, p. 255

This Discuss the Ideas guides students through the process involved in choosing the right bank account. Students will have to consider their spending and banking habits and decide which account best suits their needs.

SOLUTIONS

1. Answers will vary. Some typical transactions include: making purchases from stores, depositing paycheques and gifts, withdrawing cash, paying cell phone or internet bill.
2. You might need to write a cheque if you need to pay someone and you do not have cash, or if you want to pay a bill by mail. People often use cheques to pay rent.
3. You might want a full-service account if you do not like technology; if you need to purchase foreign currency or traveller's cheques often; or if you have a business.

   You might want a savings account if you are saving up for something and you want to spend the savings on day-to-day items
4. In order to choose an appropriate bank account, you must know approximately what your average minimum balance will be each month and how many transactions of each type you would normally do each month, such as number of cheques, withdrawals, and regular deposits. Also, you should consider whether you need to buy foreign currency or traveller's cheques on a regular basis.

Mental Math and Estimation

STUDENT BOOK, p. 257

This mental math question is a direct application of concepts covered thus far.

SOLUTION

Jenna makes a total of 21 transactions. She is allowed 10 free self-service transactions, and she must pay $0.50 for each transaction after that.

Students can calculate the number of transactions over the 10 that are free by mentally subtracting 10 from 21. Eleven transactions must be paid for. Two transactions will cost $1.00. Therefore, 10 transactions will cost $5.00, and 11 will cost $5.50.

Jenna's total service charge for that month will be $5.50.

ACTIVITY 6.1
COMPARING BANKING SERVICES

STUDENT BOOK, p. 258

T This is an activity where students can go online to investigate details of specific services at a financial institution in their community. This activity could be done at home. Mind maps can take many forms. Encourage students to be creative. Students who have difficulty writing down their ideas in Activities 6.1, 6.2, and 6.3 may join a group with a student who feels comfortable taking the role of "recorder."

SOLUTIONS

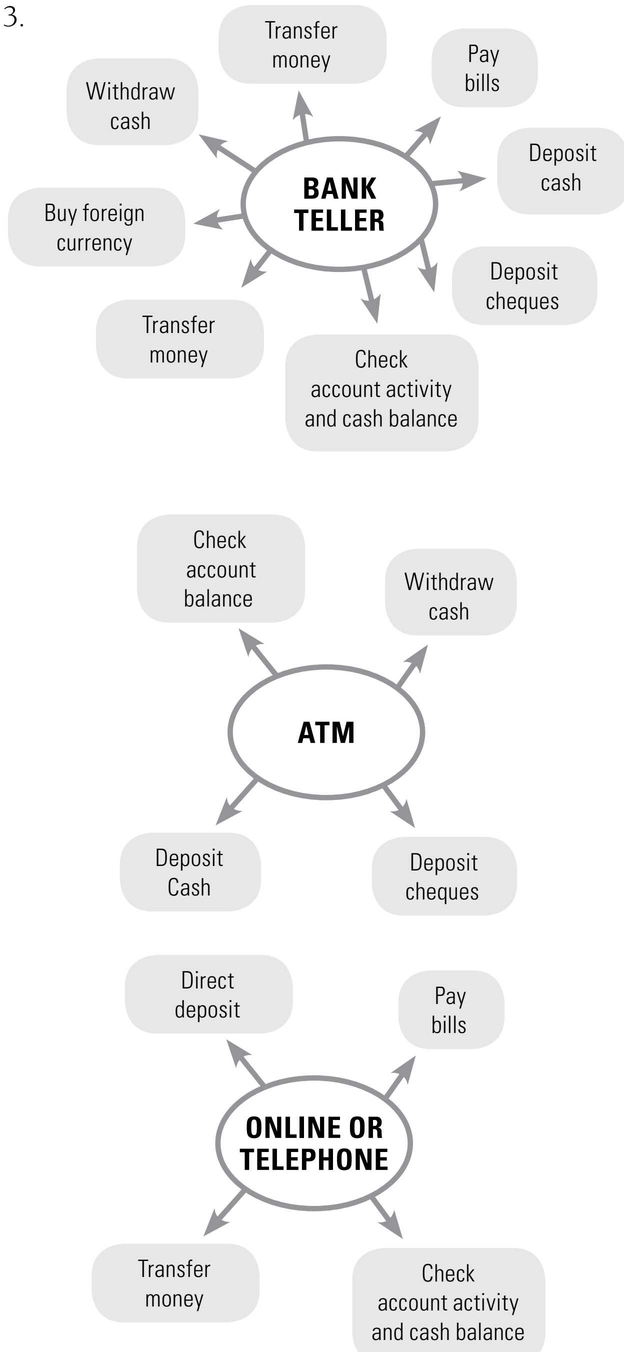
1. a) Student answers will vary. Answers could include the following: savings account, chequing account, RESPs, RRSPs, mortgages, credit cards, bill payments, wire transfers, cheque cashing, investment accounts, tax-free savings accounts, as well as many other services.

   b) Transactions that can be made using the internet or telephone banking include: paying bills; arranging direct deposit of paycheques; transferring money; setting up automatic bill payments; and checking account activity and account balances.

   c) Transactions that must be done in person include: purchasing traveller's cheques and foreign exchange transactions.
2. Advantages of online banking include: it is quick and convenient; you can set up automatic payments for bills on due dates; you can view your current account balance; it allows faster bill payments; service available 24/7; you can access your account from any location with internet access; you can identify fraudulent activity much more quickly than waiting for paper bank statements.

   Disadvantages of online banking include: technical difficulties—system may unexpectedly go down or be unavailable during maintenance; can be difficult for those who are not technically savvy; concerns about security, including identity theft; not all financial institutions offer online all the services you need; no live personal help if something goes wrong; usernames, passwords, and security questions may be difficult for some people to remember.

3.



ACTIVITY 6.2
USING A BANK CARD

STUDENT BOOK, p. 258

Many students may already have bank cards and be familiar with their uses. Nevertheless, consumer awareness about the potential dangers of bank cards is important, especially for young people. This activity teaches crucial life skills.

Students can work in small groups to discuss the questions.

SOLUTIONS

1. It is best to use your bank card at a branch of your own bank because you will not be charged a fee from another bank.
2. Advantages: you don't have to carry cash; your bank account is debited right away, so there are no future bills to pay; with care, bank cards are secure.

   Disadvantages: there are sometimes fees for bank card transactions; it is easy to get carried away in your spending; sometimes merchants or other institutions do not accept bank cards; if you don't protect your personal information, your bank card can be subject to fraud.
3. a) The ATM receipt shows that $60.00 was withdrawn. It also shows the amount of money left in the account, and the date of the withdrawl.

   b) The complete bank card number is not shown so that other people cannot get your personal information if you lose the receipt.

   c) It is a good idea to keep your ATM receipts as a record of your transactions and to avoid someone using the information to commit fraud.

Example 2

STUDENT BOOK, p. 260

This example is the first instance in this chapter where personal record keeping is covered. It provides an opportunity to discuss with students how important it is to keep personal records, and why they should also keep track of their bank charges.

ACTIVITY 6.3 PROTECTING YOUR PERSONAL AND FINANCIAL INFORMATION

STUDENT BOOK, p. 262

SOLUTIONS

1. Most financial institutions use the following security measures for online and electronic banking: firewalls and cookies; several levels of login, such as passwords, PINs, and authentication questions; instructions on how to report suspicious "phishing" emails; chip technology in bank access cards and bank credit cards; voiceprint verification for telephone banking; cheque imaging.
2. Some ways to ensure the security of your personal and financial information include: keeping your computer safe using firewalls and up-to-date antivirus and anti-spyware software; learning how to recognize fraud; protecting your PIN by not sharing the number with anyone; not lending your bank cards to anyone; making sure your card is in view the entire time it is being used; remembering to take your card with you after you have used it at a store or an ATM; never providing your credit card number over the phone unless you are dealing with a reputable company or you initiated the call yourself; never sending your card number by email.
3. Tips may include:
   - Report a lost or stolen credit card immediately.
   - Don't keep your social insurance card in your wallet.
   - Never give personal information to anyone who phones you up if you don't know them.
   - Check your credit card bills and bank statements as soon as you receive them.
   - Use your ATM card wisely. Memorize and cover up your PIN as you type it in.
   - Do not use names of family members, birthdates, or other personal information for your PIN or password.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 262

You can select and assign a minimum number of questions for students to complete. Challenge the stronger students to complete all the exercises. Students will need the Northwest Bank of Canada table on p. 256 of the student resource and provided as Blackline Master 6.3 (p.405) to complete questions 1 to 4.

SOLUTIONS

1. Marc should choose a bonus savings account. He will get interest on his savings, and can keep his day-to-day transactions separate.
2. Jannik should choose a value account. He won't have to pay the monthly fee and will only pay for 5 extra transactions a month, so he will only pay \$2.50 per month. His monthly minimum is not enough to waive the fee for the self-serve account, so he would have to pay \$10.90/month, which is greater than \$2.50 with the value account.
3. a) Calculate Kyra's service charges.
   - 2 full-service transactions (utility bill payment, traveller's cheques)

     $2 \times \$1.00 = \$2.00$

- ATM charge of $1.50 from Northwest Bank of Canada
- ATM charge of $1.50 ATM from a non-Northwest Bank of Canada machine

  Total service charges:
  $2.00 + $1.50 + $1.50 = $5.00

b) Calculate total expenses for the month.

   $250.42 + $650.00 + $100.00 + $60.00 + $20.00 + $100.00 + $102.24 + $43.20 + $50.00 + $7.35 + $99.95 + $36.35 + $5.00 = $1524.51

   Calculate her closing balance.

   $2150.23 – $1524.51 = $625.72

c) If Kyra deposits $800.00, calculate her new balance.

   $800.00 + $625.72 = $1425.72

   Kyra needs a monthly balance of $1500.00 in order to have her monthly account fee waived. She will have to pay the monthly fee.

4. a) It is important to keep your PIN secure because if people know your PIN, they can go to an ATM and take money from your account.

   b) Ways to protect your personal banking information include: covering up the ATM keypad when entering your PIN; never telling anyone your PIN; not using your birthday/address/phone number as a PIN; never writing down your PIN.

5. Answers will vary. Some possible situations include:

   - Bank card: store purchases, ATM withdrawals and deposits
   - Online banking: automatic cheque deposit, pay bills, statements, transfers
   - Telephone banking: automatic cheque deposit, pay bills, statements, transfers
   - Bank teller: deposits, withdrawals, traveller's cheques, foreign currency, pay bills
   - ATM: deposits, cash withdrawals, balance
   - Cheque: pay bills, pay for items at stores, pay people

6. a) Calculate how much he spends on fees per week.

   $1.50 + $1.50 = $3.00

   Multiply by 52 to calculate how much he would spend in 1 year.

   $3.00 × 52 = $156.00

   b) Answers will vary. Possible answers include:

   - withdrawing more money at one time
   - using only ATMs belonging to his bank
   - using his bank card to make purchases (depending on the nature of his account)

Extend Your Thinking

7. Timothy withdraws $60.00 USD. Using the exchange rate given, calculate the value in Canadian dollars.

   $60.00 USD = 60 × $1.05 CAD

   $60.00 USD = $63.00 CAD

   He is also charged a $1.00 USD service charge.

   $1.00 USD = $1.05 CAD

   His bank charges him $1.50 CAD for the transaction.

   Calculate Timothy's total deductions in Canadian dollars.

   $63.00 + $1.05 + $1.50 = $65.55 CAD

   $65.55 will be deducted from Timothy's account.

6.2 Simple and Compound Interest

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 264–275

Some students may need to review the prior skills required for this section. Review questions are available on Blackline Master 6.6 (p. 409).

MATH *ON THE JOB*

STUDENT BOOK, p. 264

You can introduce this Math on the Job by explaining the concepts of start-up costs and operating costs to students. Some students, especially those interested in trades, may be interested in becoming self-employed, or may already have experienced this. Students may have held summer jobs doing yard work or making and selling goods. If so, ask students if they had to purchase anything before they could start working. What did they purchase? How much did they have to spend? How did they obtain the money to cover their start-up costs? Did they work alone or share the work with a partner?

You can also discuss the advantages and disadvantages of taking out a loan. What other scenarios can students think of where it might be necessary to take out a loan?

SOLUTION

Multiply the amount of the monthly payment by 12.

$1698.43 × 12 = $20 381.16

The total cost of the loan would be $20 381.16.

EXPLORE THE MATH

STUDENT BOOK, p. 264

Introduce students to the simple interest formula ($I = Prt$). Explain what each variable means. Ensure that students understand that they have to express the annual rate, r, as a decimal. Review how to convert percentages to decimals. Blackline Master 6.6 (pp. 409–411) contains review questions to practise this skill.

Inform students that:

- time must always be expressed in years; and
- the simple interest formula only calculates the interest on the principal.

Work Examples 1 and 2 ensuring that students understand the concepts and skills. You may want to check their understanding by asking them the following question.

What is the simple interest and final amount for an investment of $4000.00 at 4.50% per annum invested for 45 days?

SOLUTION

$I = Prt$

$I = \$4000 \times 0.045 \times (45 \div 365)$

$I = \$22.19$

Be certain that students understand that when they see rates expressed per annum, "per annum" means that it is the rate per year.

ACTIVITY 6.4
COMPARING SIMPLE INTEREST AND COMPOUND INTEREST

STUDENT BOOK, p. 269

This activity investigates the difference between simple and compound interest. Students will need to access the internet to research Canada Savings Bonds. If they have trouble locating the information, direct them to: http://csb.gc.ca/home/.

Have students work in small groups so that they can help each other. Teachers may have to circulate around the class to help students with this activity.

If students need more practise with compound interest in order to understand the concept, a remedial activity is provided in Blackline Master 6.4 (p. 406).

SOLUTIONS

PART A

1. Regular-Interest bonds pay interest annually on the anniversary of the issue date until maturity or redemption, by direct deposit into the investor's bank account, or by mailing the investor an interest cheque. With Compound-Interest bonds, the interest is automatically reinvested annually.

2. a)

Interest period	*Interest rate per annum*	*Value of investment on which interest is calculated ($)*	*Interest earned during period ($) (I = Prt)*	*Value of Investment at end of period ($)*
1	2.75%	$1000.00	$1000.00 × 0.0275 × 1 = $27.50	$1000.00 + $27.50 = $1027.50
2	3.10%	$1000.00	$1000.00 × 0.0310 × 1 = $31.00	$1027.50 + $31.00 = $1058.50
3	2.50%	$1000.00	$1000.00 × 0.0250 × 1 = $25.00	$1058.50 + $25.00 = $1083.50
4	1.00%	$1000.00	$1000.00 × 0.0100 × 1 = $10.00	$1083.50 + $10.00 = $1093.50

   b) The total interest earned is the final value minus the principal.

   $1093.50 – $1000.00 = $93.50

   Over 4 years, the CSB earned $93.50 interest.

3. If you redeem a Canada Savings Bond within 3 months following its issue date, you will not receive any interest. You will only receive the original value of the bond.

PART B

Interest period	*Interest rate per annum*	*Value of investment on which interest is calculated ($)*	*Interest earned during period ($) (I = Prt)*	*Value of Investment at end of period ($)*
1	2.75%	$1000.00	$1000.00 × 0.0275 × 1 = $27.50	$1000.00 + $27.50 = $1027.50
2	3.10%	$1027.50	$1027.50 × 0.0310 × 1 = $31.85	$1027.50 + $31.85 = $1059.35
3	2.50%	$1059.35	$1059.35 × 0.0250 × 1 = $26.48	$1059.35 + $26.48 = $1085.83
4	1.00%	$1085.83	$1085.83 × 0.0100 × 1 = $10.86	$1085.83 + $10.86 = $1096.69

1. The interest is calculated using the simple interest formula ($I = Prt$).

2. You would calculate the investment value at the end of each year by adding the interest to the investment value at the beginning of that year.

3. The final value of the investment is $1096.69.

   Calculate the interest earned.

   $1096.69 – $1000.00 = $96.69

   The interest earned is $96.69.

4. The investment earns $93.50 in simple interest and $96.69 in compound interest. Therefore, compound interest is the better investment. Calculate the difference.

   $96.69 – $93.50 = $3.19

   Compound interest earns $3.19 more than simple interest.

5. Simple interest is used to calculate the interest for each period. When compound interest is applied, the principal for the next period includes the simple interest earned in the previous period.

6. Answers will vary, depending on individual students' reasons for investing.

ACTIVITY 6.5

THE RULE OF 72

STUDENT BOOK, p. 270

Point out to students that the Rule of 72 estimates the time taken for an investment to double in value; it is not an exact calculation.

SOLUTIONS

1. The Rule of 72 can be expressed with the following formula.

   Years to double investment = 72 ÷ interest rate

   $$y = 72 \div r$$

2. $y = 72 \div r$

   $10 = 72 \div r$

   $r = 72 \div 10$

   $r = 7.2\%$

   You would need to invest your money at an interest rate of 7.2%.

Extension

This activity guides students through calculations so that they can discover the Rule of 72 for themselves. Remind students that the rule gives only the approximate times to double an investment, which is why answers are rounded to the nearest $100.00. Have them use the rule with different values for the principal and verify that it works, no matter what the principal is.

1. The Rule of 72 is a rule for approximating the time taken for an investment to double when it is compounded annually. To discover this rule, use the compound interest formula to calculate the final value of a $1000.00 investment compounded annually for the following rates and terms. Round to the nearest $100.00.

   a) 4.00% for 18 years

   b) 6.00% for 12 years

2. In question 1, approximately how many times the original investment are each of the final amounts?

3. How are the rates and the times related?

4. Create a rule for determining the approximate time in years for an investment to double.

5. Use your rule to find out how long it would take for $1000.00 to double at 3.00% per annum. Verify that your rule works by using the compound interest formula.

SOLUTIONS

1. a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   $A = \$1000.00\left(1 + \frac{0.04}{1}\right)^{1 \times 18}$

   $A = \$2025.82$

   $A \approx \$2000.00$

   b) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   $A = \$1000.00\left(1 + \frac{0.06}{1}\right)^{1 \times 12}$

   $A = \$2012.20$

   $A \approx \$2000.00$

2. They are double the original investment.

3. When you multiply the time and the percentage rate, the answer is 72. If you divide 72 by the percentage rate, you get the time.

4. Divide 72 by the annual percentage rate to get the time in years for an investment to double.

5. $72 \div 3 = 24$ years

Mental Math and Estimation

STUDENT BOOK, p. 271

SOLUTION

The Rule of 72 can be written as follows.

$y = 72 \div r$

Students can round the interest rate of 1.95% to 2%.

Students can round 72 down to 70. They can then use the formula to approximate that half of 70 is 35.

An investment invested at a rate of 1.95% would take over 35 years to double. If students divide 72 by 2, they will arrive at a more accurate answer of 36 years.

DISCUSS THE IDEAS

GUARANTEED INVESTMENT CERTIFICATES

STUDENT BOOK, p. 271

SOLUTIONS

1. Calculate how much interest Vyanjana would earn with each option.

Option 1:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$5000.00\left(1 + \frac{0.01125}{12}\right)^{12}$$

$$A \approx \$5056.54$$

$I = A - P$

$I = \$5056.54 - \$5000.00$

$I = \$56.54$

Option 2a:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$5000.00\left(1 + \frac{0.00875}{12}\right)^{12}$$

$$A \approx \$5043.93$$

$I = A - P$

$I = \$5043.93 - \$5000.00$

$I = \$43.93$

Option 2b:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = P\left(1 + \frac{0.0005}{12}\right)^{6}$$

$$A \approx \$5001.25$$

$I = A - P$

$I = \$5001.25 - \$5000.00$

$I = \$1.25$

Option 3:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$5000.00\left(1 + \frac{0.0125}{1}\right)^{1}$$

$$A = \$5000.00\ (1.0125)^{1}$$

$$A \approx \$5062.50$$

$I = A - P$

$I = \$5062.50 - \$5000.00$

$I = \$62.50$

2. If Vyanjana knows that she definitely will not need to access the money for the full year, she should choose option 3 because it pays the most interest.

   If Vyanjana thinks she might need the money before the end of the year, she should choose option 2. She will earn less interest, but she will be able to access her money if she needs it.

   Vyanjana should not choose option 1. Like option 3, it does not allow her to access her money during the year, but it earns less interest than option 3.

ACTIVITY 6.6

THE EFFECT OF DIFFERENT COMPOUNDING PERIODS

STUDENT BOOK, p. 272

This activity does not take very long to do, especially if students use the compound interest formula.

Students who have difficulty understanding how to apply the compound interest formula may work on Blackline Master 6.4 (p. 406), which provides practice in determining the correct values of the variables used in the formula.

SOLUTIONS

1.

Interest period	*Final value of investment (A)*	*Interest (I)*
Annually	$\$4000.00\left(1+\frac{0.03}{1}\right)^{(1\times 2)} \approx \$4243.60$	\$243.60
Semi-annually	$\$4000.00\left(1+\frac{0.03}{2}\right)^{(2\times 2)} \approx \$4245.45$	\$245.45
Quarterly	$\$4000.00\left(1+\frac{0.03}{4}\right)^{(4\times 2)} \approx \$4246.40$	\$246.40
Monthly	$\$4000.00\left(1+\frac{0.03}{12}\right)^{(12\times 2)} \approx \$4247.03$	\$247.03
Daily	$\$4000.00\left(1+\frac{0.03}{365}\right)^{(365\times 2)} \approx \$4247.34$	\$247.34

2. The daily compounding period yields the most interest. The annual compounding period yields the least interest. Knowing this, you would choose an investment which is compounded the most times per year to accumulate the most interest.

3. This question might be challenging for some students who have not had experience developing spreadsheets. You can provide them with a spreadsheet template similar to those below. Students can use it to calculate compound interest. Ask students to use their spreadsheet to determine the answer to Example 3 (p. 268 of the student book). Ask students to compare their spreadsheets with the interest table in Example 3 (p. 268 of the student book). Students who are familiar with spreadsheets can work with students who have not used them.

   A compound interest table can be created using a spreadsheet program. The example below is for an investment of \$4000.00 compounded semi-annually at a rate of 3.00% for 2 years.

   - Enter the principal into cell B1.
   - Enter the percent interest rate per year in cell D1.
   - Enter the number of compound periods per year in cell F1.
   - Enter the labels in row 3.
   - Format rows 4 and 5 with the formulas shown below.
   - Copy row 5 into the number of rows needed for the investment's term (in this case, 2 more rows for a total of 4 compounding periods).

   Note: The \$ signs in cells C4 and C5 allow the formulas to be copied down without changing the value of the rate per period as a decimal and the number of compounding periods.

	A	B	C	D	E	F
1		4000		3		2
2						
3	PERIOD	BALANCE AT START OF PERIOD	INTEREST EARNED	BALANCE AT END OF PERIOD		
4	1	=B1	=B1*$D$1/$F$1/100	=B4+C4		
5	=A4+1	=D4	=B5*$D$1/$F$1/100	=B5+C5		
6						
7						
8						
9						

Below is the resulting compound interest table.

	A	B	C	D	E	F
1		4000		3		2
2						
3	PERIOD	BALANCE AT START OF PERIOD	INTEREST EARNED	BALANCE AT END OF PERIOD		
4	1	4000.00	60.00	4060.00		
5	2	4060.00	60.90	4120.90		
6	3	4120.90	61.81	4182.71		
7	4	4182.71	62.74	4245.45		

A compound interest spreadsheet can be constructed using the operations indicated in the spreadsheet below.

A simple spreadsheet can be made to calculate the investment value at the end of each period for a given principal, interest rate, number of compounding periods, and time.

The known values are entered as indicated in row 2 of the spreadsheet.

The interest period can be 1 (annually); 0.5 (semi-annually); $\frac{1}{12}$ (monthly);or $\frac{1}{365}$ (daily).

The formula in cell C2 calculates the interest earned using the simple interest formula.

The formula in cell D2 adds the principal to the interest.

The formula in cell A3 adds 1 to the previous period number.

The formula in cell B3 enters the investment value at the end of the previous period.

The formula in cell C3 calculates the interest earned using the simple interest formula.

The formula in cell D3 adds the investment value at the start of the period to the interest earned.

Row 3 is copied down the appropriate number of cells (e.g., the time of the investment multiplied by the length of the interest period in years). This action calculates the investment value at the end of the final interest period.

The overall interest can be calculated by subtracting the principal from the investment value at the end of the final interest period.

	A	B	C	D
1	*Interest period*	*Investment value at beginning of period*	*Interest earned*	*Investment value at end of period*
2	1	Enter the value of the principal in this cell.	= B2* (enter the interest rate as decimal) *(enter the length of the interest period in years)	= B2+C2
3	= A2+1	= D2	= B3*(enter the interest rate as decimal) *(enter the length of the interest period in years)	= B3+C3

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 272

1. a) $I = Prt$

   $I = \$2000.00 \times 0.03 \times 2$

   $I = \$120.00$

   b) Calculate the sum of Gerard's principal and the interest earned.

   $A = \$2000.00 + \$120.00$

   $A = \$2120.00$

2. a)
$$I = Prt$$
$$\$50.00 = P \times 0.03 \times 2$$
$$50.00 = P \times 0.06$$
$$50.00 \div 0.06 = P$$
$$P = \$833.33$$

   b) Use the simple interest formula, with t in years.
$$I = Prt$$
$$\$180.00 = \$5000.00 \times 0.02 \times t$$
$$180.00 = 100.00t$$
$$180.00 \div 100.00 = t$$
$$1.8 = t$$

   Multiply by 12 to convert years to months.

   $t = 1.8$ years $\times$ 12 months

   $t = 21.6$ months

   c)
$$I = Prt$$
$$\$120.00 = \$4000.00 \times r \times 3$$
$$120.00 = 12\,000.00r$$
$$120.00 \div 12\,000.00 = r$$
$$0.01 = r$$
$$1.0\% = r$$

3. a) $I = Prt$

   $I = \$1500.00 \times 0.065 \times 0.5$

   $I = \$48.75$

   b) Calculate Mei Lin's total payment by adding the principal plus the interest.

   $A = \$1500.00 + \$48.75$

   $A = \$1548.75$

   c) Divide Mei Lin's total payment by 6 months.

   $\$1548.75 \div 6 = \$258.13$

   She will pay $258.13 a month.

4.
$$y = 72 \div r$$
$$18 = 72 \div r$$
$$18r = 72$$
$$r = 72 \div 18$$
$$r = 4$$

   To double your investment in 18 years, you would need an interest rate of 4% per annum.

5. a) There are 4 interest periods because the loan is semi-annual, which means interest is calculated twice a year for 2 years.

   b)

Interest period	*Investment value at beginning of period ($)*	*Interest earned ($) (I = Prt)*	*Investment value at end of period ($)*
1	$1200.00	$1200.00 × 0.026 × 0.5 = $15.60	$1200.00 + $15.60 = $1215.60
2	$1215.60	$1215.60 × 0.026 × 0.5 = $15.80	$1215.60 + $15.80 = $1231.40
3	$1231.40	$1231.40 × 0.026 × 0.5 = $16.01	$1231.40 + $16.01 = $1247.41
4	$1247.41	$1247.41 × 0.026 × 0.5 = $16.22	$1247.41 + $16.22 = $1263.63

   c) The value of the investment at the end of 2 years is $1263.63.

   d) Calculate the interest earned over 2 years by subtracting the principal from the investment value.

   $\$1263.63 - \$1200.00 = \$63.63$

   The interest earned is $63.63.

ALTERNATIVE SOLUTION

Part c) can also be solved using the compound interest formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$1200.00\left(1 + \frac{0.026}{2}\right)^{(2 \times 2)}$$

$$A = \$1263.63$$

The value at the end of the investment period is \$1263.63. The interest earned is \$63.63.

6. a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$2000.00\left(1 + \frac{0.0380}{2}\right)^{(2 \times 4)}$$

$$A \approx \$2325.00$$

$I = A - P$

$I = \$2325.00 - \$2000.00$

$I = \$325.00$

b) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$1500.00\left(1 + \frac{0.0260}{4}\right)^{(4 \times 3)}$$

$$A \approx \$1621.27$$

$I = A - P$

$I = \$1621.27 - \$1500.00$

$I = \$121.27$

c) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$6000.00\left(1 + \frac{0.0220}{12}\right)^{(12 \times 2)}$$

$$A \approx \$6269.64$$

$I = A - P$

$I = \$6269.64 - \$6000.00$

$I = \$269.64$

d) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$3560.00\left(1 + \frac{0.120}{12}\right)^{\left(12 \times \frac{3}{12}\right)}$$

$$A \approx \$3570.69$$

$I = A - P$

$I = \$3570.69 - \$3560.00$

$I = \$10.69$

7. **Option 1:**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$2500.00\left(1 + \frac{0.020}{1}\right)^{2}$$

$$A \approx \$2601.00$$

Option 2:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$2500.00\left(1 + \frac{0.020}{2}\right)^{(2 \times 2)}$$

$$A \approx \$2601.51$$

$\$2601.51 - \$2601.00 = \$0.51$

The second option is the better investment by \$0.51.

Extend Your Thinking

8. a) Answers will vary. Sample solutions are for a typical interest rate of 3.00% per annum, compounded semi-annually with a principal of \$4 000 000.00.

i) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$4\,000\,000.00\left(1 + \frac{0.03}{2}\right)^{(2 \times 1)}$$

$$A \approx \$4\,120\,900.00$$

$I = A - P$

$I = \$4\ 120\ 900.00 - \$4\ 000\ 000.00$

$I = \$120\ 900.00$

ii) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.03}{2}\right)^{\left(2 \times \frac{1}{12}\right)}$

$A \approx \$4\,009\,938.07$

$I = A - P$

$I = \$4\ 009\ 938.07 - \$4\ 000\ 000.00$

$I = \$9938.07$

iii) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.03}{2}\right)^{\left(2 \times \frac{1}{365}\right)}$

$A \approx \$4\,000\ 326.34$

$I = A - P$

$I = \$4\ 000\ 326.34 - \$4\ 000\ 000.00$

$I = \$326.34$

b) i) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.04}{2}\right)^{(2 \times 1)}$

$A \approx \$4\,161\,600.00$

$I = A - P$

$I = \$4\ 161\ 600.00 - \$4\ 000\ 000.00$

$I = \$161\ 600.00$

Calculate the change from part a).

$161 600.00 – $120 900.00 = $40 700.00

ii) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.04}{2}\right)^{\left(2 \times \frac{1}{12}\right)}$

$A \approx \$4\,013\,223.56$

$I = A - P$

$I = \$4\ 013\ 223.56 - \$4\ 000\ 000.00$

$I = \$13\ 223.56$

Calculate the change from part a).

$13 223.56 – $9938.07 = $3285.49

iii) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.04}{2}\right)^{\left(2 \times \frac{1}{365}\right)}$

$A \approx \$4\,000\ 434.05$

$I = A - P$

$I = \$4\ 000\ 434.05 - \$4\ 000\ 000.00$

$I = \$434.05$

Calculate the change from part a).

$434.05 – $326.34 = $107.71

c) Change compound period to monthly.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4\,000\,000.00\left(1 + \frac{0.03}{12}\right)^{(12 \times 1)}$

$A \approx \$4\,121\,663.83$

$I = A - P$

$I = \$4\ 121\ 663.83 - \$4\ 000\ 000.00$

$I = \$121\ 663.83$

If the interest is compounded more frequently, the amount of interest earned increases.

d) Increasing the rate by 1.00% increases the interest for 1 year from $120 900.00 to $161 600.00, a difference of $40 700.00.

Increasing the compound period to monthly increases the interest for 1 year from $120 900.00 to $121 663.88, a difference of $763.88.

Increasing the interest rate has the largest effect on interest.

THE ROOTS OF MATH

THE CANADIAN DOLLAR

STUDENT BOOK, p. 275

This activity provides a brief history of Canadian currency.

Students may be unfamiliar with the Canadian political history mentioned in this Roots of Math. You may want to review that, before 1867, the area that is now Canada was made up of provinces that were not governed by a central government. The Dominion of Canada was created in 1867, between the provinces of Ontario, Quebec, New Brunswick, and Nova Scotia, and a central government was formed.

The British monetary system of pounds and pence may be unfamiliar to students. If others have been to the United Kingdom or know about its currency, have them explain it to the class. One British pound is now equal to 100 pence. Prior to 1971, 1 pound was divided into 20 shillings, and each shilling was divided into 12 pence, so there were 240 pence to 1 pound.

The Royal Canadian Mint has resources for teachers about the history of Canadian currency:

http://www.mint.ca/store/mint/learn/teachers-corner-1100018

The Bank of Canada provides free resources about counterfeit bill detection:

http://www.bankofcanada.ca/en/banknotes/education/index_schools.html

Information about accessibility features can be found on this page of the Bank of Canada website:

http://www.bankofcanada.ca/en/banknotes/accessibility.html

The Currency Museum of the Bank of Canada also offers education materials:

http://www.currencymuseum.ca/home/learning-center/classroom-resources/

SOLUTIONS

1. In the long term, the cost of the toonie is much lower than the cost of the $2.00 bill. The $2.00 bill had only a 1-year life, so the cost to supply the currency for 20 years was $1.20 (20 times 6 cents). The cost of a toonie, which lasts 20 years, is only $0.16.
2. Security features of Canadian bills include:
   - a metallic (holographic) stripe
   - a ghost image (watermark)
   - metallic dashes printed on the bill shift from gold to green when tilted
   - a see-through number indicating the value of the bill appears when held up to light
   - raised ink on some elements of the bill
   - under UV (fluorescent) light, text appears

   Accessibility features of Canadian bills include:
   - a tactile feature of raised dots on one corner of the bill
   - large, high-contrast numerals identify the denomination (dark numeral on pale background and white number on dark background)
   - the various denominations are printed in contrasting colours

   The Bank of Canada also provides a hand-held bank note reader that informs the user of the value of the bill.

6.3 Credit Cards and Store Promotions

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 276–287

MATH *ON THE JOB*

STUDENT BOOK, p. 276

After reading through the Math on the Job, ask students what they think deferred payment means. Use these definitions to arrive at a final definition that the class will use. Ensure students know that deferred payment is a form of credit. You could also mention that deferred payment is often used as part of sales promotions, and ask students to identify businesses that use deferred payment to promote products.

To lead into a discussion about credit, you could have students brainstorm different types of credit. Next, have small groups or pairs of students construct a list of the advantages and disadvantages of using a credit card. Have students share their lists with the class. Next, ask students to brainstorm and share ideas on how credit cards can be used effectively.

SOLUTIONS

1. To calculate the cost of the phone on the deferred payment plan, multiply the advertised price by 1.08.

   $995.00 × 1.08 = $1074.60

   The phone will cost $1074.60 on the deferred payment plan.

2. Answers will vary. Sample answers may include the following. The customer may not have enough money saved to pay for the phone in one payment. The customer wants to use his money for other purposes for six months. In six months, the customer may have a larger income, which will allow him to pay for the phone. The customer needs the phone immediately, but wants to pay for it later.

EXPLORE THE MATH

STUDENT BOOK, p. 276

Read through the Explore the Math section with the students. Students will likely be familiar with many of the ideas talked about here.

Ask to see if they know how old they must be to get a credit card. Why might this be?

Ask students if they have heard of payday loans. What do they know about them? Do the advertisements make them sound appealing?

DISCUSS THE IDEAS

USING A CREDIT CARD

STUDENT BOOK, p. 277

Students can work in groups and brainstorm what they already know about credit cards. They may not know all the answers. Encourage them to find out what they don't know by going online at home or school.

SOLUTIONS

1. Answers will vary. Possible answers include: American Express, Visa, MasterCard, Hbc (Hudson's Bay Company/The Bay), Shell Oil, Husky, Sears, Future Shop, Best Buy.
2. A minimum payment is the smallest monthly amount you have to pay on your credit card account.
3. The credit limit is the maximum amount of credit that the credit card company will allow you to use. It is a spending limit on your credit card. The credit card company will base your credit limit on how likely they think it is that you will be able to repay your credit. If you are able to pay off your bills quickly, they will give you a higher credit limit.

4. Bank credit cards and store credit cards are very similar. Often stores offer discounts if their credit card is used. It's easier to qualify for a store card than a bank card. Store cards can only be used in branches of that store. Bank credit cards are more universally accepted throughout the world. The interest rates on store cards are often higher than on bank cards.

   Answers will vary. Possible answers include:

   - Bank of Montreal (BMO) MasterCard No Fee Air Miles card; no annual fee; 19.50% interest rate.
   - Hbc Credit Card; no annual fee; 28.80% interest rate.

5. Advantages of using a credit card: convenient; fits into your wallet easily; don't have to carry cash; don't have to pay straight away; some credit cards have consumer insurance; some credit cards have air miles or other benefits; build your credit rating if you pay your bill regularly.

   Disadvantages: If you don't pay off your full credit card bill each month, you pay interest on the unpaid balance; interest rates are high; some credit cards charge fees; it is easy to get into debt and hard to get out of debt.

Example 1

STUDENT BOOK, p. 277

You can find a simple interest calculator at the link below:

www.easycalculation.com/simple-interest.php

ACTIVITY 6.7
UNDERSTANDING CREDIT CARD STATEMENTS

STUDENT BOOK, p. 279

SOLUTIONS

1. The previous balance on Laurie's credit card was $1428.00.
2. Laurie's payment of her previous balance was marked as a negative number because $1428.00 was subtracted from the previous balance.
3. The value of purchases ($2211.52) was calculated by adding up all the purchases detailed in column 4.
4. No, Laurie did not have to pay interest in March because she had no outstanding balance. You only pay interest on your previous balance if you do not pay in full by the payment due date.
5. The minimum payment is calculated by taking 5% of $2211.52.

   $0.05 \times \$2211.52 = \$110.58$
6. Laurie will have to pay $2211.52 in April in order to avoid paying interest. If she pays off her current balance, she will have a zero balance and will not have to pay interest.
7. Available credit is calculated by subtracting the new balance from the credit limit.

ACTIVITY 6.8

UNDERSTANDING DEBT REPAYMENT

STUDENT BOOK, p. 281

SOLUTIONS

Students will need to search online for a credit card minimum payment calculator. There is one available at the following link:

http://www.fcac-acfc.gc.ca/iTools-iOutils/CreditCardCalculator-eng.aspx

1.

Account balance	*Annual interest rate*	*Min. payment %*	*Min. payment factor*	*Total interest*	*Time to pay off debt*
\$1000.00	18.00%	5%	\$10.00	\$382.42	70 months
\$1000.00	20.00%	5%	\$10.00	\$445.28	72 months

2. It takes 2 months longer to pay off the balance at 20.00%. Calculate how much more you will pay in interest.

   \$445.28 – \$382.42 = \$62.86

   You will pay \$62.86 more in interest.

Mental Math and Estimation

STUDENT BOOK, p. 281

SOLUTION

To estimate 5.00% of Chezhian's credit card balance, \$1002.93 can be rounded to \$1000.00.

Students can calculate that 1.00% of \$100.00 is \$1.00; therefore, 1.00% of a number 10 times greater (\$1000.00) is \$10.00. Since 1.00% of \$1000.00 is \$10.00, students can multiply this number by 5 to determine that 5.00% of \$1000.00 equals \$50.00.

5.00% of \$1000.00 is \$50.00. This is greater than \$10.00, so Chezhian's minimum payment will be about \$50.00.

DISCUSS THE IDEAS

MANAGING CREDIT AND DEBT

STUDENT BOOK, p. 281

This Discuss the Ideas will allow students to discover the dangers of too much debt if you don't have a plan for repayment.

Credit card debt is a major problem in Canada. In September 2009, outstanding credit card debt reached \$78 billion in Canada (Toronto Star, December 2009, quoting Equifax Canada). In 2009, there were 4.5 bankruptcies per 1000 people aged 18 and older in Canada, up from 3.2 in 2000 (Office of the Superintendent of Bankruptcy Canada).

SOLUTIONS

1. Answers will vary. Some long-term consequences of having too much debt include:
   - Because of compounding interest, if you can't pay your bills you will accumulate even more debt and may never be able to pay off what you owe.
   - You may not be able to rent housing. Landlords will often check your credit history before renting to you.
   - You may not get a job or promotion. Some employers check your credit history before hiring or giving promotions.
   - It might cost more for you to get insurance. Insurance companies sometimes check your credit history before setting your insurance rate.
   - If you don't pay your bills, credit card companies can raise your interest rates. If you try to get a new loan or credit card, the company will offer you a higher interest rate.
   - Your creditors may have your wages garnished in order to receive their payments. This means part of your paycheque will be paid directly to your creditors.

2. Answers will vary. Some ways of reducing accumulated debt include:
    - Reduce your spending. Prioritize your expenses and look for ways to save money. Use your savings to start paying back your debt.
    - When you do have money to put towards debt, use it to pay off loans or bills with the highest interest rates first.
    - Stop using credit cards, which have high interest rates. If you need to borrow money, try to use other loans that have lower interest rates.
    - Contact your lenders and negotiate lower interest rates.
    - Track your spending. In order to reduce your debt, you need to have a clear picture of where your money is going.
    - If you are unable to repay your debt through any other means, the last resort is to file for bankruptcy. There are serious consequences to bankruptcy.
3. Information about credit counselling can be found at the following websites:

    http://www.creditcounsellingcanada.ca/

    http://www.nomoredebts.org/

    Credit counselling agencies offer the following services:
    - Credit counselling: they will review your budget and help you manage your personal finances.
    - Debt consolidation: they will contact your creditors to establish payment plans and negotiate reduced interest rates on your debts.
    - Financial education: they provide resources and expertise on healthy money management.

ACTIVITY 6.9
BUY NOW! NO DOWN PAYMENT REQUIRED!

STUDENT BOOK, p. 283

This activity is based in the context of purchasing a flat-screen TV, so it will be a useful exercise which can inform students' project work. Sales promotions can be very seductive, especially to young people. It is important to be very clear about just exactly what various sales are offering. Remind students of the well-known phrase, "Let the buyer beware."

Students should be reminded that if they do not have the money to pay immediately, or to pay with monthly payments, unless they expect a radical change in their future income, then committing to a buy now—pay later scheme is likely unwise. They could end up having to pay increasing interest amounts, and it could possibly affect their credit rating.

SOLUTIONS

1. Calculate the total cost for each option.

    Option 1: $1245.65

    Option 2: $1500.00 + $35.00 + $75.00 = $1610.00

    Option 3: 24 × $70.00 = $1680.00
2. Option 1, cash payment, is the least expensive payment option.
3. Answers will vary. Advantages and disadvantages of each option might include the following.

    Option 1: cheapest, but you need all the cash right now.

    Option 2: you can have the TV immediately. You only have to pay $110.00 up front. But you have to come up with $1500.00 before 365 days have passed or get charged 12.00% interest.

    Option 3: The most expensive but with affordable monthly payments. $0 to pay up front.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 284

1. a) Calculate the interest on the unpaid balance.

   $I = Prt$

   $I = \$345.67 \times 0.20 \times (30 \div 365)$

   $I = \$5.68$

   Calculate 5% of the unpaid balance plus interest.

   $(\$345.67 + \$5.68) \times 0.05 = \$17.57$

   This is more than $10.00, so the minimum payment is $17.57.

   b) Calculate the interest on the unpaid balance.

   $I = Prt$

   $I = \$55.75 \times 0.18 \times (31 \div 365)$

   $I = \$0.85$

   Calculate 5% of the unpaid balance plus interest.

   $(\$55.75 + \$0.85) \times 0.05 = \$2.83$

   This is less than $10.00, so the minimum payment is $10.00.

2. $I = Prt$

   $\$16.22 = \$1032.05 \times r \times (31 \div 365)$

   $16.22 = 87.65r$

   $16.22 \div 87.65 = r$

   $0.185 \approx r$

   Convert the interest rate to a percentage.

   $0.185 \times 100 = 18.5$

   The interest rate is 18.5% per annum.

3. $I = Prt$

   $\$7.75 = P \times 0.2150 \times (19 \div 365)$

   $7.75 = 0.01119P$

   $7.75 \div 0.01119 = P$

   $692.58 \approx P$

   The unpaid balance on Sanaa's credit card was $692.58.

4. a) Unpaid balance = $505.50 – $100.00

   Unpaid balance = $405.50

   $I = Prt$

   $I = \$405.50 \times 0.1890 \times (21 \div 365)$

   $I = \$4.41$

   b) Phil will owe the interest on the unpaid balance plus the interest on the new purchase plus the unpaid balance.

   Calculate the interest on the new purchase (daily interest over 6 days).

   $I = Prt$

   $I = \$160.40 \times 0.1890 \times (6 \div 365)$

   $I = \$0.50$

   Add the two interest amounts plus the unpaid balance.

   $\$4.41 + \$0.50 + \$405.50 = \$410.41$

   As of August 21, Phil will owe $410.40.

5. a) Days in August = 31 – 10

   Days in August = 21 days

   Days in September = 28 days

   Total days = 49 days

   b) $I = Prt$

   $I = \$300.00 \times 0.255 \times (49 \div 365)$

   $I = \$10.27$

   c) $A = P + I$

   $A = \$300.00 + \$10.27$

   $A = \$310.27$

6. Calculate the total cost of the refrigerator on the deferred payment plan:

   $\$1099.99 + \$40.00 + \$60.00 = \$1199.99$

   Subtract the cash price plus delivery from the total deferred payment plan price.

   $\$1199.99 - (\$729.99 + \$40.00) = \$430.00$

   Brian will pay $430.00 in interest if he pays using the deferred payment plan.

7. Calculate the cost of each payment option.

Option 1:

$\$895.99 \times 1.12 = \$1003.51$

Option 2:

$6 \times \$190.00 = \$1140.00$

Option 3:

$I = Prt$

$I = \$1003.51 \times 0.195 \times (15 \div 365)$

$I = \$8.04$

total cost = $1003.51 + $8.04

total cost = $1011.55

Option 1 is the least expensive, so if Arlene can afford it, she should pay cash. If she does not have the cash on hand, she should use her credit card and pay off the balance as soon as possible.

T

8. a) Students will need to find an online credit card minimum payment calculator. They can use the one they found for Activity 6.8 (p. 382 of this resource).

   Students can use the calculator to find that it will take Guy 1 year and 3 months (15 months) to pay off his debt.

   b) Students will need to use the online credit card minimum payment calculator to enter the information for what Guy owes, the annual interest rate, and the fixed payment into the appropriate cells. Students should use Option C: fixed payment. The minimum payment is not required for this question. The information students enter into the credit card calculator should look like this:

   Credit Card Balance: $1533.26

   Annual Interest Rate: 18%

   Option C: Fixed Monthly Payment: $120.00

   Guy will pay $181.81 in interest.

9. a) Calculate Pasha's purchases.

   $\$78.85 + \$62.34 + \$345.55 + \$42.50 + \$68.76 + \$25.00 + \$25.00 = \$648.00$

   b) Interest is calculated over 31 days.

   c) Calculate Pasha's unpaid balance by subtracting his payment from his pervious balance.

   $\$245.86 - \$100.00 = \$145.86$

   d) Calculate the interest on his unpaid balance.

   $I = Prt$

   $I = \$145.86 \times 0.195 \times (31 \div 365)$

   $I = \$2.42$

   e) New balance = $145.86 + $648.00 + $2.42

   New balance = $796.28

   f) Calculate 5.0% of the new balance.

   $\$796.28 \times 0.05 = \$39.81$

   This is greater than $10.00, so Pasha's minimum payment is $39.81.

   g) Calculate his credit still available by subtracting his new balance from his credit limit.

   $\$3500.00 - \$796.28 = \$2703.72$

Extend Your Thinking

10. a) Total cost of the bed:

   $\$675.00 + \$35.00 + \$50.00 = \$760.00$

   b) i) Simple interest:

   $I = Prt$

   $I = \$675.00 \times 0.19 \times 1.0$

   $I = \$128.25$

   Total paid = $675.00 + $128.25 + $35.00 + $50.00

   Total paid = $888.25

ii) Compound interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$675.00\left(1 + \frac{0.19}{12}\right)^{12}$$

$$A \approx \$815.03$$

Total paid = \$815.03 +\$35.00 + \$50.00

Total paid = \$900.03

PUZZLE IT OUT

WHO HAS WHAT?

STUDENT BOOK, p. 287

Read the text out loud as a class. Ask students how they might start solving it. Allow students to work in groups of three or four, challenging them to be the first group to get the correct answer. Ensure that the class understands what is being asked for: namely, that each person will be matched with exactly one bank account and exactly one credit card. No two people have the same bank account or credit card.

Have students write their solutions on the overhead or blackboard. Discuss the results as a class. Alternatively, you could have each group of students hand in their written work. Encourage students not to just write down the answer, but to try and communicate how they got their answer.

SOLUTION

Here is a sample solution. There may be other ways to solve this puzzle.

Label the people P, Q, and R.

Label the bank accounts SS, FS, and SAV.

Label the credit cards B, S, and G.

You cannot solve this problem using the clues sequentially. You have to use parts of each clue at different steps of the logic sequence.

Step 1: Use Clue 1, part 1, to determine that Quinn has a self-service account.

	P	Q	R
ACCOUNT	FS or SAV	SS	FS or SAV
CREDIT CARD			

Step 2: Use Clue 3, part 1, to determine that Pat has a full-service account (the self-service account is taken by Quinn, and he does not have a savings account). By elimination, Rachel has the savings account.

	P	Q	R
ACCOUNT	FS	SS	SAV
CREDIT CARD			

Step 3: Use Clue 2 to determine that Pat has the store credit card.

	P	Q	R
ACCOUNT	FS	SS	SAV
CREDIT CARD	S		

Step 4: Use Clue 1, part 2, to determine that Rachel has the bank card (the store card is taken, and she doesn't have the gas card because she has the savings account). By elimination, Quinn has the gas card.

	P	Q	R
ACCOUNT	FS	SS	SAV
CREDIT CARD	S	G	B

Note: Clue 3, part 2 isn't necessary to solve the puzzle.

Pat has the full-service account and the store credit card.

Quinn has the self-service account and the gas credit card.

Rachel has the savings account and the bank credit card.

6.4 Personal Loans, Lines of Credit, and Overdrafts

TIME REQUIRED FOR THIS SECTION: 2 CLASSES

STUDENT BOOK, pp. 288–299

MATH *ON THE JOB*

STUDENT BOOK, p. 288

You can use this Math on the Job to introduce the topic of personal loans, while making connections to student goals. Students are at an age where they may have more control over their personal finances. They may have jobs. They may be considering purchasing more expensive items, such as their first car or stereo equipment. They may also be thinking of how they will pay for post-secondary education.

You may wish to look for nonverbal cues to determine whether or not students are comfortable talking about personal finance.

Encourage students to express their goals for the future. Is there a financial aspect to any of these goals? Will students have to borrow money, or have they experienced borrowing money to accomplish a goal? Will any of their future goals require securing a loan?

If students are comfortable sharing, encourage them to tell the class about their experiences connected to borrowing money. You could ask students who have borrowed money to describe how long they had to pay the money back and why, as well as how the amount of the installments was decided.

Students who have borrowed money will understand how the mathematical operations taught in this section apply in the real world. Their explanations may help other students make the connection between the information in the textbook and its practical use in their own lives.

SOLUTION

Multiply the number of months by the monthly payment.

(4 × 12) × \$277.71 = \$13 330.08

The customer will pay \$13 330.08.

Calculate how much was paid in interest.

\$13 330.08 – \$12 000.00 = \$1330.08

The customer paid \$1330.08 in interest.

EXPLORE THE MATH

STUDENT BOOK, p. 288

Read aloud, or have students read through, the information in Explore the Math. Introduce students to the various terms used when considering loans. Remind them that a finance charge is the difference between the amount you borrow and the total amount you pay back.

The most common type of loan is called an annuity or installment loan, which students saw in section 6.3 when they were purchasing items on an installment plan. Students should be aware of the difference between secured and unsecured loans, and have an understanding of the meaning of the Bank of Canada's prime lending rate—a concept that is seen frequently in the media.

DISCUSS THE IDEAS

PERSONAL LOANS

STUDENT BOOK, p. 289

SOLUTION

1. The interest rate on a secured loan is usually lower than the interest rate on an unsecured loan because, if you default on the loan, the lender will be compensated by claiming the collateral.

2. A financial institution will usually lend money based on the value of the collateral. If the collateral is a car, the value of the loan will be lower than if the collateral were a house. This is because the lender wants to be able to get back the value of the money they lent out if you default on the loan.
3. Some assets used as collateral include: vehicles, real estate, cash bank accounts, investments, insurance policies, and valuables such as jewellery.

Example 1

STUDENT BOOK, p. 290

ALTERNATIVE SOLUTION

Note that the value of the annual interest rate in the alternative solution is approximately 417.1%, while the value in the original solution is 416.1%. This 1% difference occurs because in the original solution the value of r was rouded to 4 decimal places. If the value of r had been taken as 0.0114285714 and this value had been multiplied by 365, the annual interest rate would have been calculated as 4.171428571, or 417.1%.

Example 2

T STUDENT BOOK, p. 291

Students can also use online technology to determine the monthly payment. Students can perform an internet key word search for "loan payment calculator." The link below is to such a calculator:

www.bankrate.com/calculators/mortgages/loan-calculators.aspx

Students can enter the applicable values into the calculator to determine that the monthly payment is $111.08.

ACTIVITY 6.10
COMPARING TYPES OF LOANS

STUDENT BOOK, p. 293

Have the students read through the information provided at the start of this activity or read through it with them, ensuring that they understand the difference between an overdraft and a line of credit. Note that banks offer overdraft protection with different fees. Some charge a monthly fee, while others charge interest on the amount overdrawn. Overdraft protection is a kind of line of credit; it is a loan of an unspecified amount that can be used when required.

When students have read the information, have them answer the questions in pairs or groups.

SOLUTIONS

1. Calculate Craig's NSF charges.

   $\$35.00 \times 2 = \$70.00$

   Craig will be charged $70.00 in NSF fees.

2. Calculate how much interest Craig would pay.

   $\$261.00 - \$225.00 = \$36.00$

   Calculate the interest rate.

$$I = Prt$$
$$\$36.00 = \$225.00 \times r \times 10$$
$$36 = 2250r$$
$$36 \div 2250 = r$$
$$0.016 = \mathrm{r}$$

   Convert the interest rate to a percent.

   $0.016 \times 100 = 1.6$

   The daily interest rate is 1.6%.

3. Calculate the interest charged on the overdraft.

   $I = Prt$

   $I = \$225.00 \times 0.0008 \times 10$

   $I = \$1.80$

   Craig would be charged $1.80 plus the overdraft fee.

   $\$1.80 + \$5.00 = \$6.80$

   Craig would pay $6.80 for the overdraft.

4. Calculate the interest charged on the line of credit.

   $I = Prt$

   $I = \$225.00 \times 0.04 \times (10 \div 365)$

   $I = \$0.25$

   Craig would have to pay $0.25 interest.

5. The best way to cover a financial shortfall is to borrow money on a line of credit. A line of credit usually has a much lower interest rate than overdraft protection or a payday loan.

6. When applying for a line of credit, it is best to negotiate for the lowest possible interest rate. Because interest is charged starting on the day of the withdrawal, a lower interest rate will save you money.

Extension

Just in case any students consider taking a payday loan, this activity, and the Build Your Skills questions associated with it, should be enough to convince them that, although it may be fast and convenient, it really is a bad idea.

You need $300.00 right away to fix your car. A payday loan company asks you to write a cheque dated for 10 days from now for $300.00 plus $17.50 for each $100.00 borrowed.

a) How much is the interest on this loan?

b) What does it actually cost you to fix your car?

c) Calculate the daily interest rate and the annual interest rate for this loan.

d) What are the advantages and disadvantages of payday loans?

e) Use the internet to research various payday loans. Describe your findings.

SOLUTIONS

a)
$$\text{total interest} = \$17.50 \times 3$$
$$\text{total interest} = \$52.50$$

b)
$$\text{total cost} = P + I$$
$$\text{total cost} = \$300.00 + \$52.50$$
$$\text{total cost} = \$352.50$$

c)
$$I = Prt$$
$$\$52.50 = \$300.00 \times r \times 10$$
$$\$52.50 = \$3000.00r$$
$$\$52.50 \div \$3000.00 = r$$
$$0.0175 = r$$
$$r = 1.75\% \text{ per day}$$
$$1.75 \times 365 \text{ days} = 638.75\% \text{ per year}$$

d) Advantages: You get the cash quickly without a credit check.

   Disadvantages: Payday loans are very expensive with extremely high interest rates. They are difficult to pay off.

e) Answers will vary.

Mental Math and Estimation

STUDENT BOOK, p. 294

SOLUTIONS

1. $\$25.00 \times 2 = \$50.00$
2. $\$25.00 \times 4 = \$100.00$
3. $\$25.00 \times \frac{1}{2} = \$12.50$

ACTIVITY 6.11

USING CREDIT EFFECTIVELY

STUDENT BOOK, p. 294

This activity demonstrates that although a particular purchase option may seem appealing at first glance, it is important to carefully consider all options and to calculate exactly what each might cost before making a decision. Note that sometimes a decision has to be made to choose an option based on a person's financial circumstances and their ability to repay a loan.

Students will need a personal loan payment calculator table or an online calculator to complete this activity. A table is provided on p. 292 of the student resource and Blackline Master 6.5 (p. 408) for ease of reference.

SOLUTIONS

PART A

1., 2., and 3. Answers will vary. Some typical answers are:

Option 1: First pay off the $100.00 on your credit card. You will then have $1000.00 cash left to buy the ATV. You could save $200.00 a month for 9 months, and then pay for the ATV with cash. If you don't use your credit card, you will then be debt-free in 9 months. The total you will pay is $2800.00 plus $100.00 to pay off the credit card. You will have to wait to get your ATV.

Option 2: Buy the ATV at the dealer's special offer of $250.00 a month. You will need to supplement the $200.00 monthly savings with $50.00 a month taken from your current savings. You will need $600.00 (12 × $50.00) over the year to do this. You will have an extra $500.00 cash, so first pay off your credit card debt. The total you will pay with this option is $3000.00 ($250.00 × 12) plus $100.00 to pay off the credit card. You can get your ATV immediately.

Option 3: Use the $1100.00 cash plus take a loan of $1700.00 from your bank, and then pay the dealer $2800.00 cash. The loan will cost you $146.71 a month for 12 months, which is equal to $1760.52. The total you will pay for the ATV is $2860.52 ($1760.52 plus $1100.00). You can afford to pay off the credit card loan over 12 months. You can get your ATV immediately.

Option 4: Pay for the ATV with a down payment of $1100.00 plus the balance of $1700.00 with your credit card. Pay off the credit card by making minimum monthly payments, so you have extra cash to spend each month. You get your ATV immediately, but because you are paying off your credit card with minimum monthly payments, you will end up paying much more in the long run.

Even though option 1 is the least expensive, I choose option 2 because it is the least expensive way to get the ATV immediately.

Part B

Some possible strategies are:

- Never pay off credit card debts by only paying minimum payments.
- Always pay off the balance of your credit card each month to avoid interest.
- Be careful to check special offers because they could cost you more money.
- Decide whether you can wait to purchase an item in order to avoid borrowing.
- Never take out a payday loan.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 295

For questions 1 to 3, use the personal loan payment calculator table (Blackline Master 6.5, p. 408) to determine the monthly payment.

1. a) Monthly payment = $31.11

   Total amount paid = $31.11/month × 36 months

   Total amount paid = $1119.96

   Finance charge = $1119.96 – $1000.00

   Finance charge = $119.96

b) Monthly payment = \$24.18 per \$1000.00 borrowed

Monthly payment = \$24.18 × (\$2500.00 ÷ \$1000.00)

Monthly payment = \$60.45

Total amount paid = \$60.45/month × 48 months

Total amount paid = \$2901.60

Finance charge = \$2901.60 – \$2500.00

Finance charge = \$401.60

c) Monthly payment = \$20.76 per \$1000.00 borrowed

Monthly payment = \$20.76 × (\$3000.00 ÷ \$1000.00)

Monthly payment = \$62.28

Total amount paid = \$62.28/month × 60 months

Total amount paid = \$3736.80

Finance charge = \$3736.80 – \$3000.00

Finance charge = \$736.80

2. a) \$1565.45 – \$500.00 = \$1065.45

Amy will have to borrow \$1065.45.

b) Monthly payment = \$87.10 per \$1000.00 borrowed

Monthly payment = \$87.10 × (\$1065.45 ÷ \$1000.00)

Monthly payment = \$92.80

Amy's monthly payment will be \$92.80.

c) Calculate Amy's total loan payment.

\$92.80/month × 12 months = \$1113.60

Amy's loan will cost her \$1113.60.

d) Calculate the cost of the computer, which will be the sum of the loan payments plus down payment.

\$1113.60 + \$500.00 = \$1613.60

Amy will pay \$1613.60 for the computer.

3. Calculate the cost of each payment option.

Option 1: Borrowing from the bank.

Monthly payment = \$43.87 per \$1000.00 borrowed

Monthly payment = \$43.87 × (\$8500.00 ÷ \$1000.00)

Monthly payment = \$372.90

Cost = \$372.90/month × 24 months

Cost = \$8949.60

Advantage: Cindy can afford the monthly payments.

Disadvantage: The total paid is \$449.60 more than the cash price of the snowmobile.

Option 2: Credit card.

Calculate the interest, 22.5% of \$8500.00.

0.225 × \$8500.00 = \$1912.50

Cost = \$8500.00 + \$1912.50

Cost = \$10 412.50

If she pays her credit card balance in 1 year, the snow machine will cost her approximately \$10 412.50.

Calculate the minimum payment for the first month.

\$8500.00 × 0.05 = \$425.00

Advantage: If she can pay off her credit card quickly, this might be a good option, especially if she is offered air travel rewards or some other type of reward. But it looks like Cindy will not be able to afford to do this.

Disadvantages: She does not have the cash to pay off her purchase to avoid interest charges. Her first minimum payment will be \$425.00, which she cannot afford. The total cost of the snow machine is significantly higher than options 1 or 3.

She should avoid this option.

Option 3: Line of credit.

Monthly payment = \$43.65 per \$1000.00 borrowed

Monthly payment = \$43.65 × (\$8500.00 ÷ \$1000.00)

Monthly payment = \$371.03

Cost = \$371.03/month × 24 months

Cost = \$8904.72

Advantage: The line of credit option gives her flexibility and she doesn't have to apply again for a new loan if she wants one at a later date. This is the least expensive option.

Disadvantage: The total paid is \$404.72 more than the cash price of the snow machine.

4. Toby will have to pay the returned-cheque fee twice.

   Fees = 2 × \$25.00

   Fees = \$50.00

   Toby will pay \$50.00 in charges for his NSF cheques.

5. a) \$560.00 – \$500.00 = \$60.00

   If he takes the payday loan, Ramon will pay \$60.00 interest.

   b) Calculate the daily interest rate.

   $$I = Prt$$

   $$\$60.00 = \$500.00 \times r \times 10$$

   $$\$60.00 = (\$5000.00)r$$

   $$\$60.00 \div \$5000.00 = r$$

   $$0.012 = r$$

   Convert to a percent.

   $$0.012 \times 100 = 1.2\% \text{ per day}$$

   Calculate the annual interest rate.

   $$1.2 \times 365 = 438\% \text{ per year}$$

6. An overdraft occurs when you withdraw more than you have in your account. You are charged a fee for being overdrawn, and your bank will not make any payments which you cannot cover. You can get overdraft protection, which means that the bank will pay for your overdrawn withdrawals, but you must still pay a fee.

   A line of credit is a negotiated loan with your bank which allows you to withdraw funds up to an agreed-upon limit. You pay interest until these funds are repaid. Unlike an overdraft, a fee is not charged to withdraw funds from a line of credit.

   When choosing between an overdraft and a line of credit, there are advantages and disadvantages to both. An overdraft is inexpensive or free to put in place, and is easily and quickly arranged. However, you might be charged an interest rate higher than that on a line of credit. A secured line of credit will often offer a better interest rate, but it also offers you the opportunity to spend money that you don't have. Because a line of credit offers a better interest rate, it is probably the better option.

Extend Your Thinking

7. a) Calculate the monthly payment for each option.

   Option 1: \$300.00 a month

   Option 2:

   Monthly payment = \$87.22 per \$1000.00 borrowed

   Monthly payment = \$87.22 × (\$3842.00 ÷ \$1000.00)

   Monthly payment = \$335.10

   Option 3: \$300.00 a month

   b) Calculate the total cost for each option.

   Option 1:

   Cost = down payment + (12 × monthly payment) + administration fee

   Cost = \$500.00 + (\$300.00 × 12) + \$25.00

   Cost = \$4125.00

Option 2:

Cost = 12 × monthly payment

Cost = 12 × $335.10

Cost = $4021.20

Option 3:

Cost = $3842.00

c) **Option 1:** He can buy it immediately and doesn't have to apply for a loan. It is the most expensive way.

Option 2: This is cheaper than option 1 and he can get the kayak right away. He will need to get a loan.

Option 3: This is the cheapest way, but Josh will have to wait to buy the kayak.

REFLECT ON YOUR LEARNING

FINANCIAL SERVICES

STUDENT BOOK, p. 297

In this chapter, students have gathered, described, and compared information about the various financial services commonly available from financial institutions. They should now be able to demonstrate an understanding of simple and compound interest by solving problems involving these concepts. They have learned about different methods of borrowing and the associated costs. With the new skills and knowledge, students are now equipped to make informed decisions about accessing and managing their money.

Have students review and reflect on their knowledge by asking them some of the following questions:

1. In what ways can technology help you make informed decisions about managing your money?
2. List some different methods you could use to describe the relationship between simple interest and compound interest.
3. Describe some different types of circumstances that might justify someone taking out a personal loan.
4. What are some of the things you can tell about someone's financial situation by looking at their credit card statements?
5. How would you explain to a friend the cost of carrying an outstanding balance on a credit card?
6. How would you go about establishing and maintaining a good credit rating?

There are many sources of information teachers and students can use to research financial institutions, credit cards, and other credit options. Students can visit actual banks and ask for information pamphlets, or can go online to do research. The following websites may help students get started:

- Financial Consumer Agency of Canada—The Money Belt

  http://www.themoneybelt.ca

- Canada's Office of Consumer Affairs—Privacytown

  http://www.ic.gc.ca/eic/site/oca-bc.nsf/eng/ca01304.html

- Canadian Consumer Information Gateway—Home Page

  http://consumerinformation.ca/app/oca/ccig/main.do?language=eng

- CBC Market Place: "No Money Down"

  http://www.cbc.ca/marketplace/pre-2007/files/money/nointerest/index.html

- Government of Canada Competition Bureau—Price-related Representations

  http://www.ic.gc.ca/eic/site/cb-bc.nsf/eng/00522.html

- Bankruptcy Canada Debt Management Plans and Credit Counselling

  http://www.bankruptcy-canada.ca/credit-counselling/debtManagement.htm

- Consumer Protection BC

  http://consumerprotectionbc.ca/businesses-payday-lenders-home

- Canadian Bankers Association Welcome to Your Money

  http://www.yourmoney.cba.ca/students/

PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 298

SOLUTIONS

1.

	Value	*Self-service*	*Full-service*	*Bonus savings*
Month 1				
Approximate minimum monthly balance: $900.00 (opening balance)				
Monthly fee	$3.90	$10.90	$24.50	$0
Transaction charges	8 × $0.50 = $4.00	$0	$0	16 × $1.25 = $20.00
Month 2				
Approximate minimum monthly balance: $900.00 plus deposits, minus withdrawals and service charges from Month 1 (opening balance)				
Monthly fee	$0	$10.90	$24.50	$0
Transaction charges	8 × $0.50 = $4.00	$0	$0	16 × $1.25 = $20.00
TOTAL	$11.90	$21.80	$49.00	$40.0

2.

Bank card	*Online banking*	*Bank teller*	*ATM*	*Cheque*
• withdraw cash • deposit cash or cheques • check account balance • purchase item at store • pay bills • transfer money between accounts	• deposit paycheques • pay bills • check account balance • transfer money between accounts	• pay bills • buy traveller's cheques • withdraw cash • deposit cash or cheques • transfer money between accounts • deposit paycheques • obtain foreign currency • check account balance	• pay bills • withdraw cash • deposit cash or cheques • transfer money between accounts • deposit paycheques • check the account balance	• pay bills • purchase items from a store

3.

$$\begin{aligned} I &= Prt \\ \$60.00 &= \$3000.00 \times 0.03 \times t \\ \$60.00 &= \$90.00t \\ \frac{\$60.00}{\$90.00} &= t \\ \frac{2}{3} &= t \end{aligned}$$

It will take $\frac{2}{3}$ of a year, or 8 months, to earn $60.00 interest.

4. a) $I = Prt$

$I = (\$800.00)(0.10)(1)$

$I = \$80.00$

The interest to be paid over 1 year is \$80.00.

b) Total = principal + interest

Total = \$800.00 + \$80.00

Total = \$880.00

The total amount to be paid back is \$880.00.

c) Calculate the monthly payment amount.

\$880.00 ÷ 12 = \$73.33

The monthly payments will be \$73.33.

5. a)

Period	Value at beginning of period	Interest ($I = Prt$)	Value at end of period
1	\$1000.00	\$1000.00 × 0.0425 = \$42.50	\$1000.00 + \$42.50 = \$1042.50
2	\$1030.00	\$1042.50 × 0.0425 = \$44.31	\$1042.50 + \$44.31 = \$1086.81
3	\$1060.90	\$1086.81 × 0.0425 = \$46.19	\$1086.81 + \$46.19 = \$1133.00

b) Calculate the total interest after 3 years.

I = total value – principal

$I = \$1133.00 - \$1000.00$

$I = \$133.00$

The total interest earned after 3 years is \$133.00.

6. Use the Rule of 72.

72 ÷ interest rate = time in years

72 ÷ 2.00% = 36 years

It will take about 36 years for the investment to double in value.

7. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = \$4000.00\left(1 + \frac{0.045}{2}\right)^{(2 \times 17)}$

$A = \$4000.00(1.0225)^{34}$

$A \approx \$8523.40$

The investment will be worth \$8523.40 after 17 years.

8. a) Calculate the total of Henry's purchases.

\$65.00 + \$245.89 + \$52.00 + \$105.65 + \$65.00 + \$12.50 = \$546.04

b) The unpaid balance is the previous balance minus the payment.

\$527.55 – \$200.00 = \$327.55

c) $I = Prt$

$I = \$327.55 \times 0.195 \times (31 \div 365)$

$I = \$5.42$

The interest on the unpaid balance is \$5.42.

d) The new balance is the unpaid balance from the previous statement plus the new purchases and interest.

\$327.55 + \$546.04 + \$5.42 = \$879.01

The new balance is \$879.01.

e) Calculate 5% of the new balance.

\$879.01 × 0.05 = \$43.95

The minimum payment for this statement is \$43.95.

f) Available credit is equal to the credit limit minus the new balance.

\$4500.00 – \$879.01 = \$3620.99

Henry has \$3620.99 in credit available.

9. a) For a loan at 5.00% per annum with an amortization period of 2 years, the monthly payment is \$43.87 per \$1000.00 borrowed.

   Monthly payment = \$43.87 × (\$10 500.00 ÷ \$1000.00)

   Monthly payment = \$460.64

   Amortization period of 4 years:

   Monthly payment = \$23.03 per \$1000.00 borrowed

   Monthly payment = \$23.03 × (\$10 500.00 ÷ \$1000.00)

   Monthly payment = \$241.82

   \$460.64 – \$241.82 = \$218.82

   The 2-year loan has the higher monthly payment, by \$218.51.

   b) Calculate the total cost of each option.

   2-year loan:

   \$460.64/month × 24 months = \$11 055.36

   With an amortization period of 2 years, the total cost of the loan is \$11 055.36.

   4-year loan:

   \$241.82/month × 48 months = \$11 607.36

   With an amortization period of 4 years, the total cost of the loan is \$11 607.36.

   The 4-year loan has an overall higher cost.

   \$11 607.36 – \$11 055.36 = \$552.00

   The 4-year loan costs \$552.00 more than the 2-year loan.

   c) Nadine may have to take the 4-year loan because she may not be able to afford the monthly payments for the 2-year loan.

10. Calculate the total cost of each payment option.

Option 1:

(\$115.00 × 5) + \$25.00 = \$600.00

Option 2:

$I = Prt$

$I = \$575.00 \times 0.05 \times (30 \div 365)$

$I = \$2.36$

Cost = \$575.00 + \$2.36

Cost = \$577.36

Option 3:

Solve for r.

The interest is 1.2% per day.

$1.2 \times 365 = 438$

The interest is 438% per year.

Convert to a decimal.

$r = 438 \div 100$

$r = 4.38$

Solve for t.

$$t = \frac{30}{365}$$

Solve for n.

$n = 365$

Insert the values into the compound interest formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$575.00\left(1 + \frac{4.38}{365}\right)^{\left(365 \times \frac{30}{365}\right)}$$

$$A = 575(1.012)^{30}$$

$$A = 822.40$$

Cost = \$822.40

Option 2 is the least expensive option, so Joseph should pay using his personal line of credit.

SAMPLE CHAPTER TEST

Name: ______________________ Date: ______________________

Part A: Multiple Choice

Choose the best response to each of the following questions.

1. Which transaction cannot be made using internet banking?
   a) salary payments
   b) cash withdrawal
   c) bill payment
   d) money transfer

2. Chantal has a part-time job during the school year and works full time during the summer. She has saved $5000.00 for her college fund. She put it in a savings account which earns interest at 2.50% per annum, compounded semi-annually. How much interest will she earn after 3 years?
   a) $187.50
   b) $386.92
   c) $375.00
   d) $384.45

3. On April 25, Ian withdraws $1000.00 cash on his credit card. This withdrawal appears on his monthly statement issued May 2. Ian does not pay off this amount until May 13. His credit card company's simple interest rate is 20.00% per annum starting on the day of the withdrawal. Calculate the interest he must pay.
   a) $6.03
   b) $6.58
   c) $10.41
   d) $10.96

Part B: Short Answer

4. Jasmine has a Value Account at Northwest Bank of Canada. The bank account offers 10 free self-service transactions. Additional self-service transactions cost \$0.50 each, and teller-assisted transactions cost \$1.00 each.

   She keeps a record of her transactions in her own record book. In April, she made the following transactions:

   - 6 ATM cash withdrawals from a Northwest Bank of Canada branch
   - ATM deposits for \$450.00 and \$100.00
   - 4 internet bill payments
   - Purchase of US traveller's cheques

   How much did Jasmine pay in service fees?

5. If the simple interest is \$75.00 on an investment at 2.50% per annum for 3 years, what is the principal?

6. Estimate how long it would take an investment of \$2000.00 to be worth \$4000.00 at 3.00% per annum, compounded annually.

7. An investment of $7000.00 is made with interest at 4.50% per annum, compounded annually for 2 years. Determine the value of the investment at the end of 2 years.

8. The unpaid balance on Saba's credit card is $205.50 with interest charges of $4.20. If the new charges total $354.80, what is the minimum payment? The minimum payment is 5% of the new balance or $10.00, whichever is greater.

9. You need $200.00 right away to pay your cell phone bill. A payday loan company asks you to write a cheque for $200.00 plus pay a fee of $9.50 up front for each $100.00 borrowed, and you have 5 days to repay the loan.

   a) What does it actually cost you to pay your cell phone bill?

   b) What is the annual interest rate for this loan?

Part C: Extended Answer

10. Marco has found his dream motorcycle at a dealership in Nanaimo, BC.

    The dealer will sell it for $16 450.00 cash or for $470.00 a month for 3 years with a down payment of $1000.00. Marco can get a loan from his bank at 7.00% for 2 years. He cannot afford to pay the whole amount in cash, but he does have $1000.00 in savings. He is considering his payment options.

    Option 1: Take the dealer's offer of $470.00 a month with $1000.00 down payment.

    Option 2: Borrow $15 450.00 from the bank, add it to his $1000.00 savings, and pay the dealer in cash. His monthly payment to the bank will be $691.85.

    Option 3: Borrow $16 450.00 from the bank, pay the dealer in cash, and keep his $1000.00 in savings. His monthly payment to the bank will be $736.63.

    Marco creates a chart to help him decide which option is best.

	Cash	*Monthly payment ($)*	*Total paid for the loan ($)*	*Total paid for the motorcycle ($)*
Option 1	$1000.00			
Option 2	$1000.00			
Option 3				

Complete the table. Discuss the advantages and disadvantages of each option. Which option would you choose? Explain your choice.

SAMPLE CHAPTER TEST: SOLUTIONS

Part A: Multiple Choice

1. b) A cash withdrawal cannot be made through internet banking.
2. b) Chantal earns \$386.92 interest in 3 years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$5000.00\left(1 + \frac{0.0250}{2}\right)^{(2 \times 3)}$$

$$A = \$5000.00(1 + 0.0125)^6$$

$$A = \$5000.00(1.0125)^6$$

$$A \approx \$5386.92$$

$I = A - P$

$I = \$5386.92 - \$5000.00$

$I = \$386.92$

3. c) Ian pays \$10.41 interest. He pays interest for 19 days (6 days in April and 13 days in May).

$I = Prt$

$I = \$1000.00 \times 0.20 \times (19 \div 365)$

$I = \$10.41$

Part B: Short Answer

4. Jasmine made 12 self-service transactions and 1 full-service transaction. She gets 10 free self-service transactions, so will be charged for 2 self-service transactions and 1 full-transaction.

service fees = (2 × \$0.50) + \$1.00

service fees = \$2.00

Jasmine will pay \$2.00 in service fees.

5. Convert 2.5% to a decimal by dividing by 100.

2.5 ÷ 100 = 0.025

Substitute the known values of the variables into the simple interest formula.

$$I = Prt$$

$$\$75.00 = P \times 0.025 \times 3$$

$$\$75.00 = 0.075P$$

$$\frac{\$75.00}{0.075} = \frac{0.075P}{0.075}$$

$$\$1000.00 = P$$

The principal is \$1000.00.

6. Use the Rule of 72 to estimate how long it will take for an investment to double in value.

72 ÷ interest rate = time in years

72 ÷ 3 = 24

It would take approximately 24 years for an investment at 3.00% per annum to double in value.

7. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$7000.00\left(1 + \frac{0.0450}{1}\right)^{(1 \times 2)}$$

$$A = \$7000.00(1.045)^2$$

$$A \approx \$7644.18$$

The value of the investment after 2 years is \$7644.18.

8. Calculate Saba's new balance.

\$205.50 + \$4.20 + \$354.80 = \$564.50

Calculate 5% (0.05) of the new balance.

0.05 × \$564.50 = \$28.23

This is greater than \$10.00, so Saba must pay \$28.23.

9. a) You must pay \$200.00 plus the fee for each \$100.00 borrowed.

cost of loan = \$200.00 + (2 × \$9.50)

cost of loan = \$200.00 + \$19.00

cost of loan = \$219.00

The loan actually costs you \$219.00.

b) Use the simple interest formula.

$I = A - P$

$I = \$219.00 - \$200.00$

$I = \$19.00$

$I = Prt$

$\$19.00 = \$200.00 \times r \times (5 \div 365)$

$\$19.00 = 2.7397r$

$19 \div 2.7397 = r$

$6.935 = r$

Convert the interest rate to a percentage.

$6.935 \times 100 = 693.5\%$

The annual interest rate is 693.5%.

Part C: Extended Answer

10. Calculate the total cost of each option.

Option 1:

cost = ($470.00/month × 36 months) + $1000.00

cost = $16 920.00 + $1000.00

cost = $17 920.00

Option 2:

cost = ($691.85/month × 24 months) + $1000.00

cost = $16 604.40 + $1000.00

cost = $17 604.40

Option 3:

cost = ($736.63/month × 24 months)

cost = $17 679.12

cost = $17 679.12

Option	*Cash payment ($)*	*Monthly payment ($)*	*Total paid for poan ($)*	*Total paid for motorcycle ($)*
1	$1000.00	$470.00	$470.00 × 36 = $16 920.00	$16 920.00 + $1000.00 = $17 920.00
2	$1000.00	$691.85	$691.85 × 24 = $16 604.40	$16 604.40 + $1000.00 = $17 604.40
3	–	$736.63	$736.63 × 24 = $17 679.12	$17 679.12

Option 1 has the cheapest monthly payment but costs the most overall.

Option 2 is the cheapest overall and has the second lowest monthly payment.

Option 3 has the most expensive monthly payment but does not cut into savings.

Student selection of payment option will vary depending on circumstances.

BLACKLINE MASTER 6.1

CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ What items will you buy? How much will each cost including tax?	
❒ What is the total cash price for your home entertainment centre?	
❒ How long will it take you to save up and pay by cash?	
❒ What is the total interest paid, and the time taken to pay off your debt if the minimum payment only is made each month?	
❒ What are the advantages and disadvantages of paying by credit card?	
❒ How much would you pay using a store promotion?	
❒ How much would you pay using a personal loan?	
❒ How have you decided to pay?	
❒ Why did you choose this method of payment?	

BLACKLINE MASTER 6.2

BUYING A HOME ENTERTAINMENT CENTRE: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research;
- the accuracy of your calculations;
- the effectiveness of your uses of technology for research;
- the creativity you brought to planning and presenting your project; and
- your completion of all the assigned tasks on time.

How do you feel you have done overall, given the criteria above?

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel?

Are there areas in which you could improve?

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

If you had to do the project over again, what would you do differently?

BLACKLINE MASTER 6.3

NORTHWEST BANK OF CANADA ACCOUNT SELECTOR

Name: ______________________ Date: ______________________

NORTHWEST BANK OF CANADA SERVICE PACKAGES

	Value Account	Self-service Account	Full-service Account	Bonus Savings Account
Monthly Fee	$3.90	$10.90 Students and Youth (under 18) save 50% on the monthly fee	$24.50	No fee
Fee waived on minimum monthly balance	$1000.00	$1500.00	$2000.00	
Transactions covered by monthly fee: • cheques • withdrawals • bill payments • debit purchases • transfers to other Northwest Bank of Canada accounts	10 self-service	25 self-service	40 self-service or teller-assisted No annual fee for a credit card	2 debit transactions
Charge for additional transactions not covered by monthly fee	Self-service $0.50 each Teller-assisted $1.00 each	Self-service $0.50 each Teller-assisted $1.00 each	Self-service $0.25 each	Self-service or teller-assisted $1.25 each
Non-Northwest Bank of Canada ATM withdrawals	$1.50 each	$1.50 each		
Interest				Daily interest that grows with your balance

Transaction Types

Self-service: Any transaction that does not require a bank teller. This includes withdrawals, deposits, cheques, money transfers, direct payment purchases, and transactions made at an ATM, by telephone, or online.

Teller-assisted: Includes all transactions that require a teller, such as in-branch withdrawals, transfers, in-branch bill payments, and traveller's cheque and foreign currency purchases.

BLACKLINE MASTER 6.4

ACTIVITY 6.6—THE COMPOUND INTEREST FORMULA

Name: ______________________ Date: ______________________

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A =$

$P =$

$r =$

$n =$

$t =$

If the investment or loan is compounded	*The value of* n *is*
annually	1
semi-annually	
quarterly	4
monthly	
daily	

Term of the investment	*Value of* t
4 years	
6 months	
1 month	
5 months	
1 day	
20 days	

Interest Rate	*Value of* r
10%	
4%	
3.8%	

BLACKLINE MASTER 6.4: SOLUTIONS

ACTIVITY 6.6—THE COMPOUND INTEREST FORMULA

Name: ______________________ Date: ______________________

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = the final amount, or the final value of the investment

P = the principal, or starting value of the investment

r = the annual interest rate expressed as a decimal

n = the number of compounding periods in 1 year

t = the time in years

Interest Rate	*Value of* r
10%	0.10
4%	0.04
3.8%	0.038

If the investment or loan is compounded	*The value of* n *is*
annually	1
semi-annually	2
quarterly	4
monthly	12
daily	365

Term of the investment	*Value of* t
4 years	4
6 months	$\frac{6}{12}$
1 month	$\frac{1}{12}$
5 months	$\frac{5}{12}$
1 day	$\frac{1}{365}$
20 days	$\frac{5}{365}$

BLACKLINE MASTER 6.5

ACTIVITY 6.11—PERSONAL LOAN PAYMENT CALCULATOR TABLE

Name: ______________________ Date: ______________________

PERSONAL LOAN PAYMENT CALCULATOR: MONTHLY PAYMENT PER $1000.00 BORROWED (INTEREST COMPOUNDED MONTHLY)					
Interest rate (%)	*Term in years*				
	1	*2*	*3*	*4*	*5*
3.00	84.69	42.98	29.08	22.13	17.97
3.25	84.81	43.09	29.19	22.24	18.08
3.50	84.92	43.20	29.30	22.36	18.19
3.75	85.04	43.31	29.41	22.47	18.30
4.00	85.15	43.42	29.52	22.58	18.42
4.25	85.26	43.54	29.64	22.69	18.53
4.50	85.38	43.65	29.75	22.80	18.64
4.75	85.49	43.76	29.86	22.92	18.76
5.00	85.61	43.87	29.97	23.03	18.87
5.25	85.72	43.98	30.08	23.14	18.99
5.50	85.84	44.10	30.20	23.26	19.10
5.75	85.95	44.21	30.31	23.37	19.22
6.00	86.07	44.32	30.42	23.49	19.33
6.25	86.18	44.43	30.54	23.60	19.45
6.50	86.30	44.55	30.65	23.71	19.57
6.75	86.41	44.66	30.76	23.83	19.68
7.00	86.53	44.77	30.88	23.95	19.80
7.25	86.64	44.89	30.99	24.06	19.92
7.50	86.76	45.00	31.11	24.18	20.04
7.75	86.87	45.11	31.22	24.29	20.16
8.00	86.99	45.23	31.34	24.41	20.28
8.25	87.10	45.34	31.45	24.53	20.40
8.50	87.22	45.46	31.57	24.65	20.52
8.75	87.34	45.57	31.68	24.77	20.64
9.00	87.45	45.68	31.80	24.89	20.76
9.25	87.57	45.80	31.92	25.00	20.88
9.50	87.68	45.91	32.03	25.12	21.00
9.75	87.80	46.03	32.15	25.24	21.12
10.00	87.92	46.14	32.27	25.36	21.25
10.25	88.03	46.26	32.38	25.48	21.37
10.50	88.15	46.38	32.50	25.60	21.49
10.75	88.27	46.49	32.62	25.72	21.62
11.00	88.38	46.61	32.74	25.85	21.74
11.25	88.50	46.72	32.86	25.97	21.87
11.50	88.62	46.84	32.98	26.09	21.99
11.75	88.73	46.96	33.10	26.21	22.12
12.00	88.85	47.07	33.21	26.33	22.24

BLACKLINE MASTER 6.6

REVIEWING PRIOR CONCEPTS

Name: ____________________ Date: ____________________

Working With Percentages

1. Convert the percent to a decimal.

   a) 25% b) 3% c) 5.5%

   d) 21.5% e) 0.4%

2. Calculate the percentage.

   a) 22% of $5350.00 b) 18.5% of $125.50

   c) 6% of $59.95 d) 1% of $ 2332.00

   e) 0.5% of $90.00

Converting Time To and From Years

3. Convert to years.

a) 8 months

b) 15 months

c) 3 weeks

d) 65 days

e) 150 days

4. Convert from years.

a) 3 years to months

b) 0.6 years to months

c) 3 years to days

d) 0.2 years to days

e) 15 years to months

Using Algebra to Solve for One Unknown Variable

5. Solve for the unknown variable.

   a) $w = xyz$

   Where:

   $w = 12$

   $x = 3$

   $y = 10$

   Solve for z.

   b) $x^2 + 3(y - 5w) = 7(3 + z) + 1$

   Where:

   $w = 3$

   $x = 5$

   $z = 2$

   Solve for y.

   c) $2w = (x - 3) + yz^3$

   Where:

   $w = 24$

   $z = 8$

   $y = 10$

   Solve for x.

BLACKLINE MASTER 6.6: SOLUTIONS

1. a) $25\% = 25 \div 100$

   $25\% = 0.25$

   b) $3\% = 3 \div 100$

   $3\% = 0.03$

   c) $5.5\% = 5.5 \div 100$

   $5.5\% = 0.055$

   d) $21.5\% = 21.5 \div 100$

   $21.5\% = 0.215$

   e) $0.4\% = 0.4 \div 100$

   $0.4\% = 0.004$

2. a) $22 \div 100 = 0.22$

   $0.22 \times \$5350.00 = \$1177.00$

   b) $18.5 \div 100 = 0.185$

   `$0.185 \times \$125.50 = \$23.22$

   c) $6 \div 100 = 0.06$

   $0.06 \times \$59.95 = \$3.60$

   d) $1 \div 100 = 0.01$

   $0.01 \times \$2332.00 = \$23.32$

   e) $0.5 \div 100 = 0.005$

   $0.005 \times \$90.00 = \$0.45$

3. To convert from months, divide by 12.

   To convert from weeks to years, divide by 52.

   To convert from days to years, divide by 365.

   a) 8 months = $8 \div 12$

   8 months = 0.67 years

   b) 15 months = $15 \div 12$

   15 months = 1.25 years

   c) 3 weeks = $3 \div 52$

   3 weeks = 0.06 years

   d) 65 days = $65 \div 365$

   65 days = 0.18 years

   e) 150 days = $150 \div 365$

   150 days = 0.41 years

4. To convert years to months, multiply by 12.

   To convert years to days, multiply by 365.

   a) 3 years = 3×12

   3 years = 36 months

   b) 0.6 years = 0.6×12

   0.6 years = 7.2 months

   c) 3 years = 3×365

   3 years = 1095 days

   d) 0.2 years = 0.2×365

   0.2 years = 73 days

   e) 15 years = 15×12

   15 years = 180 months

5. a) $w = xyz$

   $12 = 3 \times 10 \times z$

   $12 = 30z$

   $12 \div 30 = z$

   $0.4 = z$

   b) $x^2 + 3(y - 5w) = 7(3 + z) + 1$

   $5^2 + 3(y - 5 \times 3) = 7(3 + 2) + 1$

   $25 + 3(y - 15) = 36$

   $25 + 3y - 45 = 36$

   $3y - 20 = 36$

   $3y = 36 + 20$

   $y = 56 \div 3$

   $y \approx 18.7$

   c) $2w = (x - 3) + yz^3$

   $2(24) = (x - 3) + (10)(8)^3$

   $48 = (x - 3) + (10)(512)$

   $48 = x - 3 + 5120$

   $48 = x + 5117$

   $48 - 5117 = x$

   $-5069 = x$

ALTERNATIVE CHAPTER PROJECT—WISE MONEY MANAGEMENT

TEACHER MATERIALS

GOALS: To synthesize the concepts learned about financial institutions and services, credit options, store promotions, loans, and savings to create an informative presentation on wise money management for young consumers.

OUTCOME: In this project, students will create an information brochure or use presentation software to provide information on financial services, credit, and investment options to high school students.

PREREQUISITES: To complete this project, students will need to have knowledge about the various financial options available in Canada today. Students should use the internet to research banking, credit, loan, and savings options. They will summarize their findings in a handout or presentation for an audience of high school students. Students will need to be able to write clear and concise questions and answers on the topic of wise money management, and investigate loan and savings options. It would be very helpful for students to be familiar with word processing and presentation software.

ABOUT THIS PROJECT: This project provides students with a chance to write about and creatively present what they have learned in the chapter in very practical terms. This project can provide an opportunity for teachers to assess student understanding and knowledge contained within the Discuss the Ideas activities of the chapter. This project can be done in small groups, so that students can debate and discuss their work.

1. Start to plan

Introduce the project as you start the chapter. Ask students to start thinking of questions and how they might answer them in the discussion activities of each lesson. As a class, they can brainstorm various questions, or they can conduct an informal survey of students asking them what questions they think are important. Ensure that students have included questions regarding the following topics: financial institutions, credit options, store promotions, loans, and wise money management. You may wish to instruct students to create a minimum number of questions.

Students can use the student resource, the internet, or other sources to research their questions and answers.

2. Research loan options

In this section, students will research the overall cost of two different types of loans. They will begin by selecting a savings goal that would be of interest to high school students, such as purchasing a new car or computer, or saving up money for post-secondary school tuition. Students should research the cost of the item or goal.

Next, students will choose two ways of borrowing money for the savings goal, such as taking out a line of credit or using a credit card. They will calculate the overall cost of making the purchase using their selected options.

Finally, students will decide which loan option they would recommend to other students. They should base their decision on interest owed and minimum monthly payments.

3. Research investment options

In this section, students will research the information needed to inform others about wise investment strategies and planning for the future. They will investigate investment options, and calculate the interest earned on two different investments: $1000.00 invested for 35 years; and $2000.00 invested for 10 years.

Students can consider various types of investment, including bank savings accounts, Guaranteed Investment Certificates, Canada Savings Bonds, and Tax-Free Savings Accounts. There are many investment options that have not been discussed in this chapter, but that students can research for the project if they wish.

Students should analyze their interest calculations, and make a recommendation about whether others would be better off investing a small amount of money for a longer term, or a larger amount of money for a shorter term.

4. Make a presentation

Students will complete the project by creating a pamphlet or using presentation software to present the information. They should keep in mind that the presentation needs to be attractive and interesting to high school students. It should encourage others to take an interest in managing their money.

Students have the opportunity to be creative with their designs. Encourage neatness of presentation, use of colour, clarity of design, and supplementary illustrations. The information about loans and investments could be presented using graphs.

The presentation should include:

- an eye-catching title;
- a list of FAQs and answers;
- a slide or handout that shows which type of loan is a better option; and
- a slide or handout that shows whether it is better to invest a small amount of money for a longer term, or a larger amount of money for a shorter term.

Remind students of the requirements for a complete project and discuss with them how the project will be assessed. Provide students with a copy of Blackline Master 6.1A (p. 418) to give them an opportunity to reflect on the quality of their work.

ASSESSING THE PROJECT

1. Start to plan

- Check that students have questions covering the main topics of the project, and that the questions are written in their own words.
- Provide students with information on how they will be assessed for this project. You may wish to brainstorm with the students themselves when developing this assessment. What do they think is important?

2. Research loan options

- Students should select two different loan options with different interest rates. They should investigate actual interest rates, and should provide the source (for example, the name of the bank and URL for the loan information).
- Check that students' calculations of interest are accurate.
- Students should recommend that others select the loan option with the lowest overall cost.

3. Research investment options

- Students should investigate interest rates and investment terms for different savings options, and choose ones that suit the specified principals and terms.
- Students may wish to consider investment options not discussed in this chapter. Evaluate whether they have noted realistic interest rates and investment terms.
- Check that students' calculations of interest are accurate.

4. Make a presentation

- Check that students have written answers in their own words, rather than copying information directly from the internet.
- You may wish to have the projects evaluated by their peers. If so, have students set up the projects on their desks if they have created pamphlets. Allow students to circulate to view all the projects. For students with presentation software, arrange for a projector to be available so that the class can view the work of their peers.
- Ask students to self-assess their project using Blackline Master 6.1A (p. 418) and 6.1B (p. 419).

PROJECT ASSESSMENT RUBRIC: WISE MONEY MANAGEMENT				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
CONCEPTUAL UNDERSTANDING				
• Explanations show understanding of financial services available, credit card options, store promotions, loans, and wise money management • Loan and investment presentations show understanding of interest calculations	shows very limited understanding; questions and explanations are omitted or inappropriate; interest calculations are omitted or incorrect	shows partial understanding; questions and explanations are appropriate but some are inaccurate; interest calculations are partially correct	shows understanding; questions and explanations are appropriate; interest calculations are mostly correct	shows thorough understanding; questions and explanations are effective and thorough; interest calculations are correct
PROCEDURAL UNDERSTANDING				
Accurately: • gathers information from a variety of reliable sources • organizes research using an appropriate format • answers FAQs • calculates interest on loans and investments	limited accuracy; major errors or omissions For example: • questions cover only a few of the required topics • answers to FAQs are incorrect • loan and investment options are not analyzed • interest calculations are missing or incorrect	partially accurate; some errors or omissions For example: • questions cover most of the required topic • answers to FAQs are partially correct • loan and investment options are partially analyzed • interest calculations are included but partially incorrect	generally accurate; few errors or omissions For example: • questions cover all required topics and answers are complete and accurate • loan and investment options are well researched and interest calculations are correct • project is complete and accurate, but there is nothing beyond what is required	accurate and precise; very few errors or omissions For example: • shows creativity in question topics • answers to FAQs are complete, accurate, and go beyond what is required • loan and investment presentations are creative and informative
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems and explain the solutions • Correctly analyzes loan and investment options and accurately calculates interest • recommendation of loan/investment is logical and supported by calculations	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly using appropriate financial and mathematical terminology	does not present work and explanations clearly; uses few appropriate financial and mathematical terms	presents work and explanations with some clarity, using a few appropriate financial and mathematical terms	presents work well and explanations clearly, using appropriate financial and mathematical terms	presents work and explanations precisely, using a range of appropriate financial and mathematical terms

ALTERNATIVE CHAPTER PROJECT—WISE MONEY MANAGEMENT

STUDENT MATERIALS

PROJECT OVERVIEW

In this project, you will be a financial planner and will prepare a presentation for high school students about wise money management.

You will create an attractive handout or presentation that provides advice for young consumers on managing their money wisely. You may wish to use presentation software to present your project. Your handout or presentation will have three parts: a list of frequently asked questions (FAQs) about financial institutions and their answers; one presentation slide or handout that shows the advantages and disadvantages of two types of loans; and one presentation slide or handout that shows the effects of two types of investment.

GET STARTED

To begin the project, brainstorm questions that should be included in your FAQs. You should consider the following questions, and create one or two of your own.

- What kind of financial institution should I choose?
- What kind of account should I choose?
- How can I keep my personal and financial information secure?
- How can I use credit effectively?
- What advice can you give about store promotions?
- What kind of financial commitments should I avoid?

Your questions should cover the following broad topics: financial institutions, credit options, store promotions, and loans. You should also include questions on wise money management and how to avoid debt. If you wish, you can ask your classmates what questions they have about money management.

Use the internet to research answers to your questions. As you research your answers, you may find more questions to ask. Write answers to each question in your own words. Answers need to cover the basic facts, but should be no more than a few sentences or a short paragraph.

RESEARCH LOAN OPTIONS

The next step in the project is to research different loan options.

Begin by choosing something that students might take out a loan for—for example, a car, a computer, or a post-secondary school program. Research the cost.

Next, choose two ways a student could borrow money to pay the cost. Payment options include taking out a line of credit or a student loan, or using a credit card. Research and compare the costs of the two loans you have chosen. For example, what is the interest rate on the line of credit compared to the credit card?

Which option you would recommend students choose, and why?

RESEARCH INVESTMENT OPTIONS

Next, you will research the information needed to inform students about wise investment strategies and planning for the future.

Choose one type of investment, such as a Guaranteed Investment Certificate or a bank savings account. Research interest rates and terms of investment, and calculate the interest earned on two different investments:

- $1000.00 invested for 35 years; and
- $2000.00 invested for 10 years.

If students want to save money for the future, would they be better off investing a small amount of money for a longer term, or a larger amount of money for a shorter term?

MAKE A PRESENTATION

You are now ready to create your presentation. Decide whether you will present your information as a pamphlet or using presentation software.

Your presentation needs to be attractive and interesting to high school students. It should encourage them to take an interest in managing their money.

Your presentation should include:

- an eye-catching title;
- a list of FAQs and answers;
- a slide or handout that shows which type of loan is a better option; and·
- a slide or handout that shows whether it is better to invest a small amount of money for a longer term, or a larger amount of money for a shorter term.

BLACKLINE MASTER 6.1A

WISE MONEY MANAGEMENT: STUDENT SELF-ASSESSMENT

Name: ______________________________ Date: ______________________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness with which you created your FAQs and loan and investment recommendations;
- the accuracy of your calculations;
- the effectiveness of your uses of technology for completing research;
- the creativity you brought to the project; and
- your completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________________

____

____

____

Were you able to complete all aspects of the project? If not, why? Did you allot your time effectively?

____

____

In what areas did you excel? ______________________________

____

____

Are there areas in which you could improve? ______________________________

____

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

____

____

____

If you had to do the project over again, what would you do differently?

____

____

____

BLACKLINE MASTER 6.1B

ALTERNATIVE CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❐ Do your FAQs cover all the required topics (financial institutions, credit options, store promotions, loans, wise money management, how to avoid debt)?	
❐ Are the answers to your FAQs well researched and accurate?	
❐ Did you research the cost of an item or goal that a student might save money for?	
❐ Did you research two different loan or credit options and record their interest rates?	
❐ Did you calculate the interest for two different loan options? Did you calculate which option is less expensive, and make a recommendation to students about which loan to choose?	
❐ Did you research two different investment options and record their interest rates?	
❐ Did you calculate the final value of the two investment options and make a recommendation about which to choose?	
❐ Did you create an attractive presentation of all the information you gathered?	

Chapter 7

Personal Budgets

INTRODUCTION

STUDENT BOOK, pp. 300–343

This chapter of *MathWorks 11* addresses the outcomes of developing number sense and critical thinking skills for Workplace and Apprenticeship Mathematics 11. In this chapter, students will build on their knowledge of compound interest and financial mathematics to explore the concept of personal budgeting. Students will learn the vocabulary of budgeting, how to make a budget, and how to analyze a budget to meet future goals. The chart below locates this chapter within the curriculum.

NUMBER, GRADES 10–12

This chart illustrates the development of the Number strand in the Workplace and Apprenticeship pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 7 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop number sense and critical thinking skills.	**General Outcome** Develop number sense and critical thinking skills.	**General Outcome** Develop number sense and critical thinking skills.
Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:	**Specific Outcome** It is expected that students will:
Demonstrate an understanding of income, including wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.	Solve problems that involve personal budgets.	Solve problems that involve the acquisition of a vehicle by buying, leasing, and leasing to buy.
		Critique the viability of small business options by considering expenses, sales, and profit or loss.

STATISTICS, GRADES 10–12

This chart illustrates the development of the Statistics strand in the Workplace and Apprenticeship pathway through senior secondary school. The highlighted cells contain the outcomes that chapter 7 addresses.

Grade 10	Grade 11	Grade 12
General Outcome Develop statistical reasoning.	**General Outcome** Develop statistical reasoning.	**General Outcome** Develop statistical reasoning.
	Specific Outcome It is expected that students will:	**Specific Outcome** It is expected that students will:
	Solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.	Solve problems that involve measures of central tendency, including mean, median, mode, weighted mean, and trimmed mean.
		Analyze and describe percentiles.

CURRICULUM AND CHAPTER OVERVIEW

General Outcome:
Develop number sense and critical thinking skills.

Specific Outcome:
Solve problems that involve personal budgets.

General Outcome:
Develop statistical reasoning.

Specific Outcome:
Solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.

Section 7.1:
Preparing to Make a Budget

- Math on the Job
- Explore the Math
- Discuss the Ideas: Variable Income
- Mental Math and Estimation
- Activity 7.1: Balancing Income and Expenses
- Build Your Skills

Section 7.2:
The Budgeting Process

- Math on the Job
- Explore the Math
- Discuss the Ideas: Starting a Budget
- Mental Math and Estimation
- Mental Math and Estimation
- Activity 7.2: Financial Planning for a Sport or Hobby
- Build Your Skills

Section 7.3:
Analyzing a Budget

- Math on the Job
- Explore the Math
- Discuss the Ideas: Assessing Income and Expenses
- Discuss the Ideas: Pay Yourself First
- Mental Math and Estimation
- Activity 7.3: Budgeting Using Spreadsheets
- Mental Math and Estimation
- Build Your Skills
- Puzzle It Out: The Disappearing Dollar
- The Roots of Math: The Origins of the Electronic Spreadsheet

Chapter Project: Budgeting to Live Away from Home
Reflect on Your Learning
Practise Your New Skills

THE MATHEMATICAL IDEAS

PERSONAL BUDGETS

Throughout this chapter, students will work primarily with income and expense data to solve problems in basic finance. They will use algebra to set up and solve proportions, determine percent increases and decreases, and solve problems involving compound interest. Students will learn how to research and collect data to make an accurate budget, how to analyze and organize existing data to make a budget, and how to represent the data in a meaningful way. Students will use their graphing skills to produce circle graphs, which provide a powerful way to visualize categories of spending. For example, if a circle graph indicates that a person is spending 35% of their income on recreation and entertainment, it will be apparent that these items are incorrectly prioritized. When preparing a budget, essentials such as food, housing, transportation, and clothing generally account for the highest percentages of expenses.

A budget is a plan for future income and spending. Personal budgets are often inaccurate because people tend to underestimate spending and overestimate income. Therefore, the first step in creating a budget is to keep records of past income and expenses. Students will work with income and expense data that are provided, and they will research and document their own current and potential spending and income.

When preparing a budget, income and expenses must be balanced. This concept is central to this chapter on personal budgeting. In real life, a person's income and expenses are seldom exactly balanced. When making a budget, however, adjustments are made in order to balance income and expenses. This is because a budget is a plan for how you intend to "live within your means," which is to say, not exceed your income. In general, variable expenses must be reduced if the expenses are too high in a draft budget. If there is a surplus of income, it can be allocated to "savings." It is important to understand that "savings" is an expense in a budget. You are putting money away; therefore it is not available for current spending. The financial principle "pay yourself first" is based on this understanding. Paying yourself first means automatically putting aside money for savings from each paycheque, as if "savings" were a bill to be paid.

Expenses and income are often written as percents, so students will work with percents and solving proportions in order to calculate future spending and savings. Students will also activate their prior knowledge of compound interest and solving proportions in order to solve budgeting problems.

Spreadsheets may be used to help with designing and balancing budgets. Formatting the cells in a spreadsheet is advantageous when making small changes to a budget because the effect of one or more changes is automatically reflected throughout the budget. Some students may be familiar with spreadsheet use; for others, a worked example and practice questions will help them develop these skills. All calculations in this chapter can be done using a scientific calculator, but using spreadsheet technology is recommended as an option. Scientific calculators are used for calculations involving exponents, particularly with reference to compounding interest on investments or loans. Here again, spreadsheets offer a useful alternative way to solve such problems.

WHY ARE THESE CONCEPTS IMPORTANT?

- High personal debt and financial difficulties are all too common in our society; therefore, enabling students to understand the basics of financial planning will help them secure their financial future.
- Budgets provide students with a template to keep track of their income and spending, which facilitates good money management and helps avoid financial setbacks.
- Analyzing financial data, prioritizing expenses, and presenting income and expenses graphically can help students think critically and conceptually about their finances.

PRIOR SKILLS AND KNOWLEDGE

Students may be familiar with some of the ideas in this chapter as a result of life skills and career planning courses they have taken. Some students will already be working at part-time jobs and may have made budgets for themselves.

Students will likely have been exposed to the following concepts and may have developed these skills in earlier grades, or earlier in this program.

1. Concepts
   a) ratios and rates;
   b) proportional reasoning;
   c) percent;
   d) calculating averages;
   e) compound interest.
2. Mathematics Skills
   a) proportional reasoning;
   b) solving algebraic equations;
   c) identifying equivalent ratios;
   d) working with fractions, decimals, and percents;
   e) creating data tables;
   f) graphing.

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3. Technology
   a) basic calculator functions;
   b) scientific calculator functions (exponents);
   c) internet research;
   d) presentation software.

REVIEWING PRIOR CONCEPTS

Some students may benefit from reviewing concepts that have been covered in prior chapters or prior years. You may want to give some students specific review exercises in the following concepts and processes.

7.1 Preparing to Make a Budget

- expressing a numerical value as a fraction, ratio, or decimal, and converting between them;
- percent increases and decreases;
- using algebra to solve for one unknown variable; and
- proportional reasoning.

7.2 The Budgeting Process

- percent;
- averages;
- compound interest; and
- using spreadsheets.

7.3 Analyzing a Budget

- creating and interpreting graphs.

Blackline Master 7.8 contains review questions and solutions. It is found on p. 471 of this teacher resource.

PLANNING CHAPTER 7

This chapter will take four weeks of class time to complete. Class period estimates are based on a class length ranging from 60 to 75 minutes. These estimates may vary depending on individual classroom needs.

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
	301	Introduce the Chapter Project: Budgeting to Live Away from Home	20 minutes for class discussion	Blackline Master 7.1 (p. 464)
7.1	302 302 303 304	Math on the Job: Rack and Shelving Installer Explore the Math Discuss the Ideas: Variable Income Examples 1, 2	60 minutes	
7.1	306 307 308	Mental Math and Estimation Example 3 Activity 7.1: Balancing Income and Expenses	1 class	 Blackline Master 7.4 (p. 467)
7.1	309	Build Your Skills	1 class	
7.2	313 313 314	Math on the Job: Bead Maker Explore the Math Discuss the Ideas: Starting a Budget	1 class	
7.2	315 318 318 320	Example 1 Mental Math and Estimation Example 2 Mental Math and Estimation	1 class	
7.2	320 321 325	Activity 7.2: Financial Planning for a Sport or Hobby Build Your Skills Chapter Project: Gather Data	1 class	Blackline Master 7.3 (p. 466) Blackline Master 7.5 (p. 468)
7.3	326 327 327 328	Math on the Job: Illustrator/Cartoonist Explore the Math Discuss the Ideas: Assessing Income and Expenses Example 1	1 class	
7.3	329 329 332	Discuss the Ideas: Pay Yourself First Example 2 Mental Math and Estimation	1 class	
7.3	333 334	Activity 7.3: Budgeting Using Spreadsheets Mental Math and Estimation	1 class	spreadsheet software Blackline Master 7.6 (p. 469)
7.3	334 337 338	Build Your Skills Puzzle It Out: The Disappearing Dollar The Roots of Math: The Origins of the Electronic Spreadsheet	1 class	scientific calculator internet access

PLANNING FOR INSTRUCTION

Section	*Student book page*	*Lesson focus*	*Estimated time*	*Materials*
7.3	339	Chapter Project: Prepare a Budget and Make a Presentation	1 class	Blackline Master 7.2 (p. 465)
	340 340	Reflect on Your Learning Practise Your New Skills	1 class	scientific calculator Blackline Master 7.7 (p. 470)
		Chapter Test (p. 456 of this resource)	1 class	

PLANNING FOR ASSESSMENT

Purpose	*In the chapter*	*Teacher notes*
Assessment for Learning	• Chapter overview and project discussion • Math on the Job • Examples • Discuss the Ideas • Explore the Math	• Monitor how students take on responsibility and workload when participating in group work. • Review concepts and mathematical terms to clarify understanding. • Provide quick quizzes to assess student understanding. • Review student work on the project throughout to see how they are progressing, and to determine if more class time needs to be devoted to the project.
Assessment as Learning	• Explore the Math • Activities • Chapter Project	• Provide examples of strong and weak responses to guide student work. • Have students solve problems in groups and present their solutions to the class; use this as an opportunity to determine successful problem-solving strategies. • As a class, review mathematical terminology to determine any misconceptions. • Record student scores on Build Your Skills problems to assess which students need assistance or review work.
Assessment of Learning	• Activities • Chapter Project • Quizzes • Chapter Test	• Ask students to present activities to the class and use this as an opportunity to assess the level of learning that has taken place.
Learning Skills/ Mathematical Disposition	• Mathematical literacy, classroom participation, use of new mathematical skills, and use of technology	• Monitor how successfully students use data to draw up a personal budget and make budget-related calculations. • Observe how effectively students use software to make personal budgets. • Note which students participate and co-operate in group work, and which students need to improve these skills. • Encourage students to use the mathematical terms discussed in class.

CHAPTER PROJECT—BUDGETING TO LIVE AWAY FROM HOME

GOALS: To design a reasonable and accurate personal budget for living away from home.

OUTCOME: In this project, students research the costs involved with living on their own and the income they will earn in a chosen occupation. Students will then present a balanced budget to their peers that details the income and expenses associated with living on their own and working in their chosen career.

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PREREQUISITES: Students may find it helpful to use spreadsheet software in preparing their budgets, but the budget can be done manually. Students may already be familiar with the basics of spreadsheet formatting, or this can be covered in class.

Students may also use presentation software for their final presentation, so experience with this type of application will be an asset.

Students will need to have access to the internet to research income and expenses for different regions in Canada. You can guide students to use available search engines to help them with their research. Students should record their research sources (URLs) in a document, using a word processing program.

ABOUT THE PROJECT: This project is divided into three sections. Initially, students consider all the monthly expenses they may encounter when they live on their own. Then they research income and expenses for their chosen occupation and location. Finally, they design a budget that is accurate, reasonable, and balanced, and present their budget to the class.

When students present their budgets to the class, allow 3–5 minutes per student. Students should be given a few class periods to work on this project. Interim guidance can help students complete the project more successfully. This project could be completed by pairs or small groups of students where the groups would have to create a budget for one fictitious individual.

An assessment rubric for this project is included and should be handed out to students early in the project. The rubric outlines the criteria for evaluation of their project and suggests some ways in which students can reflect on their learning.

An alternative project, "Budget for a Vacation," is included on pp. 478–486.

1. Start to plan

STUDENT BOOK, p. 301

This initial part of the project allows for group brainstorming as a class. Start with a discussion of what types of expenses one would have when living alone. How much would each of these expenses cost? Discuss possible occupations for students after their schooling.

Encourage students to be realistic and reasonable in this discussion, as the project will be more rewarding when completed realistically. Explain to students that they will research actual figures with respect to income and expenses, and they will need to include sources for all of their information in their final presentation.

2. Gather data

STUDENT BOOK, p. 325

This segment of the project requires students to practise their research skills. Students will research income and expenses for their chosen career and location and record their findings. Let students know that they do not need to make an exhaustive list of their expenses. They can use the categories suggested or create one or two additional categories.

Students should also be reminded that they are not creating a budget here, so they do not need to check that income and expenses balance; this is for the next stage in the project.

3. Prepare a budget and make a presentation

STUDENT BOOK, p. 339

During the first part of this section, students will use their research to design a balanced monthly budget. Students may use calculators to prepare a manual budget and draw a circle graph by hand. However, they could also follow the instructions for using a spreadsheet, formatting the necessary cells, and create a circle graph using a spreadsheet or graphing program.

Time for this section should be divided evenly between building the budget template with spreadsheet software and adjusting the budget entries to create a reasonable and balanced budget.

Students will synthesize their planning and research activities and prepare their presentations. Their presentation can include electronic presentations, posters, or paper budgets.

Provide students with a copy of Blackline Master 7.2 (p. 465) to give them an opportunity to reflect on the quality of the work they put into their project.

ASSESSING THE PROJECT

1. Start to plan

- Discuss with students how they will make a balanced monthly budget based on researched real-life data as much as possible. Students must research all of the expenses related to the scenario and the actual income for the occupation they choose.
- Let students know how they will be graded. You can refer to the criteria on the project assessment rubric on p. 428, or use your own criteria to grade the project.

2. Gather data

- After this section of the project has been completed, you can ask students to hand in their research findings on income and expenses for their chosen occupation and location. To assess the quality of research, consider the reasonableness of the salaries, check the sources for their research, and comment on the accuracy of projected expenses.
- You can decide and inform students if this work will be part of their final grade or if it will be used as an assessment for learning. If you choose to use this step as an assessment for learning, you can hand the documents back and give general or individual suggestions on how to improve research findings.

3. Prepare a budget and make a presentation

- Give students class time in a computer lab to complete their budgets and make adjustments to balance the budget.
- Note whether students were prepared to answer questions from classmates regarding specific expenses or income for the occupation, and provide reasons why certain expenses lie outside the recommended guidelines.

PROJECT EXTENSION

As an extension, students can interview an individual who is in a situation similar to the one they researched for their project. They can then write a summary describing how their budget is similar and how their budget is different from that of the person they interviewed.

CHAPTER PROJECT ASSESSMENT RUBRIC				
	Not Yet Adequate	*Adequate*	*Proficient*	*Excellent*
CONCEPTUAL UNDERSTANDING				
• Explanations show an understanding of balanced personal budgeting	Shows very limited understanding; explanations are omitted or inappropriate	Shows partial understanding; explanations are often incomplete or somewhat confusing	Shows understanding; explanations are appropriate	Shows thorough understanding; explanations are effective and thorough
PROCEDURAL KNOWLEDGE				
• Accurately: ▪ researches expense data ▪ accounts for all expenses ▪ documents all sources ▪ constructs a balanced budget ▪ calculates appropriate percentages for the circle graph ▪ calculates appropriate angles for the circle graph ▪ draws circle graph ▪ summarizes and analyzes the budget ▪ presents project	limited accuracy; major errors or omissions For example: • expense data are not reasonable • expenses are missing from the budget • sources are not documented • budget is not balanced • most expense percentages are incorrect • circle graph is not drawn accurately • summary and analysis are incomplete • project is not presented • project is incomplete	partially accurate; some errors or omissions For example: • some expense data are not reasonable • most expenses listed in budget • some sources are not documented • budget is balanced with one or two small arithmetic errors • some expense percentages are incorrect • circle graph is drawn accurately • summary and analysis are complete • project presented with poor sequence or no visual support • project is mostly complete	generally accurate; few errors or omissions For example: • expense data are mostly reasonable • all expenses are listed in budget • most sources are documented • budget is balanced • expense percentages are correct • circle graph is drawn accurately • summary and analysis are complete • project is presented with few gaps, good sequence, and visual support • project is complete	accurate and precise; very few or no errors For example: • expense data are reasonable • all expenses are listed in budget • all sources are well documented • budget is balanced and error free • expense percentages are correct • circle graph is drawn accurately and neatly • summary and analysis are complete and well-written • presentation is thorough and has excellent visual support • project is complete
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

7.1 Preparing to Make a Budget

TIME REQUIRED FOR THIS SECTION: 3 CLASSES

STUDENT BOOK, pp. 302–312

MATH *ON THE JOB*

STUDENT BOOK, p. 302

Start this section by discussing different methods for saving. As a class, brainstorm and list these ideas. Ask students how many of them use these saving methods and determine the most popular method. Ask students to suggest positive and negative effects of the different saving methods.

For example, a positive effect of saving money in a savings account is that you will earn interest, but a negative effect is that the interest rate is much lower than with some other savings methods such as GICs.

SOLUTIONS

1. Divide to determine how much Katie will need to save each month.

   $9000.00 ÷ 12 = $750.00

   Katie will need to save $750.00 each month.

2. Divide to find Katie's monthly earnings.

   $32 000.00 ÷ 12 ≈ $2666.67

   Calculate what percent $750.00 is of this amount.

   ($750.00 ÷ $2666.67) × 100 ≈ 28%

   Katie will need to save about 28% of her monthly income.

3. If Katie were paid different amounts each month, she might decide to save a certain percentage of her monthly earnings, regardless of what they were. She could also set a savings goal for the year, and a savings goal for each three- or four-month period.

EXPLORE THE MATH

STUDENT BOOK, p. 302

In this section, students will work with records of income and expenses and learn to categorize them in standard ways used in budgets. Introduce the terms "regular income" and "variable income"; students will have encountered some concepts related to income in *MathWorks 10,* chapter 2, Earning an Income. Ask them what they think "recurring expense," "variable expense," and "unexpected expense" mean.

A key topic to emphasize in this chapter is that budgets are forward-looking. They are projections of what you expect your income and expenses will be for a given time period. Records of past income and expenses are not budgets; they are financial records that provide useful information for preparing to make a budget.

SOLUTIONS

Answers concerning the different expenses will vary. Sample answers follow.

A ski trip to Whistler can be an unexpected expense if you had not planned and saved money to go. It could also be a recurring expense if you have a ski pass and plan to go every second weekend during the winter months.

Replacing your broken eyeglasses is an unexpected expense because you don't plan on this happening.

If your records show that you are spending more than you earn, you need to find categories of expenses that you can reduce, or you need to increase your income. Usually, adjusting some variable expenses is the most straightforward way to balance a budget.

When you prepare a budget, you balance it because the aim is to account for all your income and expenses. In real life, often either expenses will be lower than income, in which case you have a surplus, or expenses will be higher than income, which means you have a deficit and are accumulating debt. In a budget document, you create a category such as "savings" or "miscellaneous expenses" that account for all your income. You don't have to spend all the money you earn; you just have to account for it in your budget. Adjusting budgets so they are balanced will be covered in more detail later in the chapter.

Examples in this section model categorizing income and expenses and calculations related to savings plans.

DISCUSS THE IDEAS

VARIABLE INCOME

STUDENT BOOK, p. 303

This discussion provides an opportunity for students to explore their prior knowledge and understanding of percentages and proportional reasoning to solve a problem. It encourages students to think through problems that they may be faced with in reality.

SOLUTIONS

Answers will vary. Sample answers follow.

1. Wayne's expenses are likely to be more consistent than his income because he will have to pay for expenses such as rent, food, utility bills (telephone, cable, internet) and transportation each month. These expenses are likely to be fairly consistent from month to month. He will also have variable expenses such as purchasing new items, which he can buy when he has more income.
2. A record of his past income and expenses helps Wayne determine the categories for his budget and estimate what the expenses will be. Without such records, he would be guessing at how much he is spending, and he could forget to include some expenses.
3. If Wayne knows his income will be lower some months, he could save more during the months when his income is higher so that he has some savings. This will help him to meet all his expenses without going into debt. He could also delay major purchases until he has enough money saved.

Mental Math and Estimation

STUDENT BOOK, p. 306

In the previous chapter, students have worked with percentages and interest. Give them a few minutes to estimate the solution on their own. If they need assistance, let them know that by solving simpler smaller problems, they should be able to use these answers as benchmarks for their final estimate.

SOLUTIONS

Add to calculate Yuri's savings. Students can add the exact amounts in their heads and arrive at $6300.00.

$\$6120.00 + \$180.00 = \$6300.00$

Or they can round $180.00 up to $200.00 to produce the approximate amount of $6320.00.

By moving the decimal point in $6300.00, students can estimate that 10% of $6300.00 is $630.00. They can double this to find that 20 percent of Yuri's savings is equal to $1260.00.

$\$6300.00 \times 0.20 = \$1260.00$

ACTIVITY 7.1
BALANCING INCOME AND EXPENSES

STUDENT BOOK, p. 308

T This activity is intended to give students an opportunity to work with the new vocabulary in this section and to introduce the concept of prioritizing expenses. You may want to discuss with students the difference between "needs" and "wants," and how this can help them decide what expenses can be eliminated or reduced. You can have students brainstorm all possible expenses in a typical month. Then decide as a class what expenses are "needs" and what expenses are "wants." Finally, you can prioritize the "wants" according to importance, and decide on which ones could be reduced or eliminated altogether when faced with a financial crisis.

Copy and handout Blackline Master 7.4 (p. 467) for this activity and ask students to work in pairs to complete it.

SOLUTIONS

1.

JACKSON'S MONTHLY INCOME AND EXPENSES						
Income		*Regular or variable*	*Expenses*		*Recurring or variable*	*Priority*
Tips	$220.00	V	Rent	$500.00	R	1
Paycheque	$800.00	R	Food	$400.00	V	2
House-painting	$950.00	V	Entertainment	$75.00	V	11
Tips	$175.00	V	Loan payment	$300.00	R	4
Paycheque	$800.00	R	Cost of new bicycle	$1250.00	V	10
			Clothing	$50.00	V	9
			Car insurance	$180.00	R	5
			Gas	$150.00	V	7
			Cell phone	$74.00	V	6
			Car loan	$300.00	R	3
			Charitable donations	$38.00	V	8
Total	$2945.00		*Total*	$3317.00		

2. See table above.
3. Add to calculate Jackson's total income.

   $220.00 + $800.00 + $950.00 + $175.00 + $800.00 = $2945.00

   Add to calculate Jackson's total expenses.

   $500.00 + $400.00 + $75.00 + $300.00 + $1250.00 + $50.00 + $180.00 + $150.00 + $74.00 + $300.00 + $38.00 = $3317.00

   Subtract Jackson's expenses from his income.

   $2945.00 – $3317.00 = –$372.00

   Jackson's expenses are $372.00 higher than his income, so they are not balanced.
4. Answers will vary. See table for possible answers.
5. Answers will vary. Students might suggest that Jackson could balance his budget by saving up to buy his bike rather than paying it all out of one month's income. He could also reduce his spending on entertainment, clothing, and gas.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 309

1. Answers may vary for the prioritization of the expenses.

MONTHLY INCOME AND EXPENSES					
Regular		*Variable*		*Unexpected*	
Priority	Expense	Priority	Expense	Priority	Expense
1	Rent: $760.00	5	Birthday gift: $42.00	4	Muffler: $135.00
8	Savings: $50.00	7	New TV: $799.00		
2	Insurance: $71.00	6	Gas for car: $68.00		
3	Telephone bill: $55.00				

2. Divide to calculate how much Doug saves every month.

   $3500.00 ÷ 12 months = $291.67 per month.

   Let x be the number of months that Doug needs to save.

   $$\frac{x}{\$2200.00} = \frac{1}{\$291.67}$$

   Multiply both sides by $2200.00

   $$x = \frac{2200}{291.67}$$

   $x \approx 7.54$

   Doug needs to save for about 8 months.

3. a)

KIM'S MONTHLY INCOME AND EXPENSES			
Income			*Regular or variable*
	Job 1	$700.00	Variable
	Job 2	$350.00	Variable
	Job 3	$855.00	Variable
	Job 4	$820.00	Variable
	Job 5	$1063.00	Variable
	Total	$3788.00	
Expenses			
Priority			*Recurring or variable*
5	Transportation	$600.00	Variable
6	Entertainment	$450.00	Variable
2	Food	$500.00	Variable
1	Rent	$1500.00	Regular
3	Child care	$600.00	Regular
4	Insurance	$200.00	Regular
7	Misc.	$300.00	Variable
	Total	$4150.00	

   b) See table.

   c) Subtract to find whether her income is more or less than her expenses.

   $3788.00 – $4150.00 = –$362.00

   Kim's income is $362.00 less than her expenses.

   d) Answers will vary. See table for sample answers.

   Kim could decrease her entertainment expense by doing fewer activities and she could likely reduce her miscellaneous expense. She might also decrease her transportation expense by using public transit. These three reductions would help reduce her expenses.

4. a) Add André's weekly salary and tips.

   $340.00 + $125.00 = $465.00

   Let x be André's net monthly income.

   $x \times 12 = \$465.00 \times 52$

   $12x = \$24\ 180.00$

   Divide both sides by 12.

   $x \approx \$2015.00$

   André earns about $2015.00 per month.

   b) Multiply to determine how much should André spend on an apartment.

   For 25%:

   $2015.00 × 0.25 = $503.75

   For 35%:

   $2015.00 × 0.35 = $705.25

   André should be looking for an apartment that costs between $503.75 and $705.25 monthly.

   c) André could consider renting a room in a house, sharing an apartment with a friend, or living in a less expensive part of the town or city. Another option would be to try to increase his income.

   d) Multiply to calculate what percentage of his income would be 10–15%.

   For 10%:
   $2015.00 × 0.10 = $201.50

   For 15%:
   $2015.00 × 0.15 = $302.25

Subtract what he has already contributed.

$201.50 – $30.00 = $171.50

$302.25 – $30.00 = $272.25

André could give an additional amount between $171.50 and $272.25 and stay within the guidelines.

5. a) Find the amount of money Vanessa needs to save each month.

$1500.00 ÷ 6 = $250.00

Find the percentage of her income this amount represents.

$250.00 ÷ $1200.00 ≈ 0.208

0.208 × 100 = 20.8%

Vanessa will need to save about 20.8% of her income each month.

b) Subtract the amount to be saved from Vanessa's monthly income.

$1200.00 – $250.00 = $950.00

Vanessa will have $950.00 left to cover her monthly expenses. Whether or not this is a reasonable amount depends on Vanessa's lifestyle. If she has to pay for rent, utilities, food, and transportation, this is not a lot of money to cover those expenses. However, if she lives at home or her rent and other expenses are modest, $950.00 might be a reasonable amount to cover monthly expenses.

6. a) Divide to calculate how much Sandra needs to save each month.

$4200.00 ÷ 10 = $420.00

Let Sandra's monthly income be x.

35% of her monthly income needs to equal $420.00.

$0.35 \times x = \$420.00$

Divide both sides by 0.35

$x = \$420.00 \div 0.35$

$x = \$1200.00$

Sandra needs to earn $1200.00 per month to save $4200.00 in 10 months.

b) Multiply Sandra's net monthly income by 12 to calculate her annual income.

$1200.00 × 12 = $14 400.00

Divide by 52 to calculate her weekly income.

$14 400.00 ÷ 52 ≈ $276.92

Let the number of hours that Sandra needs to work per week be y.

$\$10.75 \times y = \$276.92$

Divide both sides by $10.75.

$y = \$276.92 \div 10.75$

$y = 25.76$

Sandra needs to work about 26 hours per week.

c) Answers will vary. Sandra could reduce her monthly expenses. She could work more hours or look for a higher paying job. She could also plan to take the program later, giving her more time to save.

7. a) Divide to calculate Marie-Hélène's monthly salary.

$55 000.00 ÷ 12 ≈ $4583.33

Variable income:

$4583.33 × 0.20 ≈ $916.67

Regular income:

$4583.33 × 0.80 ≈ $3666.66

Marie-Hélène's regular monthly income equals $3666.66 and her variable monthly income equals $916.67.

b) Marie-Hélène's annual salary:

$3666.66 × 12 ≈ $44 000.00

Her annual salary is about $44 000.00

c) Answers will vary. Marie-Hélène spends 30% of her work time on contract work but earns only 20% of her income from it. Therefore it would seem more advantageous to increase her salaried hours if that is possible.

8. a) Let Brad's monthly income be x.

$$\frac{x}{\$4720.00} = \frac{1 \text{ month}}{3 \text{ months}}$$

Multiply both sides by 4720.

$$x = \frac{\$4720.00}{3}$$
$$x \approx \$1573.33$$

Divide to determine how much Brad needs to save each month.

$\$7000.00 \div 12 \approx \$583.33$

Brad needs to save about \$583.33 per month for the car.

Determine what percentage this is of his monthly earnings.

$(\$583.33 \div \$1573.33) \times 100 = 37\%$

This is 37% of his monthly earnings.

b) Calculate how much Brad earns in June, July, and August.

$\$1573.33 \times 1.10 \approx \$1730.66$

Calculate how much Brad would save.

$(37 \times \$1730.66) \div 100 \approx \$640.34$

Brad would save approximately \$640.34 each month during June, July, and August.

c) Calculate how much Brad would earn in June, July, and August.

$\$640.34 \times 3 = \$1921.02$

Brad's savings during those months equal \$1921.02.

Add this to his savings for March to May.

$(3 \times \$583.33) + \$1921.02 = \$3671.01$

By August, Brad will have saved \$3671.01.

Subtract what he has saved from the cost of the car.

$\$7000.00 - \$3671.01 = \$3328.99$

Divide the amount still to be saved by his regular monthly savings.

$\$3328.99 \div \$583.33 \approx 5.7$

Brad needs to save for an additional 6 months, so a total of 12 months. He will have saved slightly more than the \$7000.00 he needs.

$(\$583.33 \times 9) + (\$640.34 \times 3) = \$7170.99$

Extend Your Thinking

9. a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = \$15\,000.00\left(1 + \frac{0.0450}{12}\right)^{12\times2}$$
$$A = \$15\,000.00(1.00375)^{24}$$
$$A \approx \$16\,409.85$$

After 2 years, Leanne will owe \$16 409.85.

b) Students will need a personal loan payment calculator table, an online calculator, or a loan payment calculator on a graphing calculator to complete this problem. A table is provided on p. 292 of the student resource and as Blackline Master 6.5 (p. 408).

Using the table, students should find that a 3-year loan at a rate of 4.50% per annum, compounded monthly, has monthly payments of \$29.75 per \$1000.00 borrowed.

$$\text{Monthly payment} = \frac{\$16\,409.85}{\$1000.00} \times \$29.75$$
$$\text{Monthly payment} \approx \$488.19$$

Calculate Leanne's gross monthly income.

$\$55\,000.00 \div 12 \approx \$4583.33$

Calculate what percentage of her monthly income the payments represent.

$\$488.19 \div \$4583.33 \approx 0.11$

$0.11 \times 100 = 11\%$

The monthly payments represent about 11% of Leanne's gross monthly income.

c) Monthly payment = \$18.64 per \$1000.00 borrowed

$$\text{Monthly payment} = \frac{\$15\,000.00}{\$1000.00} \times \$18.64$$
$$\text{Monthly payment} \approx \$279.60$$

Calculate what percentage of her monthly income the payments represent.

$$\$279.60 \div \$4583.33 \approx 0.06$$

$$0.06 \times 100 = 6\%$$

The monthly payments represent about 6% of Leanne's gross monthly income.

d) Option 1:

$$\text{Total payment} = \$488.19/\text{month} \times 36 \text{ months}$$

$$\text{Total payment} = \$17\,574.84$$

$$I = A - P$$

$$I = \$17\,574.84 - \$15\,000.00$$

$$I = \$2574.84$$

Option 2:

$$\text{Total payment} = \$279.60/\text{month} \times 60 \text{ months}$$

$$\text{Total payment} = \$16\,776.00$$

$$I = A - P$$

$$I = \$16\,776.00 - \$15\,000.00$$

$$I = \$1776.00$$

Leanne will pay more interest if she chooses payment option 1.

Option 1: Waiting for 2 years before making any payments is advantageous because Leanne could pay off other debts first and get established living on her own. However, she accumulates interest over the 2 years, so she will pay more interest overall.

Option 2: Paying over 60 months is advantageous because Leanne's monthly payments will be lower and she will pay less interest overall. However, she needs to begin making payments right away, and this may be difficult when starting a new career.

Extension

You can challenge students further by devising other situations where monthly payments need to be calculated, such as for car loans and mortgages.

7.2 The Budgeting Process

TIME REQUIRED FOR THIS SECTION: 3 CLASSES

STUDENT BOOK, pp. 313–325

MATH *ON THE JOB*

STUDENT BOOK, p. 313

You may begin this section by asking how many students think they would like to be self-employed. Although only personal budgets are addressed in this chapter, work-related expenses do enter into personal budgets, especially if the person is operating a home-based business.

Engage students in a discussion of the kinds of work-related expenses tradespeople in different occupations may have. For example, construction workers may need to buy safety helmets and steel-toed boots. Tradespeople often receive an allowance to purchase such mandatory equipment.

Then bring the focus back to personal budgets, emphasizing that budgets help you forecast your expenses and make decisions based on that financial information. In this Math on the Job, Carrie would not know if she can afford the vehicle loan if she had not kept a record of her income and expenses and made a budget.

SOLUTIONS

1. Answers will vary. Students might say that Carrie could draw up a budget and add this item to her expenses. If she has been putting money into savings each month, that money may need to be allocated to loan payments.
2. If Carrie didn't want to take out a loan, she could open a savings account for the car and save until she had enough to purchase it.
3. $\$450.00 \div \$2800.00 \approx 0.16$

   If Carrie's net monthly income is $2800.00, she would spend 16% of it on the loan payment.

EXPLORE THE MATH

STUDENT BOOK, p. 313

Begin by asking students where they typically spend their money. Food, clothes, entertainment, savings, and "other" may be some typical responses. Then ask students what they think might be a reasonable percentage of a total income for each category. Select a student to volunteer his/her weekly income and estimate weekly expenses for each category, then calculate the percentages.

The class should notice that people are likely to underestimate their actual spending. Discuss how this could be a problem over longer periods of time. Because people underestimate their spending, they often go over budget; eventually, this can lead to large debts and financial crisis. This is a main purpose for having a budget, and documenting past spending accurately will help in making an accurate budget.

Worked examples show budgeting procedures, including the use of spreadsheet technology.

SOLUTIONS

Answers will vary. If you prepare a conservative budget you are less likely to spend more than you budgeted, since you will have budgeted slightly more than your records show you spent in the past. If you don't estimate your expenses higher, you may find yourself spending more than you budgeted, and therefore accumulate debt.

If your budget shows a deficit when you are preparing it, you need to find some categories of expenses that can be reduced. Reductions are usually applied to variable expenses such as entertainment or miscellaneous expenses.

To make your next budget accurate, it is important to have tracked your actual expenses and compared them to what you budgeted last time. If either your income or expenses have changed since you made the last budget, you need to update the figures. You may also have found that some expenses were missing from your budget, and you should add them in. It is a good practice to estimate your expenses high and estimate your income low.

DISCUSS THE IDEAS

STARTING A BUDGET

STUDENT BOOK, p. 314

This Discuss the Ideas provides students an opportunity to start designing a budget. They have to decide how to categorize the information in a meaningful way that can generalize the finances into a useful budget document.

SOLUTIONS

1. Answers will vary. Students may suggest separating the income from the expenses, then categorizing the expenses into recurring and variable, and then calculating totals.
2. To find out if her expenses and income are balanced, total her expenses and total her income. If they are equal, they are balanced. Identify the expenses and add them. Expenses include groceries, restaurant meals, cell phone, transportation, clothes, internet and digital TV, rent, entertainment, utilities, and savings.

   Add the expenses associated with each item.

   \$300.00 + \$80.00 + \$75.00 + \$45.00 + \$25.00 + \$115.00 + \$800.00 + \$320.00 + \$65.00 + \$150.00 = \$1975.00

   Identify income amounts and add them. Income consists of the 2 paycheques.

   \$1150.00 + \$1050.00 = \$2200.00

   Subtract expenses from income.

   \$2200.00 – \$1975.00 = \$225.00

   Renée's budget is showing a \$225.00 surplus.
3. Her expenses are less than her income, so she can allocate more to savings or to other variable expenses.
4. A budget based on only one month's financial transactions will not be very accurate. A more reliable budget could be made by using income and expense data accumulated over several months. It would also be prudent to include a monthly amount for unexpected expenses.

Mental Math and Estimation

STUDENT BOOK, p. 318

SOLUTIONS

1. Students can mentally add up Rose-Marie's expenses.

   \$300.00 + \$150.00 + \$100.00 + \$85.00 + \$50.00 = \$685.00

   Students can mentally subtract Rose-Marie's expenses from income.

   \$750.00 – \$685.00 = \$65.00

   Rose-Marie's expenses are \$65.00 less than her income, so the budget is not balanced.
2. If Rose-Marie saves \$65.00 a month, she will have saved \$390.00 after 6 months.

Example 2

Some students may already be familiar with using spreadsheets, so this could be review material for them. For others, doing several budget problems using spreadsheets may be helpful.

Extension

Many different spreadsheet templates for personal budgets are available either within the spreadsheet software package or as a free download from a website. Have students experiment with different templates until they find one they like best, and then ask them to use this template for Kailea's budget.

Mental Math and Estimation

STUDENT BOOK, p. 320

SOLUTIONS

1. Audrey currently has a budget surplus of \$20.00 per month. If she subscribes to the new cell phone plan that costs \$40.00 more, she will have a deficit of \$20.00 per month. Having a deficit is referred to as being over budget.
2. When Audrey prepares her next budget, she should increase her income by \$80.00 and increase her expenses by \$40.00. To balance the budget, she can allocate the extra \$40.00 (\$80.00 – \$40.00 = \$40.00) to savings or other expenses.

ACTIVITY 7.2

FINANCIAL PLANNING FOR A SPORT OR HOBBY

STUDENT BOOK, p. 320

T Begin this activity with your class by asking the following questions:

- How much would it cost to take up skiing for the first time?
- What percent of your income should go towards hobbies or recreation?
- Why is it important to think about these expenses before beginning a new activity?

In this activity, it would be beneficial to use a computer lab so students can use the internet to research the necessary information. Also, they would be able to practise working with a spreadsheet when designing their budgets. If a lab is not available, students can complete the bulk of the activity using estimates and approximate figures, and then research more accurate information at home to complete the activity.

SOLUTIONS

Answers will vary. A sample answer follows.

1. I would like to start skiing.
2. Start-up expenses:
    - Ski equipment: \$800.00
    - Season's Pass on Grouse Mountain: \$675.00
    - Lessons (5 at \$44.00 each): \$220.00
    - Total cost: \$1695.00
3. Answers will vary. To calculate the income, students need to know the minimum wage in their province or territory. The sample answer uses the 2010 minimum wage of \$8.00/h in BC, but rates may change, so students should confirm the current rates online.

    Calculate the apprenticeship hourly wage.

    Minimum wage multiplied by 1.35.

    \$8.00 × 1.35 = \$10.80/h

    Calculate the net hourly rate.

    \$10.80 × 0.7 = \$7.56
4. See table.

MONTHLY BUDGET			
Income			
Hourly net wage	\$7.56		
Monthly hours	120		
Monthly net income	\$907.20		
Expenses			
Room and board	\$350.00		
Clothing	\$60.00		
Transportation	\$75.00		
Cell phone	\$55.00		
Recreation	\$125.00		
Savings	\$242.20		
Total	\$907.20		

5. Monthly savings are \$242.20.

    Divide the cost of the new sport by the monthly savings to calculate how long you will need to save.

    \$1695.00 ÷ \$242.20 ≈ 7

    You will need to save for approximately 7 months to cover the costs of the new sport.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 321

Use Blackline Master 7.5 (p. 468) for question 3.

1. a) Answers will vary. A sample answer could be that Stuart could save the receipts for all of his purchases. He could divide these receipts into categories such as food, entertainment, credit card payments, and so on. He could also classify his expenses as recurring, variable, and unexpected, necessary and unnecessary.

   b) Answers will vary. To get better control of his finances, Stuart could calculate how much he has to spend on necessities (such as food and rent) each month and draw up a budget designed to meet these expenses. He could devote a certain percentage of his income to these expenses. Stuart could also reduce his spending and devote this money to paying off his debts. He could also decide to devote a percentage of each paycheque to paying off his debt.

2. a) Answers will vary. A sample answer is provided. Underestimate the monthly income and overestimate the expenses.

FRANCEY'S MONTHLY BUDGET	
Income	*Expenses*
$1800.00	Rent: $650.00
	Transportation: $250.00
	Food: $350.00
	Other: $400.00
	Savings: $150.00
Total: $1800.00	*Total:* $1800.00

   Total the expenses and subtract from the income. The difference is savings.

   $1800.00 – $1650.00 = $150.00

   Francey can save $150.00 monthly.

   b) Divide the cost of the equipment by Francey's monthly savings.

   $1000.00 ÷ $150.00 ≈ 6.67

   Francey can afford the new snowboarding equipment in 7 months.

3. a) See table.

Sept. 01	–$650.00	Cheque #102: Rent • Expense • Fixed	Sept. 17	–$48.00	Gas • Expense • Variable
Sept. 01	$1400.00	Paycheque • Income • Fixed	Sept. 17	–$55.00	Pizza Palace • Expense • Variable
Sept. 03	–$212.40	SuperGroceries • Expense • Variable	Sept. 21	–$120.00	Cheque #022: Car insurance • Expense • Fixed
Sept. 05	$–45.00	Gas • Expense • Variable	Sept. 22	–$42.00	Bigger Burgers • Expense • Variable
Sept. 05	–$40.00	Cash • Expense • Variable	Sept. 22	–$15.00	Cinema Magic • Expense • Variable
Sept. 05	–$1.50	Transaction fee • Expense • Fixed	Sept. 24	–$47.45	Wow Clothing Co. • Expense • Variable
Sept. 10	–$54.25	SuperGroceries • Expense • Variable	Sept. 24	–$35.00	Cheque #023: Charity ABC • Expense • Variable
Sept. 11	–$60.00	Cash • Expense • Variable	Sept. 28	–$3.50	Wake-up Coffee • Expense • Variable
Sept. 11	–$1.50	Transaction fee • Expense • Fixed	Sept. 30	–$60.00	Cash • Expense • Variable
Sept. 15	$1400.00	Paycheque • Income • Fixed	Sept. 30	–$1.50	Transaction fee • Expense • Fixed

b) See table.

JANINE'S MONTHLY BUDGET		
Income	*Expenses*	
$2800.00	Rent	$650.00
	Food	$300.00
	Entertainment/ Restaurants	$130.00
	Car	$250.00
	Clothing	$75.00
	Donations	$35.00
	Other	$200.00
	Savings	$1160.00
Total: $2800.00	*Total*	$2800.00

c) Calculate Janine's annual savings.

$1160.00 × 12 = $13 920.00

Divide to calculate half of the balance.

$13 920.00 ÷ 2 = $6960.00

Calculate the accumulated amount of Janine's savings.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$6960.00\left(1 + \frac{0.03}{1}\right)^{(1\times10)}$$

$$A = \$6960.00)(1.03)^{10}$$

$$A \approx \$9353.69$$

4. a) Calculate Alana's gross weekly pay.

$16.00 × 8 = $128.00

$128.00 × 5 = $640.00

Alana's gross weekly pay is $640.00.

Calculate her net weekly pay.

$640.00 × 0.70 = $448.00

Alana's net weekly pay is $448.00.

b) See table.

ALANA'S WEEKLY BUDGET		
Income	*Expenses*	
$448.00	Food	$90.00
	Transportation	$25.00
	Rent	$150.00
	Clothes and personal items	$40.00
	Entertainment	$80.00
	Other	$25.00
	Savings	$38.00
Total: $448.00	*Total*	$448.00

c) Divide to determine how long it will take Alana to save $2000.00.

$2000.00 ÷ 38 ≈ 52.6

It will take Alana about 53 weeks to save $2000.00.

d) Answers will vary. Alana could reduce her spending on entertainment, put in more hours at work, take out a loan or investigate how to access possible government health programs.

5. a) Calculate what percentage $250.00 is of $3500.00.

$250.00 ÷ $3500.00 ≈ 0.07

Marc's unexpected expenses are about 7% of his net income.

b) Unexpected expenses might include car maintenance, repairing a roof leak, vet bills, or an unplanned purchase.

6. a) First calculate Devin's net hourly pay.

$10.40 × 0.7 = $7.28

Calculate how many hours he gets paid for annually.

16 h/week × 52 = 832 h

Calculate his net annual earnings.

$7.28 × 832 = $6056.96

Devin's net annual income is $6056.96.

Calculate Devin's annual expenses. For expenses given in weekly amounts, multiply by 52. For expenses given in monthly amounts, multiply by 12.

Cell phone costs $40.00 monthly.

$\$40.00 \times 12 = \$480.00$

Transportation costs $25.00 a week.

$\$25.00 \times 52 = \$1300.00$

Entertainment costs $40.00 a week.

$\$40.00 \times 52 = \$2080.00$

Clothing and personal items cost $50.00 monthly.

$\$50.00 \times 12 = \$600.00$

Total Devin's annual expenses.

$\$480.00 + \$1300.00 + \$2080.00 + \$600.00 = \$4460.00$

Subtract his annual expenses from his annual net income, and allocate the balance to savings.

annual net income – annual expenses = savings

$\$6056.96 - \$4460.00 = \$1596.96$

Devin's annual savings are $1596.96.

DEVIN'S ANNUAL BUDGET			
Net income		*Expenses*	
Annual net income	$6056.96	Cell phone	$480.00
		Transportation	$1300.00
		Entertainment	$2080.00
		Clothing and personal items	$600.00
		Savings	$1596.96
Total	$6056.96	*Total*	$6056.96

b) Divide annual savings by 12 to calculate his monthly savings.

$\$1596.96 \div 12 = \$133.08$

Divide annual savings by 52 to calculate his weekly savings.

$\$1596.96 \div 52 \approx \$30.71$

Devin saves approximately $30.71 weekly, $133.08 monthly, and $1596.96 annually.

c) Calculate Devin's additional annual cell phone expense.

Multiply the increase in the monthly cost by 12.

$\$20.00 \times 12 = \$240.00$

Divide the added expense by his previous savings and multiply by 100 to calculate the percent decrease in savings.

$\dfrac{\$240.00}{\$1596.96} \times 100 \approx 15$

Devin's annual savings will decrease by 15% if his monthly cell phone costs increase by $20.00.

7. a) Calculate Elaine's total weekly expenses.

$\$150.00 + \$400.00 = \$550.00$

Calculate how much Elaine would earn if she worked for 20 or 30 hours at $15.00/h.

$20 \times \$15.00 = \$300.00$
$30 \times \$15.00 = \$450.00$

If she worked 20 hours, she would earn $300.00 and if she worked 30 hours, she would earn $450.00. This wage would not cover her expenses. Based on this, students might say they would not advise Elaine to accept the job.

b) Calculate 70% of Elaine's hourly wage.

$\$15.00 \times 0.70 = \$10.50$

After deductions, Elaine's hourly wage is $10.50.

Calculate her net weekly income.

$25 \times \$10.50 = \$262.50$

Elaine's net weekly income is $262.50.

c) Divide to determine how many hours Elaine would have to work.

$\$550.00 \div \$10.50 \approx 52.4$

Elaine would have to work about 53 hours each week to cover her expenses. Based on this information, it would not be advisable for Elaine to take the job, given that she can work a maximum of 30 hours each week.

Extend Your Thinking

8. a) Calculate Josif's weekly gross pay.

$\$17.50 \times 40 = \$700.00$

Multiply to calculate his weekly net pay

$\$700.00 \times 0.7 = \$490.00$

Multiply by 52 and divide by 12 to calculate his monthly net pay.

$\frac{\$490.00 \times 52}{12} \approx \$2123.33$

Josif's net monthly income is approximately \$2123.33.

JOSIF'S DRAFT MONTHLY BUDGET

Income		*Expenses*	
Regular monthly net pay	\$2123.33	Housing	\$1200.00
Net overtime pay	\$0.00	Food	\$650.00
		Gas	\$200.00
		Educational savings	\$300.00
		Misc.	\$200.00
Total	\$2123.33	*Total*	\$2550.00
Draft deficit	–\$426.67		

Josif's draft budget shows a deficit of \$426.67. This means his expenses exceed his income by \$426.67.

b) Calculate Josif's hourly overtime wage.

$1.5 \times \$17.50 = \$26.25$

Calculate his net hourly overtime wage.

$\$26.25 \times 0.7 \approx \$18.38$

Divide to find out how many hours of overtime Josif must work each month.

$\$426.67 \div \$18.38 \approx 23.21$

JOSIF'S MONTHLY BUDGET

Income		*Expenses*	
Regular monthly net pay	\$2123.33	Housing	\$1200.00
Net overtime pay	\$426.67	Food	\$650.00
		Gas	\$200.00
		Educational savings	\$300.00
		Misc.	\$200.00
Total	\$2550.00	*Total*	\$2550.00

Calculate Josif's weekly overtime hours.

Multiply his monthly overtime hours by 12 and divide by 52.

$\frac{(23.21 \times 12)}{52} \approx 5.34$

Josif needs to work about 6 hours of overtime each week to balance his budget. His income will be slightly higher than his expenses, and he can allocate the difference to "Misc. expenses"

c) Answers will vary. Josif could reduce the amount of money he has budgeted under "Misc. expenses." He could also try to reduce his food costs.

7.3 Analyzing a Budget

TIME REQUIRED FOR THIS SECTION: 5 CLASSES

STUDENT BOOK, p. 326–343

MATH *ON THE JOB*

STUDENT BOOK, p. 326

Have students read the Math on the Job as a group or individually. Discuss how Byron might be able to afford his new computer. Look at the circle graph and discuss what sections appear too large. Have students calculate the actual money spent per month on each category, then discuss again. Are there any expenses that cannot be changed? What are the variable expenses, and what expenses are of low priority?

SOLUTIONS

1. Calculate the actual amount of money spent on each expense category before Byron decreases his miscellaneous expenses. Convert the percentage to a decimal by dividing by 100, then multiply by Byron's monthly net pay.

Housing:	0.40 × $3800.00 = $1520.00
Car:	0.16 × $3800.00 = $608.00
Food:	0.18 × $3800.00 = $684.00
Entertainment:	0.04 × $3800.00 = $152.00
Charitable donations:	0.08 × $3800.00 = $304.00
Miscellaneous:	0.11 × $3800.00 = $418.00
Savings:	0.03 × $3800.00 = $114.00

   Next, decrease Byron's "Miscellaneous" expenses to 5% and increase his savings to 9%.

   Other: 0.05 × $3800.00 = $190.00

   Savings: 0.09 × $3800.00 = $342.00

   He will be able to save $342.00 with his new budget.

2. Calculate how much he will save in 2 years.

   $342.00/month × 24 months = $8208.00

   Byron needs $11 500.00 for the computer and the car. At this rate of savings, he will not be able to afford both.

3. Calculate how much additional money Byron could save every month from his new income.

   $4800.00 ÷ 12 = $400.00

   Calculate how much he can save in total each month.

   $342.00 + $400.00 = $742.00

   With this new monthly income, Byron can save $742.00 per month.

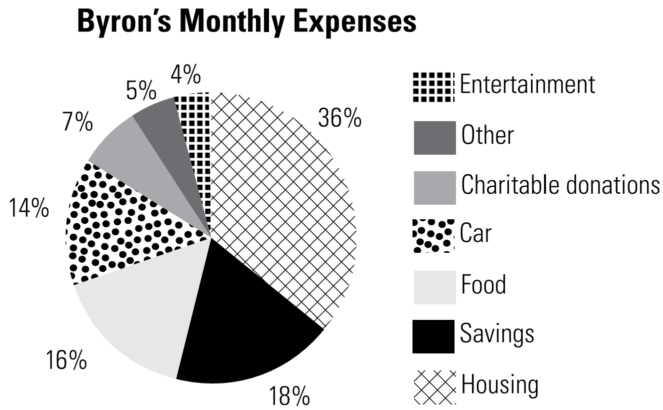
   Calculate what percentage each category represents of his new net monthly income. His monthly income is now $4200.00 ($3800.00 plus $400.00).

Housing:	$1520.00 ÷ $4200.00 ≈ 0.36 0.36 × 100 = 36%
Car:	$608.00 ÷ $4200.00 ≈ 0.14 0.14 × 100 = 14%
Food:	$684.00 ÷ $4200.00 ≈ 0.16 0.16 × 100 = 16%
Entertainment:	$152.00 ÷ $4200.00 ≈ 0.04 0.04 × 100 = 4%
Charitable donations:	$304.00 ÷ $4200.00 ≈ 0.07 0.07 × 100 = 7%
Other:	$190.00 ÷ $4200.00 ≈ 0.05 0.05 × 100 = 5%
Savings:	$742.00 ÷ $4200.00 ≈ 0.18 0.18 × 100 = 18%

   To construct a circle graph manually, determine how many degrees each percentage of expenses equals.

Housing:	0.36 × 360° ≈ 130°
Car:	0.14 × 360° ≈ 50°
Food:	0.16 × 360° ≈ 58°
Entertainment:	0.04 × 360° ≈ 14°
Charitable donations:	0.07 × 360° ≈ 25°
Other:	0.05 × 360° = 18°
Savings:	0.18 × 360° ≈ 65°

Draw a circle and use a protractor to measure the degrees for each category of expense.



ALTERNATIVE SOLUTION

T This problem can be solved using a spreadsheet program.

Enter Byron's monthly expenses into a spreadsheet, and use the graphing function to create a circle graph. The graph should have the same divisions as shown above.

EXPLORE THE MATH

STUDENT BOOK, p. 327

Begin by discussing how technology has made budgeting easier. When using spreadsheets for budgets, you can make changes to your budget and the spreadsheet will instantly calculate the results for these changes in the entire budget. Ask students how a circle graph like the one used for Byron in Math on the Job can help you visualize your spending.

Students may suggest that the circle graph helps to see how some expenses compare to others, or they may notice that the circle graph shows how individual expenses compare to the whole. In this section, students will create circle graphs using technology, and use these circle graphs to analyze spending and adjust budgets accordingly. Students will use algebra and equations with proportions to calculate how much spending needs to change to effect percent changes in the circle graph.

A key point to reinforce is the importance of tracking actual income and expenses and comparing those figures with the budgeted amounts to determine whether you are staying on budget. A budget is only useful if you consult it and adjust your spending if necessary to stay on track.

The examples in this section explore spending guidelines and show how to adjust a budget to achieve spending and savings goals.

SOLUTIONS

Answers will vary. If Byron actually spends 25% of his income on food, then he should budget for that in his next budget and reduce other expenses to balance his budget. Alternatively, he could determine how to spend less on food, for example, by eating out less often. For budgeting purposes, however, he should project somewhat higher than the amount his actual financial records show.

DISCUSS THE IDEAS

ASSESSING INCOME AND EXPENSES

STUDENT BOOK, p. 327

This discussion presents an opportunity to talk about spending guidelines, their usefulness and limitations. Ask students when it might not be appropriate to follow spending guidelines too closely? For example, if someone is earning a very low income, it might be inappropriate to spend a maximum of 15% on food. If your net monthly income is \$1100.00, 15% of that would be \$165.00. This is a very low amount, since it comes to about \$5.50 per day.

Discussing the advantages of paying loans semi-monthly rather than monthly provides an opportunity for students to apply the concept of compounding interest. In the case of a loan repayment, less interest will accumulate if payments are made more frequently, so the total loan (principal and interest) will be paid off sooner.

SOLUTIONS

a) Aisha is likely spending a larger percentage of her monthly income on basic needs than Sylvester. Because they both live in the same community, their expenses are likely similar, but Aisha has a lower income, so a larger percentage of it will be spent on basic needs. Another way to look at this is to say that the person with the higher income generally has more disposible income, which is income not needed for basic expenses.

b) Calculate Aisha's new net semi-monthly pay.

$\$22\ 900.00 \div 24 \approx \$954.17$

Calculate the difference in income.

$\$954.17 - \$750.00 = \$204.17$

Calculate what percent the difference in income is of Aisha's original income.

$\$204.17 \div \$750.00 \approx 0.27$

$0.27 \times 100 = 27\%$

Aisha's income will increase by approximately 27%.

Because of her increased income, she might make some of the following budget adjustments:

- her housing costs are unlikely to change, unless she wants to move into a more expensive housing unit;
- she could spend more on food, transportation, entertainment, or other expenses;
- she could put more money into savings.

c) Sylvester will pay off his student loan sooner if he pays $75.00 semi-monthly. If he makes payments more often, he will pay less interest.

In his budget, Sylvester would have to take into account the total interest payment on his loan based on the two payment options. With semi-monthly payments, the loan will be paid off sooner and he can allocate the money saved to other items.

DISCUSS THE IDEAS

PAY YOURSELF FIRST

STUDENT BOOK, p. 329

Ask students if anyone knows what pay yourself first means. This could lead to interesting interpretations, especially if students are not familiar with the term.

SOLUTIONS

1. If you apply the pay-yourself-first strategy and find that you are over budget at the end of the month, you could make the following adjustments:
   - reduce your expenses in other areas (food, transportation, entertainment);
   - reduce the amount of money you are putting into savings at the beginning of the month.
2. Credit card debt has a very high interest rate. It is always best to pay off credit card debt before putting money into savings because the interest earned on the savings is certainly lower than the interest being paid on the debt.

Mental Math and Estimation

STUDENT BOOK, p. 332

Students will need to prioritize their own spending and decide what category they would need to reduce in order to afford the new laptop. If they need assistance, ask them how many days are in a year, and what if they saved $2.00 every day? Tell them that by solving simpler, smaller problems, they should be able to use these answers as benchmarks for their final estimate.

SOLUTIONS

1. $2.00 every day would amount to about $700.00 after a year.

   $3.00 every day would amount to about $1000.00 after a year.

   So, you would need to save about $2.50 per day to afford the new laptop.

2. Some suggested expenses that could be eliminated to save this amount include snacks, coffee, lunches purchased at a café (bring from home instead), and bus fare (walk or ride bicycle instead).

ACTIVITY 7.3

BUDGETING USING SPREADSHEETS

STUDENT BOOK, p. 333

T In this activity, students will explore the manipulation of budgets manually and using a spreadsheet to achieve specific monetary goals.

Students will need to use a spreadsheet program. They will use the program to create a circle graph (called a pie chart in some spreadsheet programs). Students will also need to write formulas in the spreadsheet cells. This will allow them to adjust entries in the budget without having to recalculate monthly and annual totals.

You may want to use Blackline Master 7.6 (p. 469), instructions for formatting a spreadsheet, as a refresher for students who need to review using a spreadsheet for budgeting.

This activity assumes that GICs purchased over the course of 3 years will all have the same interest rate of 3.0% per year. In reality, interest rates change over time and from issuer to issuer.

SOLUTIONS

Part A

1. First, calculate Duma's total expenses.

   $1800.00 + $800.00 + $500.00 + $300.00 + $100.00 = $3500.00

   Calculate the percentage that each expense represents of the total, rounding to the nearest percent.

   Housing: $1800.00 ÷ $3500.00 ≈ 0.51
   0.51 × 100 = 51%

   Food: $800.00 ÷ $3500.00 ≈ 0.23
   0.23 × 100 = 23%

   Transportation: $500.00 ÷ $3500.00 ≈ 0.14
   0.14 × 100 = 14%

   Child care: $300.00 ÷ $35.00 ≈ 0.09
   0.09 × 100 = 9%

   Other: $100.00 ÷ $3500.00 ≈ 0.03
   0.03 × 100 = 3%

   Compare these percentages to the spending guidelines.

   Duma is spending too much on housing and food. Because housing accounts for over half her income, reducing this cost should be the first priority.

2. Calculate 35% of Duma's expenses.

   0.35 × $3500.00 = $1225.00

   Calculate the difference between what she is currently spending on housing and her new housing budget.

   $1800.00 – $1225.00 = $575.00

   Duma can save $575.00 per month if she reduces her housing budget to 35%.

3. If Duma puts $575.00 into savings each month, calculate how much she would be able to deposit into a GIC after 12 months.

   $575.00 × 12 months = $6900.00

   At the end of year 2, the $6900.00 will have earned a year's worth of interest and she will have another $6900.00 to deposit into the GIC.

   Balance at end of year 2 = ($6900.00 × 1.03) + $6900.00

   Balance at end of year 2 = $7107.00 + $6900.00

   Balance at end of year 2 = $14 007.00

   At the end of year 3, the money from year 2 will have earned more interest and she will have another $6900.00 to deposit into the GIC.

   Balance at end of year 3 = ($14 007 × 1.03) + $6900.00

   Balance at end of year 3 = $14 427.21 + $6900.00

   Balance at end of year 3 = $21 327.21

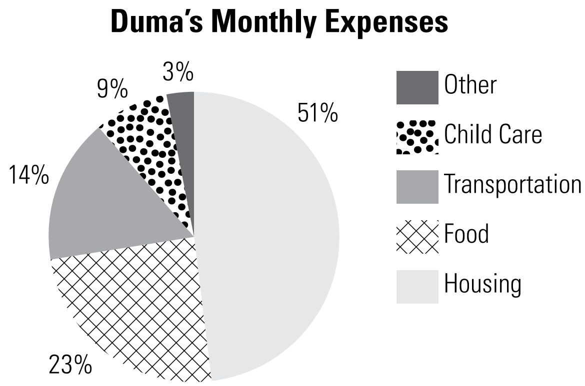
   After 3 years, Duma will have $21 327.21 in savings. She will have more than is needed to buy the car.

   $21 327.21 – $12 000.00 = $9327.21

She will have \$9327.21 extra in savings.

Part B

1. Copy the figures from Duma's monthly budget into a spreadsheet program and use the graphing function to create a circle graph.



The circle graph confirms the percentages calculated in Part A, question 1.

2.

	A	B	C
1	**Duma's Monthly Budget**		
2			
3	**Income**		
4	Wage	\$3500.00	
5			
6	**Expenses**		
7	Housing	\$1800.00	
8	Food	\$800.00	
9	Transportation	\$500.00	
10	Child care	\$300.00	
11	Other	\$100.00	
12	Savings	=\$B\$4–sum(B7:B11)	
13			
14			
15	Annual savings	=12*B12	
16			
17	**Savings after 3 years**	=B15*(1.03)^2+B15*(1.03)^1+B15	
18			

Format the "boxed" cells of the spreadsheet to help with adjusting Duma's budget.

B12: this cell needs to be the amount of money left after paying her other expenses. It is equal to income (cell B4) minus the sum of the other expenses (B7:B11).

B15: this cell is the total of 12 months of savings. It is equal to 12 times monthly savings (cell B12).

B17: this cell is the accumulation of 3 years of savings. The first year will have 2 years to accumulate compound interest, the second year will have 1 year to accumulate compound interest, and so on. It is equal to annual savings (cell B15) times $(1.03)^2$ plus annual savings times $(1.03)^1$ plus annual savings (cell B15).

3. Change Duma's housing expense to \$1225.00.

7	Housing	\$1225.00

The savings cell should change to \$575.00.

12	Savings	\$575.00

The annual savings cell should change to \$6900.00.

15	Annual savings	\$6900.00

The cell showing savings after 3 years should change to \$22 400.16.

17	**Savings after 3 years**	\$21 327.21

Duma will have saved \$21 327.21 in 3 years, so she would be able to afford the new vehicle.

Mental Math and Estimation

STUDENT BOOK, p. 334

SOLUTION

1. To estimate Francine's net monthly income, round her net annual salary to \$36 000.00 and divide by 12.

   \$36 000.000 ÷ 12 = \$3000.00.

   Francine makes about \$3000.00 per month.

2. To estimate 5% of \$3000.00, first calculate 10% of \$3000.00 by moving the decimal one place to the left. 10% of \$3000.00 is \$300.00. 5% will be half this amount, or \$150.00. Francine will save about \$150.00 per month.

   To estimate how much she will save per year, you will need to estimate 5% of \$36 000.00. First calculate 10% of \$36 000.00 by moving the decimal one place to the left. 10% of \$36 000.00 is \$3600.00. 5% will be half this amount, or \$1800.00. Francine will save about \$1800.00 per year.

BUILD YOUR SKILLS: SOLUTIONS

STUDENT BOOK, p. 334

1. a) To calculate Lucas's new transportation expense, set up a proportion and let x be the new amount of money he spends.

$$\frac{\$540.00}{28} = \frac{x}{20}$$

$$20 \times \frac{\$540.00}{28} = \frac{x}{20} \times 20$$

$$\$385.71 \approx x$$

   Lucas reduces his transportation expense to \$385.71 per month.

   b) If Lucas cannot use public transportation to get to his job at the mine, he can consider one of the following options to lower his transportation expense:

   - carpooling with a co-worker and splitting the transportation cost; or
   - driving a more fuel-efficient vehicle.

   c) Let Lucas's total income be x. 28% of x is \$540.00. Set up a proportion to solve for x.

$$\frac{28}{100} = \frac{\$540.00}{x}$$

$$x \times 100 \times \frac{28}{100} = \frac{\$540.00}{x} \times 100 \times x$$

$$28x = \$54\,000.00$$

$$x = \frac{\$54\,000.00}{28}$$

$$x \approx \$1928.57$$

   Lucas's total monthly income is \$1928.57.

2. a) Let Gurjinder's total income be x. 57% of x is \$650.00. Set up a proportion to solve.

$$\frac{57}{100} = \frac{\$650.00}{x}$$

$$x \times 100 \times \frac{57}{100} = \frac{\$650.00}{x} \times 100 \times x$$

$$57x = \$65\,000.00$$

$$x = \frac{\$65\,000.00}{57}$$

$$x \approx \$1140.35$$

   Gurjinder's total monthly income is \$1140.35. To calculate Gurjinder's monthly savings, convert 3% to a decimal, 0.03, and multiply by her income.

   $0.03 \times \$1140.35 \approx \$34.21$

   Gurjinder saves \$34.21 per month.

ALTERNATIVE SOLUTION

   Let Gurjinder's total income be x. 57% of x is \$650.00. So, 0.57 multiplied by x equals \$650.00.

$$0.57x = \$650.00$$
$$x = \$650.00 \div 0.57$$
$$x \approx \$1140.35$$

   Multiply by 0.03 to calculate her savings.

   $0.03 \times \$1140.35 \approx \$34.21$

   Gurjinder saves \$34.21 per month.

   b) Calculate how much of her income is used for housing expenses.

   57% + 11% = 68%

   Calculate 68% of Gurjinder's total income.

   $0.68 \times \$1140.35 \approx \$775.44$

   Gurjinder's monthly housing expense is \$775.44.

   c) Gurjinder is currently spending 68% of her income on housing. Calculate the difference between her spending and the guidelines.

   68% – 35% = 33%

   Gurjinder is exceeding the guidelines by 33% of her income. Calculate the dollar value.

   $0.33 \times \$1140.35 \approx \$376.32$

Gurjinder's spending on housing exceeds the guidelines by \$376.32.

3. a) Answers will vary. Putting money into a savings account at a lower interest rate while paying a higher rate on a debt is not a sound financial strategy. It is advisable to pay off debts that have a high interest rate as quickly as possible, even if it means reducing savings.

   b) Answers will vary. Yun could consider putting the amount that he has been saving towards the debt. He could consider reducing some of his variable expenses until the debt is paid off.

4. a) After 1 year, Gérard has saved \$600.00. He does not earn interest on this amount yet, because he buys the GIC only at the end of the year.

   Calculate Gérard's savings after 2 years.

   $A_2 = P\left(1 + \frac{r}{n}\right)^{nt} + \$600.00$

   $A_2 = (\$600.00)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$600.00$

   $A_2 = (\$600.00)(1.035) + \$600.00$

   $A_2 = \$621.00 + \$600.00$

   $A_2 = \$1221.00$

   After year 3:

   $A_3 = P\left(1 + \frac{r}{n}\right)^{nt} + \$600.00$

   $A_3 = (A_2)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$600.00$

   $A_3 = (\$1221.00)(1.035) + \$600.00$

   $A_3 \approx \$1263.74 + \$600.00$

   $A_3 \approx \$1863.74$

   After year 4:

   $A_4 = P\left(1 + \frac{r}{n}\right)^{nt} + \$600.00$

   $A_4 = (A_3)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$600.00$

   $A_4 = (\$1863.74)(1.035) + \$600.00$

   $A_4 \approx \$1928.97 + \$600.00$

   $A_4 \approx \$2528.97$

   Gérard's savings after 4 years will be \$2528.97.

ALTERNATIVE SOLUTION

T This problem can be solved using a spreadsheet.

- Enter balance after 1 year, \$600.00, into cell B1.
- Format cell B2 with the equation for calculating the balance after 2 years. The calculated sum is shown in cell C2.
- Format cell B3 with the equation for calculating the closing balance after 3 years.
- Use the same method for formatting cell B4.

	A	B	Calculated Sum
1	After 1 year	\$600.00	
2	After 2 years	"=B1*1.035+600"	\$1221.00
3	After 3 years	"=B2*1.035+600"	\$1863.74
4	After 4 years	"=B3*1.035+600"	\$2528.97

Gérard's savings after 4 years will be \$2528.97.

   b) Calculate how much Gérard would be able to invest in a GIC per year if he increased his savings by 5%.

   $\$600.00 \times 1.05 = \$630.00$

   Gérard would invest \$630.00 per year in a GIC.

   $A_1 = \$630.00$

   After year 2:

   $A_2 = P\left(1 + \frac{r}{n}\right)^{nt} + \$630.00$

   $A_2 = (A_1)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$630.00$

   $A_2 = (\$630.00)(1.035) + \$630.00$

   $A_2 = \$652.05 + \$630.00$

   $A_2 = \$1282.05$

   After year 3:

   $A_3 = P\left(1 + \frac{r}{n}\right)^{nt} + 630.00$

   $A_3 = (A_2)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$630.00$

   $A_3 = (\$1282.05)(1.035) + \$630.00$

   $A_3 \approx \$1326.92 + \$630.00$

   $A_3 \approx \$1956.92$

After year 4:

$$A_4 = P\left(1 + \frac{r}{n}\right)^{nt} + \$630.00$$

$$A_4 = (A_3)\left(1 + \frac{0.035}{1}\right)^{1\times 1} + \$630.00$$

$$A_4 = (\$1956.92)(1.035) + \$630.00$$

$$A_4 \approx \$2025.41 + \$630.00$$

$$A_4 \approx \$2655.41$$

If he increases his savings by 5%, Gérard's savings balance would be $2655.41 after 4 years.

ALTERNATIVE SOLUTION

T This problem can be solved using a spreadsheet. Use the same equations as in part a), but replace $600.00 with $630.00.

5. a) Spending $200.00 a month on food amounts to only $6.45 to $6.67 per day, depending on whether the month has 30 or 31 days. This is very low in most situations. With Marlene's net monthly income of $1500.00, $200.00 is 13%, which is within the recommended guidelines, but because her monthly income is low, she should be spending a higher percentage on food.

   b) To reduce her expenses, Marlene could reduce her photography expenses while she is saving for a new camera. Or she could reduce her clothing expense or her "other" expense.

   c) When she prepares a new budget, Marlene should adjust her expenses to reflect the fact that they are higher than she had budgeted for. She should overestimate expenses for the new budget. Marlene also needs to incorporate paying off the credit card debt, and unless she can find other categories of expenses to reduce, she may need to consider increasing her income.

6. a) Calculate Vern's monthly take-home pay.

   $38 940.00 ÷ 12 = $3245.00

   Calculate Vern's total current expenses.

   $1500.00 + $900.00 + $600.00 + $245.00 = $3245.00

   Vern is currently on budget, but he is not paying off the debt or accumulating any savings. To pay off a $6400.00 debt and save $8500.00 for a house down payment would require $14 900.00.

   Ignoring interest charged on the debt and potential interest earned on savings, calculate the monthly amounts that would be added to Vern's budget.

   $14 900.00 ÷ 48 = $310.42

   He would need to lower his expenses by $310.42 per month.

   Calculate what percent of his net monthly income this represents.

   $310.42 ÷ $3245.00 × 100 = 9.6%

   Vern would need to save 9.6% of his monthly income.

   Examine his expense categories. He could probably reduce his variable expenses by 9.6% and therefore pay off the debt and save for the down payment.

   b) Calculate Vern's monthly debt payment (9% of his take-home pay).

   $3245.00 × 0.09 = $292.05

   Calculate his annual debt payment.

   $292.05 × 12 = $3504.60

   Vern would have paid $3504.60 after 1 year.

   c) Vern could reduce his variable expenses most easily. These are cell/internet/cable, food, and possibly transportation.

   d) Vern should put some money into savings each month because unexpected expenses may come up and he doesn't appear to have an emergency fund or any savings.

PUZZLE IT OUT

THE DISAPPEARING DOLLAR

STUDENT BOOK, p. 337

To solve this problem, encourage students to use logic and identify the different amounts of money received by each of the three customers and the busboy. Students can work in small groups to solve the puzzle. After arriving at an answer, you can discuss the different answers with the class, and identify successful strategies for solving the puzzle.

SOLUTION

There is no missing money. The figure of $59.00 has little to do with the true cost of the meal. After the money was returned, the three customers paid $19.00 each (3 × $19.00 = $57.00) plus $2.00 for the waiter, or a total of $59.00. However, the true cost of the meal was $55.00 ($60.00 minus the $5.00 that the waiter wanted returned). Three dollars was given back to the customers, and $2.00 went to the busboy, giving a total of $5.00. Add $5.00 to $55.00, and all of the money is accounted for.

THE ROOTS OF MATH

THE ORIGINS OF THE ELECTRONIC SPREADSHEET

STUDENT BOOK, p. 338

T To complete these questions, students will need to understand how to use an electronic spreadsheet. Before completing this section, you can do a class poll to determine how many students are comfortable using electronic spreadsheets, and review this information, if necessary.

SOLUTIONS

1. Answers will vary. Sample answers are provided.
    - Working with calculators is easier if you don't have much experience using spreadsheets.
    - Using spreadsheets is usually faster and more accurate once you are familiar with the program.
    - The electronic spreadsheet is more suited to a complicated budget because it does mathematical operations for you. If you change one figure in the monthly expenses column, the spreadsheet will automatically adjust the total monthly expenses to reflect the change.
2. If you do calculations manually, you might introduce errors by typing wrong numbers on the calculator or forgetting a decimal point.

    When using an electronic spreadsheet, errors may occur if a cell is formatted incorrectly or an item is entered in the wrong cell.

REFLECT ON YOUR LEARNING

PERSONAL BUDGETS

STUDENT BOOK, p. 340

Ask students to review and reflect on the list of new skills and knowledge they have encountered in this chapter. Students may need some prompting to identify these. Prompting questions might include the following:

1. What is a personal budget?
2. Why is it important to track your spending before preparing a budget?
3. How would you explain the concept of a conservative budget to someone who was unfamiliar with it?
4. If someone is consistently spending over their budget, what could they do to balance their budget?
5. Explain how circle graphs can help in a budget analysis.
6. How can a budget help in planning for a future expense?
7. Why is it important to compare actual expenses to budgeted amounts?

Confident students could be asked to lead a discussion on these points.

PRACTISE YOUR NEW SKILLS: SOLUTIONS

STUDENT BOOK, p. 340

For question 3, you may wish to give students Blackline Master 7.7 (p. 470).

1. Calculate how much Johan saves per month.

   $3000.00 ÷ 12 = $250.00 per month

   Calculate how many months it will take him to save $1850.00 by dividing by the amount he saves per month.

   $1850.00 ÷ $250.00 = 7.4

   Johan will need to save for 8 months.

2. a) Calculate Shaira's monthly savings. Convert 1.5% to a decimal, 0.015, and multiply by her monthly income.

   $906.00 × 0.015 = $13.59

   Shaira saves $13.59 per month.

   Calculate her annual savings.

   $13.59 × 12 = $163.08

   Shaira saves $163.08 annually.

ALTERNATIVE SOLUTION

   Calculate her annual income and multiply by the percent savings.

   ($906.00 × 12) × 0.015 = $163.08

   b) Calculate the percentage of fees Shaira will pay (100% – 40% = 60%).

   $395.00 × 0.60 = $237.00

   Divide her portion of the fees by her monthly savings.

   $237.00 ÷ $13.59/month ≈ 17.44 months

   She would need to save for 18 months or 1.5 years.

3. a) Some answers may vary. Cost of cell phone/internet may be a recurring expense if Roberto has a fixed rate plan. How to prioritize entertainment, recreation, and "other" may also vary. See table.

b) See table.

MONTHLY BUDGET						
Net income		*Regular or variable*	*Expenses*		*Recurring or variable*	*Priority*
Salary	$2550.00	Regular	Food	$425.00	Variable	2
Commission	$825.00	Variable	Housing	$850.00	Recurring	1
Contract work	$350.00	Variable	Transportation	$500.00	Variable	4
			Entertainment	$500.00	Variable	9
			Life insurance	$75.00	Recurring	3
			Cell phone/internet	$175.00	Recurring	5
			Recreation	$300.00	Variable	7
			Savings	$500.00	Variable	8
			Other	$400.00	Recurring	6
Total	$3725.00		*Total*	$3725.00		

c) If Roberto's income decreases, he needs to decrease his expenses in order to balance his budget. He could spend less on recreation, "other," entertainment, or cell phone/internet. If necessary, he could also decrease his savings.

4. a) Calculate Martina's gross monthly income.

$44 600 ÷ 12 = $3716.67

Calculate the percent of the gross income that the net monthly income represents.

$2415.00 ÷ $3716.67 × 100 ≈ 65%

If Martina's net income is 65% of the gross, then deductions account for 35% of her gross income.

b) Martina's net monthly income is $2415.00.

Calculate her target monthly savings.

$1800.00 ÷ 6 = $300.00

Calculate the percent of Martina's net monthly income that her target monthly savings represents.

$300.00 ÷ $2415.00 × 100 ≈ 12.4%

Martina needs to save 12.4% of her net monthly income.

c) Calculate Martina's new monthly income.

$2415.00 + $96.60= $2511.60

Calculate her new monthly savings.

12.4% × $2511.60 ≈ $311.44

Calculate the savings that Martina will have accumulated after 6 months.

6 × $311.44 = $1868.64

Martina's savings after 6 months will be $1868.64.

5. a) A conservative budget for Zabo will underestimate his income ($2400.00 monthly) and overestimate his expenses.

ZABO'S CONSERVATIVE MONTHLY BUDGET			
Income	$2400.00	*Expenses*	
		Rent	$650.00
		Food	$350.00
		Transportation	$320.00
		Sports	$200.00
		Entertainment	$150.00
		Savings	$130.00
		Clothing	$200.00
		Charitable donations	$150.00
		Other	$250.00
Total	$2400.00	*Total*	$2400.00

b) Divide Zabo's savings goal by the amount he saves per month.

$1250.00 ÷ $130.00 ≈ 9.62

Zabo needs to wait 10 months to buy the bike.

6. a) Calculate Zabo's savings after one year.

Multiply his monthly savings by 12.

$130.00 × 12 = $1560.00

b) Calculate the balance of the GIC after 10 years. Note that the savings after 1 year will be $1560.00 and interest will be compounded for 9 years.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \$1560.00\left(1 + \frac{0.0275}{1}\right)^{1\times 9}$$

$$A = \$1560.00(1.0275)^9$$

$$A = \$1991.41$$

Zabo's GIC will have a balance of $1991.41.

7. a) To make a conservative budget, overestimate Trevor's expenses and underestimate his income.

Calculate Trevor's net income (70% of his gross income), assuming he works 40 hours per week.

40 × $15.85 × 0.70 = $443.80

Round this down to $440.00.

TREVOR'S WEEKLY BUDGET				
Income			*Expenses*	
Hourly wage	$15.85		Food	$100.00
Hours worked	40		Transportation	$30.00
Overtime hours worked	0		Rent	$200.00
			Cell phone	$50.00
Total gross income	$634.00		Sponsor a child	$10.00
			Miscellaneous	$50.00
Net income	$440.00		*Total*	$440.00

b) Trevor's budget will balance if he works 40 hours per week.

c) At 40 hours per week Trevor's budget is balanced, so he needs to work several hours more per week at the overtime rate to save $75.00.

Calculate how many overtime hours yield $75.00.

Let x represent the number of overtime hours.

$$x \times (\$15.85 \text{ x } 1.5) \times 0.7 = \$75.00$$
$$16.643\,x = 75$$
$$x \approx 4.5$$

Trevor needs to work 4.5 hours of overtime, which would usually be rounded up to 5 hours. He needs to work 45 hours per week to save an extra $75.00 per week.

8. a) Calculate what percentage each category represents of Trevor's monthly income.

Food: $100.00 ÷ $440.00 ≈ 0.23
0.23 × 100 = 23%

Transportation: $30.00 ÷ $440.00 ≈ 0.07
0.07 × 100 = 7%

Rent: $200.00 ÷ $440.00 ≈ 0.45
0.45 × 100 = 45%

Cell phone: $50.00 ÷ $440.00 ≈ 0.11
0.11 × 100 = 11%

Sponsor a child: $10.00 ÷ $440.00 ≈ 0.02
0.02 × 100 = 2.0%

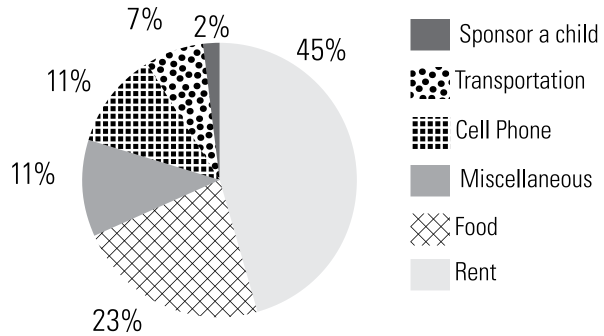
Miscellaneous: $50.00 ÷ $440.00 ≈ 0.11
0.11 × 100 = 11%

To construct a circle graph manually, determine how many degrees each percentage of expenses equals.

Food: 0.23 × 360° ≈ 83°
Transportation: 0.07 × 360° ≈ 25°
Rent: 0.45 × 360° = 162°
Cell phone: 0.11 × 360° ≈ 40°
Miscellaneous: 0.11 × 360° ≈ 40°
Sponsor a child: 0.02 × 360° ≈ 7°

Note that degrees may not total 360 because of rounding.

Trevor's Monthly Expenses



b) Trevor could decrease his spending on miscellaneous and/or food by \$25.00 a week in order to afford his new rent.

c) If Trevor's actual transportation costs are \$60.00 per week, he should budget that amount or a bit more in his next budget. It is not useful to budget a lower amount than you actually spend.

9. a) Calculate how much Megan saves per year.

\$100.00 × 12 = \$1200.00

Megan does not earn interest in her first year of savings, so her balance at the end of year 1 is \$1200.00.

Calculate the interest after year 2 and year 3.

After year 2:

$$A_2 = P\left(1 + \frac{r}{n}\right)^{nt} + \$1200.00$$

$$A_2 = (\$1200.00)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$1200.00$$

$$A_2 = (\$1200.00)(1.035) + \$1200.00$$

$$A_2 = \$1242.00 + \$1200.00$$

$$A_2 = \$2442.00$$

After year 3:

$$A_3 = P\left(1 + \frac{r}{n}\right)^{nt} + \$1200.00$$

$$A_3 = (\$2442.00)\left(1 + \frac{0.035}{1}\right)^{1\times1} + \$1200.00$$

$$A_3 = (\$2442.00)(1.035) + \$1200.00$$

$$A_3 = \$2527.47 + \$1200.00$$

$$A_3 = \$3727.47$$

Megan's savings balance will be \$3727.47 after 3 years.

b) Since saving \$100.00 per month yields \$3727.47 after 3 years, and Megan wants to save \$5500.00 in that time, first estimate how much she would need to save monthly without taking interest into account.

\$5500.00 ÷ 36 months = \$152.78

Since interest will be compounded annually, she will need to save less than \$152.00 per month.

SAMPLE CHAPTER TEST

Name: ______________________ Date: ______________________

Part A: Multiple Choice

Choose the best response to each of the following questions.

1. Which of the following is the best example of a recurring expense?
   a) $250.00 per month on food.
   b) $450.00 per month on rent.
   c) $75.00 per month on entertainment.
   d) $25.00 per month on charitable donations.
2. Which scenario best describes a balanced budget?
   a) Total income is greater than total expenses.
   b) Total income is less than total expenses.
   c) Total income is equal to total expenses.
   d) Variable expenses are equal to recurring expenses.
3. When designing a conservative budget, what should be considered?
   a) Expenses should be rounded down and income should be rounded down.
   b) Expenses should be rounded up and income should be rounded up.
   c) Expenses should be rounded down and income should be rounded up.
   d) Expenses should be rounded up and income should be rounded down.
4. When designing a budget, how can you be confident that your expenses are accurate?
   a) You can visit a financial advisor and ask for his/her advice.
   b) You can estimate your future spending based upon your own experience.
   c) You can document your expenses over 6 months and calculate monthly averages.
   d) You can use the internet to research typical expenses.
5. Your expenses regularly exceed your income, and you are becoming overwhelmed with debt. You should:
   a) Prioritize and reduce some of your expenses, or find ways to increase your income.
   b) Reduce your largest expense until your budget is balanced.
   c) Reduce all of your expenses and find ways to increase your income.
   d) Make an appointment with your bank to arrange for more credit.

Part B: Short Answer

6. Izzy has two part-time jobs. She works as a cashier at a grocery store and also walks dogs on weekday mornings. She records her income and expenses for one month in the following table.

IZZY'S INCOME AND EXPENSES			
Income		*Expenses*	
Job 1	$1140.00	Food	$350.00
Job 2	$750.00	Housing	$450.00
		Transportation	$85.00
		Clothing	$50.00
		Entertainment	$50.00
		Charitable donations	$75.00
		Other	$100.00
Total		*Total*	

a) By how much does Izzy's income exceed her expenses?

b) If she allocated surplus income to savings, could she create a balanced budget? Explain your answer.

7. Liam works in a diamond mine near Yellowknife, NT. His gross annual salary is $48 000.00, and his net pay is 70% of gross. He would like to save $7500.00 to buy a car in 7 months. What percent of his net monthly income should Liam set aside to save for the car?

8. Gabrielle's monthly food expense is $475.00 and accounts for 25% of her net monthly income. How much should Gabrielle spend on food to bring the monthly percentage down to 20%?

Part C: Extended Answer

9. Steve works as a custodian in Selkirk, MB. His take-home pay is $475.00 per week.

   a) Calculate Steve's monthly income.

   b) Use this table to find the maximum amount Steve should spend on transportation each month.

SPENDING GUIDELINES	
Income	*Percent of income*
Housing	25–35%
Food	5–15%
Transportation	10–15%
Utilities	5–10%
Medical/health	5–10%
Clothing	2–7%
Personal	5–10%
Savings	5–10%
Charitable gifts	10–15%

   c) Steve is spending too much of his income on transportation. Suggest two ways by which he might lower his transportation costs.

   d) Steve has already spent $45.00 on clothing this month. Use the guidelines in the table to determine how much more Steve can spend on clothing and not exceed the guidelines.

10. Tara intends to save 30% of her net income from her daycare job to take some courses in Early Childhood Education (ECE). The courses will cost $3850.00 and begin in 8 months.

    a) What net monthly income does Tara need to earn in order to afford the tuition after just 8 months?

    b) Tara earns about $18.00 net for every hour she works. How many hours will Tara need to work per week to earn the net monthly income calculated in a)?

    c) If Tara is unable to save the tuition in 8 months, what other options could she consider?

11. Shannon is a 2nd Lieutenant in the Canadian Armed Forces and earns a net salary of approximately $3150.00 per month. She has documented and averaged her expenses over the past 6 months and they are listed below.

Housing: $800.00	Food: $215.65	Utilities: $175.00	Entertainment: $63.22
Transportation: $450.00	Charitable donations: $35.00	Cell phone/internet: $185.00	Other: $315.00

a) Use these data and the table below to design a conservative budget for Shannon. Allocate any surplus income to savings.

SHANNON'S MONTHLY BUDGET			
Income		*Expenses*	
Total		*Total*	

b) Following the budget in a), how much will Shannon save in 1 year?

c) At the end of every year Shannon takes 75% of her annual savings and buys a GIC earning 3.1% interest, compounded annually. How much will her savings be at the end of the third year?

SAMPLE CHAPTER TEST: SOLUTIONS

Part A: Multiple Choice

1. b) Rent is a recurring expense.
2. c) In a balanced budget, total income equals total expenses.
3. d) When designing a conservative budget, expenses should be rounded up and income should be rounded down.
4. c) When designing a budget, you can calculate average monthly expenses over 6 months.
5. a) If your expenses regularly exceed your income, you should prioritize and reduce some of your expenses or find ways to increase your income.

Part B: Short Answer

6. a) Calculate Izzy's total monthly income and expenses.

IZZY'S INCOME AND EXPENSES

Income		*Expenses*	
Job 1	\$1140.00	Food	\$350.00
Job 2	\$750.00	Housing	\$450.00
		Transportation	\$85.00
		Clothing	\$50.00
		Entertainment	\$50.00
		Charitable donations	\$75.00
		Other	\$100.00
Total	**\$1890.00**	*Total*	**\$1160.00**

income – expenses = surplus

\$1890.00 – \$1160.00 = \$730.00

b) Izzy's income exceeds her expenses by \$730.00. If she allocated the surplus to savings, which is an expense, her budget would be balanced because income would equal expenses.

7. Calculate Liam's annual net income.

\$48 000.00 × 70% = \$33 600.00

Calculate his net monthly income.

\$33 600.00 ÷ 12 = \$2800.00

Calculate the amount he needs to save per month.

\$7500.00 ÷ 7 ≈ \$1071.43

Calculate the proportion this represents of his net monthly income.

\$1071.43 ÷ \$2800.00 ≈ 0.383

0.383 × 100 = 38.3%

Liam needs to save 38.3% of his net monthly income.

8. Set up a proportion. Let x represent the amount that Gabrielle should spend on food.

$$\frac{\$475.00}{0.25} = \frac{x}{0.20}$$

$$0.05 \times \frac{\$475.00}{0.25} = \frac{x}{0.20} \times 0.05$$

$$\$95.00 = 0.25x$$

$$\frac{\$95.00}{0.25} = x$$

$$\$380.00 = x$$

Gabrielle should spend \$380.00 per month on food to bring her food expense down to 20% of her net income.

Part C: Extended Answer

9 a) Monthly income = \$475.00 × 52 ÷ 12

Monthly income = \$2058.33

b) Calculate transportation at 15% of \$2058.33.

0.15 × \$2058.33 ≈ \$308.75

Steve should be spending a maximum of \$308.75 on transportation.

c) Answers will vary. Steve could take public transit, ride a bike (or walk), arrange for ride sharing, or possibly sell his vehicle in order to reduce his transportation expense.

d) Calculate clothing expense at the maximum value of 7% of $2058.33.

$0.07 \times \$2058.33 \approx \$144.08$

If $144.08 is the maximum Steve should spend on clothing, subtract what he has already spent to calculate the balance.

$\$144.08 - \$45.00 = \$99.08$

Steve could spend $99.08 more on clothing and remain within the guidelines.

10. a) Set up a proportion. Let x represent Tara's income for 8 months.

$$\frac{\$3850.00}{0.30} = \frac{x}{1}$$

$$0.30 \times \frac{\$3850.00}{0.30} = 0.30x$$

$$\$38\,500.00 = 0.30x$$

$$\frac{\$38\,500.00}{0.30} = \frac{0.30x}{0.30}$$

$$\$12\,833.33 \approx x$$

Tara's income for 8 months is about $12 833.33.

Calculate her income per month.

Monthly income = $12 833.33 ÷ 8

Monthly income ≈ $1604.17

To save $3850.00 in 8 months, Tara would need a net monthly income of $1604.17.

b) Calculate how many hours Tara needs to work per month.

$\$1604.17 \div \$18.00/\text{hour} \approx 89.12$ hours

Calculate how many weeks there are in a month.

$52 \div 12 \approx 4.33$

Calculate how many hours she needs to work per week.

$89.12 \div 4.33 \approx 20.58$ hours

Tara will need to work about 21 hours per week in order to earn the required income.

c) Answers will vary. Tara could work more hours per week because 21 is not considered full-time. She could possibly work part-time at another job. Tara could wait longer before taking the courses. Saving more than 30% of her net income is probably not reasonable.

11. a) Answers will vary. Underestimate her income and overestimate her expenses. This table shows one possible conservative budget.

SHANNON'S CONSERVATIVE MONTHLY BUDGET

Income	$3000.00	*Expenses*	
		Housing	$800.00
		Food	$225.00
		Utilities	$200.00
		Entertainment	$75.00
		Transportation	$475.00
		Charitable donations	$45.00
		Cell phone/ internet	$200.00
		Other	$325.00
		Savings	$655.00
Total	$3000.00	*Total*	$3000.00

b) Answers will vary depending on the amount allocated to savings in a).

Calculate how much Shannon will save in 1 year by multiplying savings in a) by 12.

$\$655.00 \times 12 = \$7860.00$

Shannon will save $7860.00 in 1 year.

c) Calculate how much money Shannon invests every year.

Annual investment equals 75% of annual savings.

Annual investment = $0.75 \times \$7860.00$
Annual investment = $5895.00

Calculate accumulated savings plus compounded interest after 3 years.

Shannon does not earn interest during her first year of savings.

$A_1 = \$5895.00$

After year 2:

$A_2 = P\left(1 + \frac{r}{n}\right)^{nt} + \$5895.00$

$A_2 = (\$5895.00)\left(1 + \frac{0.031}{1}\right)^{1\times1} + \$5895.00$

$A_2 = (\$5895.00)(1.031) + \$5895.00$

$A_2 \approx \$6077.75 + \$5895.00$

$A_2 = \$11\,972.75$

After year 3:

$A_3 = P\left(1 + \frac{r}{n}\right)^{nt} + \$5895.00$

$A_3 = (\$11\,972.75)\left(1 + \frac{0.031}{1}\right)^{1\times1} +$

$\$11\,972.75$

$A_3 = (\$11\,972.75)(1.031) + \$5895.00$

$A_3 \approx \$12\,343.91 + \$5895.00$

$A_3 = \$18\,238.91$

Shannon will have saved \$18 238.91 by the end of the third year.

BLACKLINE MASTER 7.1

CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ Did you make a spreadsheet or table that shows a balanced monthly budget?	
❒ Did you make a circle graph that shows a breakdown of your monthly spending?	
❒ Do you have a collection of clippings of online ads for potential rental apartments in your area? Do these ads include the rental price?	
❒ Do you have a collection of advertisements or job descriptions for entry-level jobs in your field? Do these advertisements include information about salary or wage?	
❒ Have you prepared a summary of your budget that shows your income?	
❒ Does your budget summary include expenses?	
❒ If your expenses are outside the recommended guidelines, have you explained why?	
❒ Other notes	

BLACKLINE MASTER 7.2

BUDGETING TO LIVE AWAY FROM HOME: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you may want to consider the following:

- the thoroughness of your research on budget-related expenses;
- the reasonableness of your budgets;
- the accuracy of your circle graphs;
- the effectiveness of your uses of technology for research and drawing up a budget;
- the completeness of your documentation;
- the creativity you brought to presenting your project; and
- completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

____

____

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

____

____

In what areas did you excel? ______________________

____

____

Are there areas in which you could improve? ______________________

____

If you collaborated with a partner or a small group, what strengths did each person bring to the project?

____

____

____

If you had to do the project over again, what would you do differently?

____

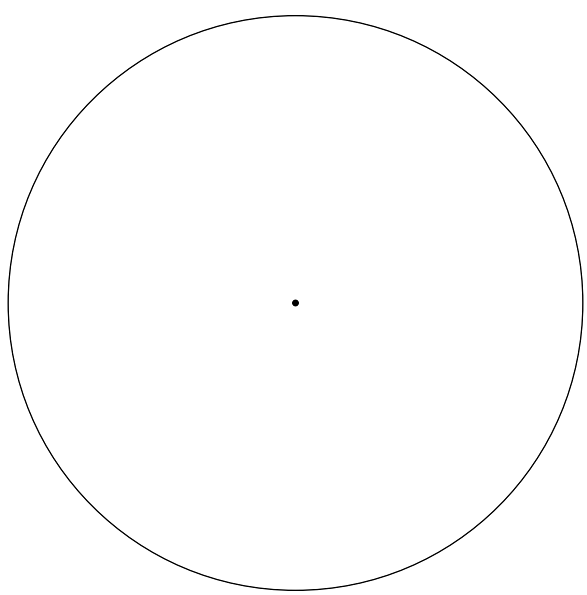
____

BLACKLINE MASTER 7.3

PREPARING AND GRAPHING A BUDGET

Name: ______________________________ Date: ______________________________

	BUDGET:				
	Income		*Expenses*		
	Total		*Total*		



BLACKLINE MASTER 7.4

ACTIVITY 7.1—BALANCING INCOME AND EXPENSES

JACKSON'S INCOME AND EXPENSES						
Income		*Regular or variable*	*Expenses*		*Recurring or variable*	*Priority*
Tips	$220.00		Rent	$500.00		
Paycheque	$800.00		Food	$400.00		
House-painting	$950.00		Entertainment	$75.00		
Tips	$175.00		Loan payment	$300.00		
Paycheque	$800.00		Cost of new bicycle	$1250.00		
			Clothing	$50.00		
			Car insurance	$180.00		
			Gas	$150.00		
			Cell phone	$74.00		
			Car loan	$300.00		
			Charitable donations	$38.00		
Total			*Total*			

BLACKLINE MASTER 7.5

SECTION 7.2, BUILD YOUR SKILLS #3

Name: ______________________ Date: ______________________

Date	Amount	Description	Income or expense	Fixed or variable
Sept 1	–$650.00	Cheque #021 Rent		
Sept 1	$1400.00	Paycheque		
Sept 3	–$212.40	SuperGroceries		
Sept 5	–$45.00	Gas		
Sept 5	–$40.00	Cash		
Sept 5	–$1.50	Transaction fee		
Sept 10	–$54.25	SuperGroceries		
Sept 11	–$60.00	Cash		
Sept 11	–$1.50	Transaction fee		
Sept 15	$1400.00	Paycheque		
Sept 17	–$48.00	Gas		
Sept 17	–$55.00	Pizza Palace		
Sept 21	–$120.00	Cheque #022 Car Insurance		
Sept 22	–$42.00	Bigger Burgers		
Sept 22	–$15.00	Cinema Magic		
Sept 24	–$47.45	Wow Clothing Co.		
Sept 24	–$35.00	Cheque #023 Charity ABC		
Sept 28	–$3.50	Wake-up Coffee		
Sept 30	–$60.00	Cash		
Sept 30	–$1.50	Transaction fee		

BLACKLINE MASTER 7.6

ACTIVITY 7.3—BUDGETING USING SPREADSHEETS

Name: ______________________ Date: ______________________

PART B

Format the boxed cells of the spreadsheet to help with adjusting Duma's budget.

B12: this cell needs to show the amount of money left after Duma pays her other expenses. It is equal to income (cell B4) minus the sum of the other expenses (B7:B11).

B15: this cell shows the total of 12 months of savings. It is equal to 12 times monthly savings (cell B12).

B17: this cell is the accumulation of 3 years of savings. The first year will have 2 years to accumulate compound interest, the second year will have 1 year to accumulate compound interest, and so on. It is equal to annual savings (cell B15) times $(1.03)^2$ plus annual savings times $(1.03)^1$ plus annual savings (cell B15).

	A	B	C
1		**Duma's Monthly Budget**	
2			
3	**Income**		
4	Wage	$3500.00	
5			
6	Expenses		
7	Housing	$1800.00	
8	Food	$800.00	
9	Transportation	$500.00	
10	Child care	$300.00	
11	Other	$100.00	
12	Savings	=$B$4–sum(B7:B11)	
13			
14			
15	Annual savings	=12*B12	
16			
17	**Savings after 3 years**	=B15*(1.03)^2+ B15*(1.03)^1+ B15	
18			

BLACKLINE MASTER 7.7

SECTION 7.3, PRACTISE YOUR NEW SKILLS #3

Name: ______________________ Date: ______________________

MONTHLY BUDGET						
Net income		*Regular or variable*	*Expenses*		*Recurring or variable*	*Priority*
Salary	$2,550.00		Food	$425.00		
Commission	$825.00		Housing	$850.00		
Contract work	$350.00		Transportation	$500.00		
			Entertainment	$500.00		
			Life insurance	$75.00		
			Cell phone/ internet	$175.00		
			Recreation	$300.00		
			Other	$400.00		
			Savings	$500.00		
Total			*Total*			

BLACKLINE MASTER 7.8

REVIEWING PRIOR CONCEPTS

Name: ______________________ Date: ______________________

Employment Income

1. Explain the difference between gross income and net income.

2. Terry earns a wage of $14.40 per hour. He is paid time and a half for any hours over 40 in a work week. Calculate Terry's gross pay if he works
   a) 25 hours in a work week
   b) 40 hours in a work week
   c) 46 hours in a work week

3. Ilona works at an electronics store and earns a base salary of \$1250.00 per month plus 1.5% commission on all sales. Calculate Ilona's gross monthly income in September when she sold \$48 000.00 in merchandise.

Percent Increase, Decrease, and Compound Interest

4. \$250.00 is increased by 8%. What is the new value?

5. \$420.00 is decreased by 30%. What is the new value?

6. \$1290.00 rises in value by 4.5%. What is the new value?

7. Joe earns \$1045.34 for a job. He needs to pay 30% in income tax.

   a) What is Joe's net pay?

   b) What percent of the gross pay is his net pay?

8. Jackie's investment gained 5.25% in value over the past year. One year ago, the investment was valued at \$13 550.00. What is the value of the investment now?

9. Rich invests $9000.00 in a GIC earning 6.5% interest per year, compounded annually. Calculate the accumulated amount after

   a) 1 year

   b) 3 years

   c) 20 years

Solving Proportions

10. Solve for x in each proportion.

    a) $\frac{x}{12} = \frac{15}{6}$

    b) $\frac{\$15.00}{x} = \frac{\$25.00}{6}$

c) $\frac{x}{15\%} = \frac{\$25.50}{30\%}$

11. Jonathan earns \$48 500.00 in 12 months. How much does he earn in 10 months?

12. If \$340.00 represents 22% of Jill's monthly income, how much is 15% of her monthly income?

BLACKLINE MASTER 7.8: SOLUTIONS

Employment Income

1. Gross income is the total money earned before any deductions such as income tax, employment insurance, and medical premiums. Net pay is the money you take home after all deductions.
2. a) Gross pay = 25 hours × \$14.40

   Gross pay = \$360.00

   b) Gross pay = 40 × \$14.40

   Gross pay = \$576.00

   c) Gross pay = regular pay + overtime

   Gross pay = (40 × \$14.40) + (1.5 × 6 × \$14.40)

   Gross pay = \$705.60
3. Calculate Ilona's commission, 1.5% of \$48 000.00.

   Convert 1.5% to a decimal, 0.015.

   Commission = 0.015 × \$48 000.00

   Commission = \$720.00

   Calculate Ilona's gross income by adding her salary plus commission.

   Gross income = salary + commission

   Gross income = \$1250.00 + \$720.00

   Gross income = \$1970.00

   Ilona's gross monthly income is \$1970.00.

Percent Increase, Decrease, and Compound Interest

4. Multiply by 1.08 (100% plus 8%).

   \$250.00 × 1.08 = \$270.00
5. Multiply by 0.70 (100% minus 30%).

   \$420.00 × 0.70 = \$294.00
6. Multiply by 1.045 (100% plus 4.5%).

   \$1290.00 × 1.045 = \$1348.05
7. a) Net pay = gross pay – income tax

   Net pay = \$1045.34 – (\$1045.34 × 0.30)

   Net pay ≈ \$1045.34 – \$313.60

   Net pay = \$731.74

   His net pay is \$731.74.

   b) Since tax corresponds to 30% of the gross pay, the net pay is 70% of the gross pay (100% minus 30%).
8. Multiply the original value by 1.0525 (100% plus 5.25%).

   Current value = \$13 550.00 × 1.0525

   Current value ≈ \$14 261.38

   Jackie's investment is now worth \$14 261.38.
9. a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   $A = (\$9000.00)\left(1 + \frac{0.065}{1}\right)^{1\times 1}$

   $A = (\$9000.00)(1.065)$

   $A = \$9585.00$

   b) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   $A = (\$9000.00)(1.065)^3$

   $A \approx \$10\ 871.55$

   c) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

   $A = (\$9000.00)(1.065)^{20}$

   $A \approx \$31\ 712.81$

Solving Proportions

10\. a) $$\frac{x}{12} = \frac{15}{6}$$ Multiply both sides by a common denominator.

$$12 \times \frac{x}{12} = \frac{15}{6} \times 12$$

$$x = 30$$

b) $$\frac{\$15.00}{x} = \frac{\$25.00}{6}$$

$$6x \times \frac{\$15.00}{x} = \frac{\$25.00}{6} \times 6x$$

$$6 \times \$15.00 = \$25.00x$$

$$\$90.00 = \$25.00x$$

$$\frac{\$90.00}{\$25.00} = \frac{\$25.00x}{\$25.00}$$

$$3.6 = x$$

c) $$\frac{x}{15\%} = \frac{\$25.50}{30\%}$$

$$\frac{x}{0.15} = \frac{\$25.50}{0.30}$$

$$0.30 \times \frac{x}{0.15} = \frac{\$25.50}{0.30} \times 0.30$$

$$2x = \$25.50$$

$$\frac{2x}{2} = \frac{\$25.50}{2}$$

$$x = \$12.75$$

11\. Calculate his monthly earnings.

$48 500.00 ÷ 12 = $4041.67

Multiply by the number of months.

$4041.67 × 10 = $40 416.70

Jonathan earns $40 416.70 in 10 months.

12\. Let x represent 15% of Jill's monthly income. Set up a proportion to solve for x.

$$\frac{\$340.00}{22\%} = \frac{x}{15\%}$$

$$\frac{\$340.00}{0.22} = \frac{x}{0.15}$$

$$\frac{\$340.00}{0.22} = \frac{x}{0.15}$$

$$0.15 \times 0.22 \times \frac{\$340.00}{0.22} = \frac{x}{0.15} \times 0.22 \times 0.15$$

$$0.15 \times \$340.00 = 0.22x$$

$$\$51.00 = 0.22x$$

$$\frac{\$51.00}{0.22} = \frac{0.22x}{0.22}$$

$$\$231.82 \approx x$$

15% percent of Jill's monthly income is $231.82.

ALTERNATIVE CHAPTER PROJECT—BUDGET FOR A VACATION

TEACHER MATERIALS

GOALS: To design two different balanced budgets for the same vacation.

OUTCOME: In this project, students will choose a vacation destination and design two different budgets that will outline their spending for this trip. One budget will have expenses totalling \$2800.00, the other will have expenses totalling \$4000.00.

T **PREREQUISITES:** Students may find it helpful to use spreadsheet software in preparing their budgets, but the budget can be done manually. Students may already be familiar with the basics of spreadsheet formatting, or this can be covered in class.

Students may also use presentation software for their final presentation, so experience with this type of application will be an asset.

Students will need to have access to the internet to research income and expenses for different regions in Canada. You can guide students to use available search engines to help them with their research. Students should record their research sources (URLs) in a document, using a word processing program.

ABOUT THIS PROJECT: This project is divided into four sections. Students will begin by researching the expenses associated with a chosen vacation. They will design two budgets that are accurate, reasonable, and balanced. One budget will be for a low expense vacation, and the other budget will be for a moderate expense vacation. The first budget has a spending limit of \$2800.00, and the second has a spending limit of \$4000.00. Both budgets should be balanced.

The completed project will include copies of the two budgets, circle graphs for each budget, a list of pre-vacation expenses, a list of possible unexpected expenses, a list of sources for the data gathered, and a summary paragraph.

A sudent self-assessment rubric (Blackline Master 7.1 C, p. 486) and project checklist (Blackline Master 7.1B, p. 485) are included and should be handed out to students early in the project. The self-assessment rubric outlines the criteria for evaluation of their project and suggests some ways in which they can reflect on their learning.

1. Start to plan

Introduce the project to your students as you begin this chapter. This initial part of the project allows for group brainstorming as a class. Start with a discussion of how much a one-week vacation could cost. Where would students like to travel? What are some expenses associated with planning a vacation?

Read the project description with your class and remind students that they will need to develop two different budgets—one with expenses of \$4000.00 and one with expenses of \$2800.00. Go through the questions in the student materials and brainstorm answers with the class. Encourage students to be realistic and reasonable in this discussion, as the project will be more rewarding when completed with a realistic intent. Explain to students that they will be researching actual figures with respect to vacation expenses, and they will need to include sources for all of their information in their final project.

2. Gather data

This segment of the project requires students to practise their research skills. Students will research expenses for the two vacations, and they will record their findings in a document. Let students know that they do not need to make an exhaustive list for their expenses. They can use the suggested categories or create one or two additional categories. Students should also be reminded that they are not creating a budget here, so they do not need to adjust their expenses and balance; this is for the next stage in the project. At the end of this segment of the project, discuss progress with your students to ensure that all requirements have been met.

3. Prepare your budgets

During this section of the project, students will use their research to design two balanced holiday budgets. Students may use calculators to prepare manual budgets. If they choose to do so, provide students with a copy of Blackline Master 7.1A (p. 484). However, they could also follow the instructions for using a spreadsheet, formatting the necessary cells. Spreadsheets are useful when students are trying to balance their budgets, because small changes in expenses will be reflected instantly in the budget balance.

Time for this section should be divided evenly between building the budget template with spreadsheet software and adjusting the budget entries to create a reasonable and balanced budget. If the budgets are being created manually, have students complete their expense entries in pencil, so that they can easily be changed as the budget is modified.

4. Hand in your project

Provide students with a copy of Blackline Master 7.1B (p. 485) to use when compiling their project materials. In this segment of the project, students will synthesize their planning and research activities and complete their final project package. Remind students of the requirements for a complete project and discuss with them how the project will be assessed. Provide students with a copy of Blackline Master 7.1C (p. 486) to give them an opportunity to reflect on the quality of work they put into their project.

ASSESSING THE PROJECT

1. Start to plan

- Discuss with students that they will make two balanced vacation budgets for a location of their choice. The budgets will be based on researched real-life data as much as possible. Students must research all the expenses related to these vacations.
- Let students know how they will be graded. You can tell them about the criteria on the project assessment rubric on p. 480, or use your own criteria to grade the project.

2. Gather data

- After this section of the project has been completed, you can ask students to hand in their research findings on expenses for their chosen vacations. These findings should be recorded in a word processing document. To assess the quality of research students have completed, consider the thoroughness of the expenses, check the sources for their research, and comment on the accuracy of their expenses.
- You can decide and inform students if this work will be part of their final grade or if it will be used as an assessment for learning. If you choose to use this step as an assessment for learning, you can hand the documents back and give general or individual suggestions on how to improve research findings.

3. Prepare your budget

- You may want to provide students class time in a computer lab to complete the templates for their budgets. Ensure that they have formatted the appropriate cells correctly. After students have a working budget template, ask them to begin making adjustments to balance the budgets.

ALTERNATIVE CHAPTER PROJECT ASSESSMENT RUBRIC				
	Not yet adequate	*Adequate*	*Proficient*	*Excellent*
Conceptual Understanding				
• Explanations show an understanding of a balanced personal budget	shows very limited understanding; explanations are omitted or inappropriate	shows partial understanding; explanations are often incomplete or somewhat confusing	shows understanding; explanations are appropriate	shows thorough understanding; explanations are effective and thorough
Procedural Understanding				
Accurately: ▪ researches expense data ▪ accounts for all expenses ▪ documents resources well ▪ constructs a balanced budget ▪ calculates appropriate percentages for the circle graphs ▪ calculates appropriate angles for the circle graphs ▪ draws circle graph ▪ summarizes and analyzes the budgets	limited accuracy; major errors or omissions For example: • expense data are not realistic • various expenses are missing from budgets • sources are not documented • few expense percentages are correct • most angles for circle graphs are incorrect • circle graphs are not drawn accurately • summary and analysis are incomplete • project is incomplete	partially accurate; some errors or omissions For example: • expense data are mostly realistic • most expenses are listed in budgets • some sources are not documented • budgets are balanced with one or two small arithmetic errors • some expense percentages are incorrect • some angles for circle graphs are incorrect • circle graphs are mostly accurate • summary and analysis are complete • project is mostly complete	generally accurate; few errors or omissions For example: • expense data are realistic • all expenses are listed in budgets • most sources are documented • budgets are balanced • expense percentages are correct • angles for circle graphs are correct • circle graphs are drawn accurately • summary and analysis are complete and well written • project is complete	accurate and precise; very few or no errors For example: • expense data are realistic • all expenses are listed in budgets • all sources are well documented • budgets are balanced and error-free • expense percentages are correct • angles for circle graphs are correct • circle graphs are drawn accurately, neatly, and with good use of colour • summary and analysis are complete, well-written, and creative • project is complete
PROBLEM-SOLVING SKILLS				
• Uses appropriate strategies to solve problems successfully and explain the solutions	uses few effective strategies; does not solve problems	uses some appropriate strategies, with partial success, to solve problems; may have difficulty explaining the solutions	uses appropriate strategies to successfully solve most problems and explain solutions	uses effective and often innovative strategies to successfully solve problems and explain solutions
COMMUNICATION				
• Presents work and explanations clearly, using appropriate mathematical terminology	does not present work and explanations clearly; uses few appropriate mathematical terms	presents work and explanations with some clarity, using some appropriate mathematical terms	presents work and explanations clearly, using appropriate mathematical terms	presents work and explanations precisely, using a range of appropriate mathematical terms

ALTERNATIVE CHAPTER PROJECT—BUDGET FOR A VACATION

STUDENT MATERIALS

PROJECT OVERVIEW

What is your travelling style? Do you like meals in good restaurants and a private room with a comfortable bed? Maybe you prefer washing your socks in the sink of a shared hostel bathroom so that you can save money for an eco-adventure tour of the surrounding mountains. In this project, you will research the cost of travelling two ways—on a limited, or shoestring budget, and on a moderate, comfortable budget.

START TO PLAN

Over the past two years, you and a friend have both been saving income from your part-time jobs. You have each started to plan a vacation you will take when you graduate from high school. You have saved $4000.00, but your friend has only managed to save $2800.00. You will plan two vacations to the same location—one for yourself, with a moderate budget, and one for your friend, with a shoestring budget.

To begin this project, decide on a desirable location and the duration of your vacation. Then, make a list of the different expenses that you and your friend will be faced with on your vacations. Think of how these items differ in price depending on the two travelling styles.

- How will you reach your holiday location? Will you travel by plane, train, bus, or automobile?
- Where will you eat?
- How much will you spend on entertainment?
- What type of accommodation will you choose?
- Are there any sightseeing tours or special activities you will take part in? What do they cost?
- What are some holiday-related expenses leading up to your trip?
- What are three unexpected expenses that you might encounter on your holiday?

FINAL PRESENTATION CHECKLIST

Your finished project will include these items:

- two spreadsheets or manual calculations showing balanced budgets that detail expenses for the two different vacations;
- a list of vacation-related expenses you and your friend will encounter before you leave;
- a list of three unexpected vacation expenses you might encounter;

- newspaper clippings or online ads of airfares or vacation packages that relate to the two different budgets you made;
- two circle graphs, one showing your expenses, the other showing your friend's expenses; and
- a written summary for your budgets, detailing expenses.

GATHER DATA

Many people do not plan for their holidays, and as a result, they go into debt during their vacations. These people sometimes take months or years to pay off the debt and interest for just one or two weeks of holidays. If planned properly, a holiday can be both enjoyable and affordable for people on different budgets.

Your task is to research how much vacations would cost on two different budgets—shoestring and moderate. Part of your final mark will be based on the accuracy and reasonableness of your budgets. Be sure to include clippings or web page references for the research you conduct. Research and write down the expenses associated with travel as well as pre-holiday preparations.

Holiday Expenses

- Accommodation: Decide on a type of accommodation appropriate for each budget (hotel, motel, camping, hostel, or other). Research the nightly rate and convert it into Canadian dollars, if you are travelling outside Canada. You can stay in more than one type of accommodation, but remember to make note of this.
- Transportation: For each budget, choose a method of transportation to get to your holiday destination. Research and record the cost of a round trip for each. If you are travelling by car, include average fuel and maintenance costs, or fuel and rental costs. Convert your expenses into Canadian dollars, if necessary.
- Food: Research food and restaurant costs for the two budgets. If you plan to make your own meals, research costs for typical groceries in the area. Convert these expenses into Canadian dollars, if necessary.
- Sightseeing or Special Events: What will you do when you are on your holiday? Will these activities depend on how much money you have to spend? Research the costs for these special events and convert them into Canadian dollars, if necessary.

Pre-Holiday Expenses

- Insurance: If you choose to travel outside Canada, you may not be covered by your current medical insurance. Research the costs of travel insurance for your trip.
- Passport: Do you need a passport? If so, do you have a valid passport? If not, find out how much it costs to obtain one.

- Vaccinations: Before you travel to some countries, it is recommended that you have certain vaccinations. Will any be required before travelling to the destination that you chose? If so, record how much they cost.
- Other: List how much it would cost to buy the items you will need for your trip, such as a guidebook, walking shoes, or luggage.

Unexpected Expenses

- Think of three unexpected expenses you might encounter during your trip, and research how much they cost. For example, if you lost your luggage, how much would it cost to replace your clothing and other belongings?

PREPARE YOUR BUDGET

Now that you have completed your research, you can use your data to create two budgets. Each will document holiday expenses for a different style of trip.

You don't have to include all of the unexpected expenses in your budgets, but you might want to put aside a certain amount of money, in case one or two should occur.

Each budget should include income and expenses, as well as a budget balance. If your expenses are initially greater than your income, you will need to make realistic adjustments to your spending plan in order to balance your budget.

HAND IN YOUR PROJECT

You are now ready to hand in your budget. How did you show that your spending is accurate and reasonable? Be sure to include the following in your presentation:

- two spreadsheets or manual calculations showing balanced budgets that detail expenses for the two different vacations;
- a list of vacation-related expenses you and your friend will encounter before you leave;
- a list of three unexpected vacation expenses you might encounter;
- newspaper clippings or online ads of airfares or vacation packages that relate to the two different budgets you made;
- two circle graphs, one showing your expenses, the other showing your friend's expenses; and
- a written summary for your budgets detailing expenses.

BLACKLINE MASTER 7.1A

SHOESTRING BUDGET AND MODERATE BUDGET

Name: ______________________ Date: ______________________

SAMPLE: SHOESTRING BUDGET	
Duration: Location:	
Expenses: Shoestring Budget	
Pre-holiday expenses	
Transportation	
Accommodation	
Food	
Entertainment	
Souvenirs	
Sightseeing	
Unexpected expenses	
Total	

SAMPLE: MODERATE BUDGET	
Duration: Location:	
Expenses: Shoestring Budget	
Pre-holiday expenses	
Transportation	
Accommodation	
Food	
Entertainment	
Souvenirs	
Sightseeing	
Unexpected expenses	
Total	

BLACKLINE MASTER 7.1B

ALTERNATIVE CHAPTER PROJECT CHECKLIST

Name: ______________________ Date: ______________________

PLANNING CHECKLIST	
❒ Did you research expenses for two different kinds of vacations?	
❒ Did you research and record pre-vacation expenses?	
❒ Have you budgeted for unexpected expenses during your vacations?	
❒ Have you drawn up two balanced budgets?	
❒ Have you made two circle graphs showing a breakdown of expenses for your two budgets?	
❒ Do you have newspaper clippings showing the prices of airfare or vacation packages?	
❒ Have you prepared a written summary for your budget detailing your expenses?	
❒ Other notes	

BLACKLINE MASTER 7.1C

ALTERNATIVE CHAPTER PROJECT: STUDENT SELF-ASSESSMENT

Name: ______________________ Date: ______________________

To evaluate how well you did on your project, you will want to consider the following:

- the thoroughness of your research;
- the completeness of your documentation;
- the accuracy of your circle graphs;
- the effectiveness of your use of technology for research, organizing, and presenting;
- the creativity you brought to planning and presenting; and
- completion of all the assigned tasks on time.

How do you feel you have done, given the criteria above? ______________________

Were you able to complete all aspects of the project? If not, why not? Did you allot your time effectively?

In what areas did you excel? ______________________

Are there areas in which you could improve? ______________________

What were the strengths and weaknesses of your presentation? ______________________

If you had to do the project over again, what would you do differently? ______________________

On which budget item would you face the biggest challenge in decreasing your spending on a shoestring budget?